PRIMARY ADVANTAGE

Maths Programme

A model of best practice

2015

PRIMARY ADVANTAGE
SCHOOLS ACHIEVING MORE TOGETHER

the learning trust
the future for education in Hackney

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MAYOR OF LONDON

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Content:

Introduction

At the individual level, mathematics underpins many aspects of our everyday activities, from making sense of information in the newspaper to making decisions about personal finances.

A good understanding of basic mathematics is essential wherever calculations, measurements, graphical interpretations and statistical analyses are necessary.

At Primary Advantage schools a substantial amount of time is devoted to the teaching and learning of mathematics to build a strong foundation for the acquisition of mathematics knowledge and skills in later years. Effective learning of mathematics requires a coherent and well structured syllabus, excellent instructional materials, and excellent teachers who use sound pedagogical strategies that are developmentally appropriate. This is at the core of the Primary Advantage Programme.

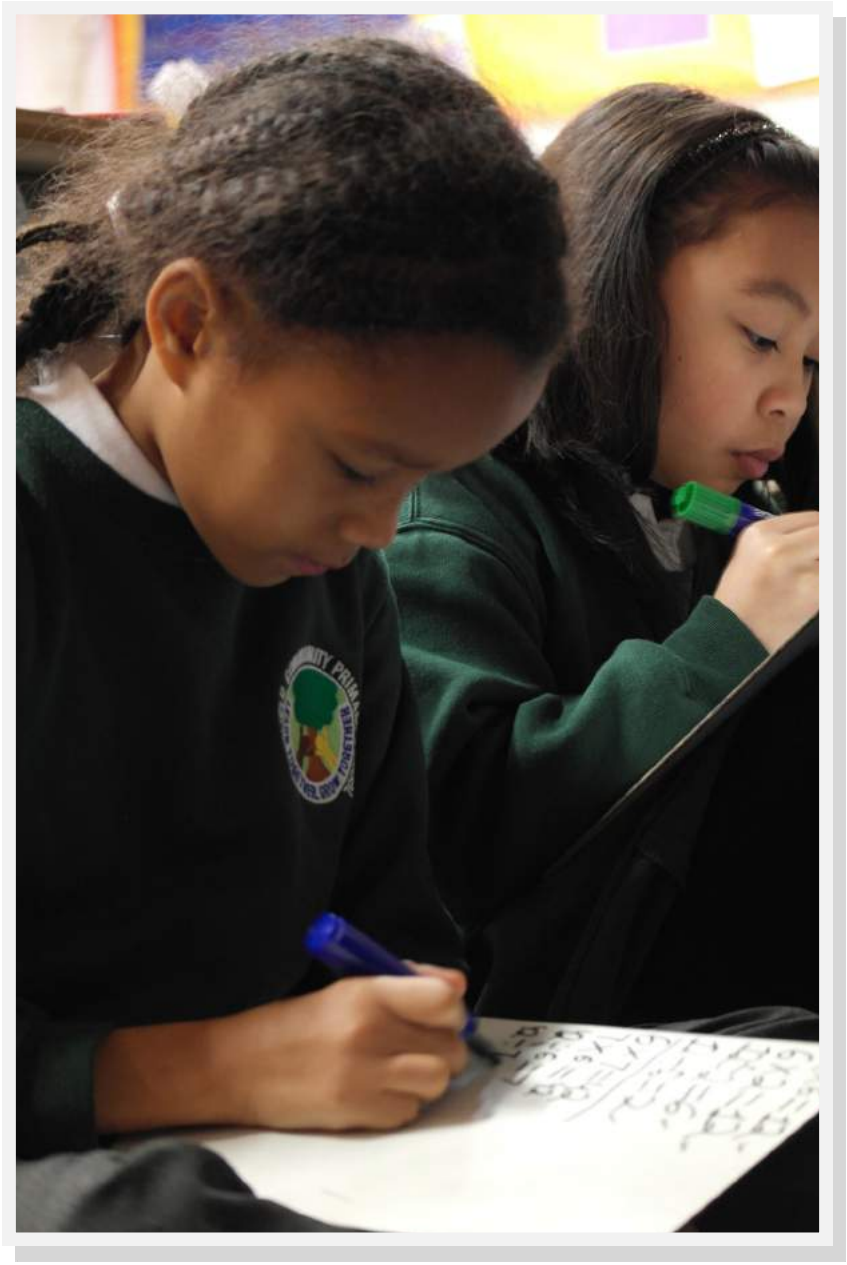
Our mathematics curriculum emphasises conceptual understanding, skills proficiency, learning of process skills and focuses on mathematical problem solving.

In sharing our experience in the development of this maths curriculum we hope to have made a positive contribution to Primary Mathematics teaching.

Acknowledgements

The programme was developed by a group of teachers from Primary Advantage Schools and without their expertise and professionalism the syllabus could not have been completed. We would like to thank Gemma Meharg, Catherine Thomas, Stephanie Saviddes, Sarah Jameson, Joanne Smith, Jo Stonehouse, Aidan Stallwood and Alyson Tyler for their input and positivity throughout the project.

We would like to thank Lucy Blewett for the 2015 edition of the programme with further thanks to Toni Mason, Anna Case, Matthew Stevenson and Izabela Jelonek for their support and input.



Aims and distinctive features

This programme aims to support you in developing the three key areas of mathematical subject knowledge.

These are:

- **Mathematical knowledge**

The programme and CPD will support in developing understanding of mathematics

- **Curriculum knowledge**

By following the programme you will come to learn exactly which areas of mathematics should be taught to each group of children you may be working with

- **Pedagogical knowledge**

The programme draws on models and images so that you can see the best ways to introduce learners to particular mathematical ideas.

The programme is practical and models best practice in primary mathematics teaching.

It will support you in the planning and delivery of lessons for the full primary range.

Distinctive features

Progression

The learning progressions within each strand are build on each other. This will help you understand which mathematical ideas should be taught to which age group of children and how the Mathematics Curriculum is developed over the primary age range.

Key concepts

These sections deal with the ideas which underpin each particular strand of mathematics covered. This allows you to see the big picture immediately and understand how the different strands knit together.

Models and images

These sections deal with the best models and images to represent the elements of mathematics in each strand. This will help you in choosing appropriate representations when planning mathematics lessons. Examples of the bar model will help you when planning for problem solving in your class.

The overview of the PA Maths Programme

Programme aim

The PA maths Programme aims to empower young people to achieve their potential, to use their knowledge of mathematical language to talk about their work and explain their findings, and ultimately use their skills to make informed and responsible choices throughout their lives.

Infusing							
Curriculum Objectives	Fluency in the fundamentals of mathematics		Reason mathematically			Solve problems	
Cross Curricular Skills	COMMUNICATION		USING MATHEMATICS			ICT	
Thinking Skills and Personal Capabilities	Managing information, working with others		Thinking, problem solving, Decision Making, Self management			Being creative	
Incorporating							
Assessment for Learning	Cleared learning intentions shared with pupil	Shared/negotiated success criteria	Ownership of learning	Taking risks for learning	Peer and self-assessment / evaluation of learning	Celebrating success	Advice what to improve and how to achieve
Promoting/Encouraging							
Learning Experiences	Investigating and problem solving Challenging and engaging	Curriculum links Supportive environment	Relevant and enjoyable Culturally diverse	Media—rich Positive reinforcement	Skills integrated Varied	Active and hands on On—going reflection	Offers choice Enquiry based
Fostering							
Attitudes and Dispositions	Personal responsibility Self-confidence	Concern for others Curiosity	Collaboration	Commitment Flexibility	Determination Resourcefulness	Open to new ideas Resilience	Respect

Primary Advantage learning dispositions

Learning mathematics extends beyond learning concepts, procedures, and their applications.

It also includes developing a disposition toward mathematics and seeing mathematics as a powerful way for looking at situations.

Disposition refers not simply to attitudes but to a tendency to think and to act in positive ways. Students' mathematical dispositions are manifested in the way they approach tasks – whether with confidence, willingness to explore alternatives, perseverance, and interest – and in their tendency to reflect on their own thinking. The assessment of mathematical knowledge includes evaluations of these indicators and students' appreciation of the role and value of mathematics.

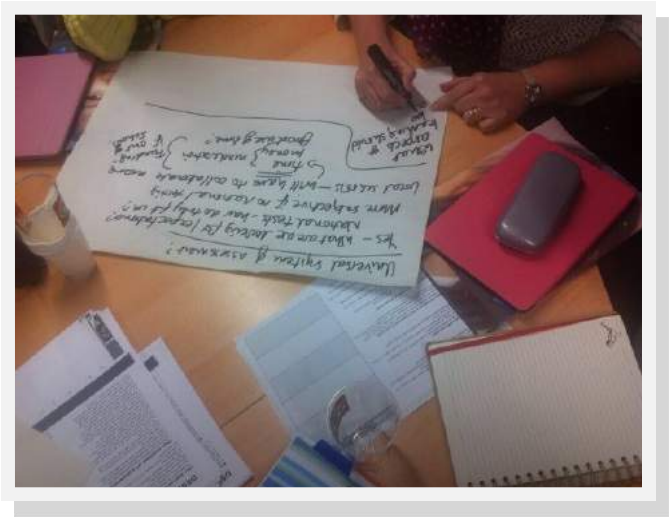
As part of the Primary Advantage programme we encourage and reward the following standards in all of our lessons.

Primary Advantage CPD

This innovative programme is designed to support teachers in developing children's mathematical understanding and enjoyment throughout the primary phase. It has been designed by a range of classroom teachers and is a model of best practice, based on experience and theoretical understanding.

The PA Maths Programme comprises of a range of complementary elements which all contribute towards the aim of high quality primary maths teaching and learning:

Eight ½ day core training CPD sessions
'How to' sessions, TA subject knowledge sessions and EYFS training sessions
The opportunity to attend modelled lessons within the PA schools
One ½ day implementation school visit
Access to the Maths Portal (including this supporting document)



Through engaging with the programme, teachers' planning and lesson delivery will develop in the following ways:

Mathematical knowledge	A deeper understanding of the underpinning structures of primary mathematics.
Curriculum knowledge	A secure understanding of best practice in terms of progression and challenge through curriculum planning.
Pedagogical knowledge	A deeper understanding of important models and images to support children's progress through a range of key mathematical concepts.

By engaging with the PA Maths Programme, teachers are demonstrating a commitment to sharing the values that underpin it, namely the belief that primary mathematics is a crucial phase in laying the foundations for lifelong numeracy skills. This drives the focus on conceptual understanding, skills proficiency, problem solving, reasoning and fluency which comprise the PA Maths Programme.

Through sharing our experiences with the wider primary community through the PA Maths Programme, we hope to make a positive contribution towards maths teaching and learning and thank you for joining us on our journey.

Problem solving, reasoning and fluency

As its underpinning aims, problem solving, reasoning and fluency are at the heart of the National Curriculum for England and Wales (DfE, 2014). By highlighting them in this way, the DfE have indicated that they should underpin the curriculum by threading through all of the teaching and learning. By using them as a lens through which to teach the content, the national curriculum will be taught in its intended manner.

The PA Maths Programme supports this approach and believes that through developing children's problem solving, reasoning and fluency skills, there will be a range of positive outcomes, including the development of children's conceptual understanding, their ability to use maths in meaningful ways and positive attitudes from the EYFS to Year 6.

Problem Solving

This can be summarised as the ability to apply mathematics to a variety of situations (Cockcroft, 1982) and PA encourage the use of 'low threshold, high ceiling' activities. These mathematical activities are designed so that the great majority of the group can begin, and then work on at their own level of engagement, but which has lots of possibilities for the participants to do much more challenging mathematics (McClure, ND). They can lead to the development of a community of practice, positive attitudes and progression through deepening subject knowledge, rather than accelerating. There is a wealth of activities on the NRich website (www.nrich.maths.org.) which can be used alongside this PA document.

Reasoning

Reasoning can be considered the glue which holds maths together. A focus on the mathematical process and a real commitment to children's understanding, as distinct from any final product, enables the development of reasoning skills in the primary classroom. The 'example questions' section of the pages in this document help support this aspect of teachers'

planning and, when used to encourage children to move from describing to explaining to justifying, it is another tool with which to challenge the higher attaining children.

Fluency

Developing children's mathematical fluency demands a focus on their efficiency, accuracy and flexibility. It requires them to know why they are doing what they are doing, and to make appropriate choices (from a toolkit of mental calculation strategies, for example). By using manipulatives (as part of a CPA approach), encouraging children to discuss their work, particularly through reasoning, and consolidating understanding across a range of meaningful contexts, children's fluency skills will develop.

These three aims are inter-related and complementary. They inform the PA Maths Programme and are deemed to be at the root of high quality maths teaching and learning.

Cockcroft, W H (1982) *Mathematics Counts: Report of the Committee of Inquiry into the Teaching of Mathematics in Schools*, London, Her Majesty's Stationery Office

CPA in the PA Maths Programme

WHY?

A commitment to CPA is intrinsic to the PA Maths Programme. It informs the pedagogy and planning of teachers using this programme as PA believe it is a supportive way of developing children's deep conceptual understanding, good progression and positive attitudes to maths.

WHAT?

CPA is an approach to teaching mathematics based on the work of Jerome Bruner (1960). Bruner's premise was that children's conceptual understanding develops from being actively engaged in their learning and making sequential process through three stages of representation: enactive, iconic and symbolic (mapped onto **c**oncrete, **p**ictorial, **a**bstract respectively). Each stage builds on the previous one, although unlike Piagetian theory, they are not age-related.

CPA therefore encompasses multiple models that approach a concept at different cognitive levels. Firstly at the concrete level, children are exposed to a range of appropriate manipulatives, for example, dienes, unifix, Numicon, egg boxes, counters, shapes, coins and dice. Use of these concrete objects engages children with their learning and can provide a 'hook' into the learning. Another advantage of this approach is that discussion is a natural by-product of active learning which is an element of good quality maths teaching and learning (Williams, 2008).

Progress into the pictorial phase is consequently underpinned by active, memorable experiences leading to deep learning. This second phase aids visualisation and the bar model is a key element of the pictorial phase of problem solving (this is explored later).

It is important to note that although the ultimate aim of a CPA strategy is to culminate in a fluent, abstract approach characterised by quick, efficient methods, the process should not be rushed. It may be necessary to return to previous phases to address children's misconceptions and consolidate their conceptual understanding.

Another key feature of the CPA process, is that although concrete objects may be perceived as too elementary for upper KS2 children (Sousa, 2007), both concrete and pictorial representations should be used at across the primary phase.

HOW?

Within the PA core CPD sessions, explanations and examples of appropriate CPA apparatus, models and images are consistently discussed. They also form the backbone of this document as each year group has clear diagrams demonstrating the progression from the concrete to pictorial to abstract. This structure informs teachers' planning and pedagogy, as does reflection on observations of any modelled lessons attended.

Two key facets of the CPA approach (the counting stick and the bar model) are discussed in depth in the following pages.

CPA Exemplified

Whilst choices around which concrete and pictorial representations are used within lessons must remain the choice of each teacher, dependent upon their individual context, there are two which PA would advocate as useful across a range of learning experiences – the counting stick and the bar model.

Counting stick

This piece of concrete apparatus embodies the ubiquitous pictorial model of the number line. Traditionally it has been used for counting on/back in ones/tens and for ordering numbers, but the counting stick is a versatile piece of equipment which can be used across a range of mathematical areas to support children's fluency and understanding.

A focus on fluency is often the genesis of counting stick use, for example children counting forwards and backwards in multiples of a given number. However, by labelling the ends of the counting stick with two numbers and asking children to label another given point, their reasoning skills are required. Careful questioning can encourage them to move from giving a basic answer to explaining and then justifying their thinking. Contextualising this by using a counting stick vertically to represent scales of temperature, length or mass can be engaging and meaningful for children.

Ideas of equivalence can be developed and consolidated through using a number line too as children begin to explore other possible labels for given points such as ¼, 0.25 and 25%. The counting stick frequently features in the models and images pages throughout this document.

Bar Model

The 'bar model' is shorthand for a systematic method of representing word problems and number relationships. It is exemplified by children sketching rectangular bars to represent relationships between known and unknown numerical quantities. In this way, it can help children identify which calculation is needed to solve a word problem or investigation. It is often this step which is problematic for children, rather than the calculation itself, so the bar model, and the discussion which supports it, can be very useful.

E.g. Ali has £10 to spend on books. He chooses one for £6, one for £1.99 and one for £1.75. How much change does he get from his £10 note?

<table>
<tr><td colspan="4">£10</td></tr>
<tr><td colspan="2">£6</td><td>£1.99</td><td>£1.75</td><td>?</td></tr>
<tr><td colspan="4">?</td><td>?</td></tr>
</table>

The bar model is a visual, flexible strategy which children can fit into their 'toolbox' of heuristics for problem solving. It can be supported by concrete apparatus such as Cuisenaire rods or unfix cubes, to enable children to access and enjoy mathematical problem solving. The bar model frequently features in the models and images pages throughout this document.

Yearly Overviews

PRIMARY ADVANTAGE

PRIMARY ADVANTAGE EARLY YEARS TEAM
EYFS - *MATHS CURRICULUM MAP*

		EYFS 1						EYFS 2					
	Order	1	2	3	4	5	6	7	8	9	10	11	12
Mathematics	Numbers	To make comparisons between quantities. ***NPV, A & S*** To use some language of quantities, such as 'more' and 'a lot'. ***NPV & A***	To use the language of more and fewer (less) to compare sets of objects. ***A & S*** To recite number names in sequence to 10. (0-10) ***NPV***	Select a small number of objects from a group 'give me one', 'give me two'. ***NPV*** To compare two groups of objects (identifying 'the same'). ***NPV*** To use number names and language. (to recognise numbers) ***NPV***	To separate a group of three or four objects in different ways. (Total is still the same) ***A & S*** To know that numbers identify how many objects are in a set. (Triad) ***NPV*** To know that a group of things changes in quantity when something is added or taken away. ***S***	To match numeral and quantity. ***NPV*** To use one to one correspondence (touch each object and give it a number 0-10) ***NPV*** To count objects in a line. ***NPV*** To create and experiment with symbols and marks representing number. ***NPV*** To respond to (and use) addition vocabulary in rhymes and games. ***A*** To respond to (and use) subtraction vocabulary in rhymes and games. ***S***	To use more/most and less/least. ***NPV*** To find the total number of items in two groups, after some are added, by counting all of them. ***A*** To find the total number of items, after some are taken away, by counting all of them. ***S*** To know that when counting a group the last number represents the quantity. ***A & S***	To count reliably (from 0-20) ***NPV*** To count objects to 10, and beginning to count beyond 10 *(Can count in a line)* ***NPV*** To use one to one correspondence (touch each object and give it a number 0-20) ***NPV*** To count actions or objects which cannot be moved. ***NPV*** To count objects in a group/ irregular arrangement of up to ten objects (same group/different group). ***NPV*** To represent numbers using fingers, marks on paper or pictures. ***NPV*** To recognise numerals. (0 to 5, 0-10 & 0-20) ***NPV*** To order numbers to 20. ***NPV***	To write numbers to 20. ***NPV*** To find/ say the number which is one more or one less than a given number. ***A & S*** Relates addition to combining two groups. ***A*** Relates subtraction to taking away. ***S*** To find **one more** or **one less** from a group of up to five objects, then ten objects. ***A & S*** Selects the correct numeral to represent 1 to 5, then 1 to 10 objects. To set out groups and find the total amount. ***Mx***	To estimate how many objects they can see and check by counting. ***NPV*** To recognise the number of objects in a small group without counting out (subitise). ***NPV*** Uses quantities and objects, to add two single-digit numbers and **count on** to find the answer. ***A*** To count on when adding to a group (holding first number in head) ***A*** To add two sets of objects which are the same (cars + cars) then different (apples + bananas) ***A*** Increase one quantity by a given amount to find the total (augmentation) ***A*** To share objects equally. ***D*** Records, using marks that they can interpret and explain. ***A & S***	To use quantities and objects, to subtract two single-digit numbers (count on or back) to find the answer. ***S*** To count backwards.(on a number line or counting stick.) ***S*** To recognise and name +, =, - signs. ***A & S*** To read an addition number sentence. ***A*** To solve an addition number sentence. ***A*** To read a subtraction sentence. ***S*** To solve a subtraction number sentence. ***S*** To group objects. ***D***	To make 10 (feel the ten-ness of ten). ***NPV*** To arrange an addition number sentence. ***A&S*** To arrange a subtraction number sentence. ***S*** To halve (an even group up to 12) ***S & D*** To solve problems involving grouping and sharing. ***F*** To share an even group of objects between 2, between 4. ***D & F*** To skip count in 2s, 5s & 10s. ***Mx*** Begin to understand odd and even. ***Mx & D*** To count up to 20 (objects/ images in an array) ***D***	Shares an even group of objects between 4. ***D*** To know number families to 5, 6 & 10. ***A & S*** To begin to identify own mathematical problems based on own interests and fascinations. ***S*** To know doubles to 10. ***A*** Begin to relate the addition of dobles to counting on (how many wheels on 2 cars? 4… 5,6,7,8 4+4=8) ***Mx*** To identify half a group of objects. ***F***

Key: Number and Place Value ***NPV***, Addition ***A***, Subtraction ***S***, Multiplication ***Mx***, Division ***D***, Fractions ***F*** and Measurement ***M***

		EYFS 1						EYFS 2					
		1	2	3	4	5	6	7	8	9	10	11	12
	Shape, Space and Measures	Can say what is different and what is the same. ***M***	Begins to categorise objects according to properties such as size (colour.) ***M*** Begins to categorise objects according to properties such as shape. ***GS***	Begins to use the language of size. ***M*** Shows an interest in shape and space by making arrangements with objects. ***M***	Experiments with capacity. (Which holds more/less) ***M*** Begins to talk about the shapes of everyday objects, e.g. 'round' and 'tall'. ***GS***	Anticipates specific time-based events such as mealtimes or home time. ***M*** Understands some talk about immediate future, e.g. 'later' or 'soon'. ***M*** Understands some talk about immediate past e.g. 'before'. ***M*** Uses money in role play. ***M***	Exchanges money for objects. ***M*** Shows awareness of similarities of shapes in the environment. Uses familiar objects and common shapes to build models. ***GS*** Beginning to use mathematical names and 'flat' 2D shapes. ***GS***	Uses positional language (below, above, next to, beside, in front, behind and on top) ***GP***	Describes their relative position such as 'behind' or 'next to'. Uses mathematical terms to describe 2d shapes. ***GS***	Orders two items by mass. (using everyday language) ***M*** **Uses everyday language to solve problems. *M*** **They recognise, create and describe patterns.** To count patterns**.** ***Mx*** Orders two or three items by length or height. ***M***	Orders two items by capacity. (using everyday language) ***M*** **Uses everyday language to compare quantities & objects. *M*** **Uses everyday language to talk about distance. *M*** Orders and sequences familiar events. ***M*** **Uses everyday language related to time (begins to identify o'clock) *M***	Measures short periods of time in simple ways. ***M*** **Uses everyday language to talk about money. *M*** Demonstrates understanding that £1 has greater value than pennies. ***M***	Know and name different coins – 1p, 2p, 5p. 10p, 20p, 50p, £1 & $2. ***M*** Can use 1p, 2p, 5p & 10p coins to make amounts up to 20p. ***M*** To identify half a shape. ***F*** To put together halves to make whole shapes. ***F*** To break an object in half. ***F*** Uses mathematical terms to describe 3d shapes. ***GS***
	Key: Number and Place Value ***NPV***, Addition ***A***, Subtraction ***S***, Multiplication ***Mx***, Division ***D***, Fractions ***F*** ,Measurement ***M,*** Geometry Shape ***GS,*** Geometry Position ***GP***												

MATHEMATICS YEARLY OVERVIEW
YEAR 1

Term	1	2	3	4	5	6	7
Autumn	**Number and place value**	**Number and place value**	**Addition**	**Addition**	**Subtraction**	**Subtraction**	
	Number and place value	**Measure –** Money	**Addition and Subtraction** *(context money)*	**Measure –** Length	**Addition and Subtraction** *(context length)*	**Geometry –** Properties of shapes	**Statistics**
Spring	**Measure -** Time	**Number and place value**	**Addition/ Subtraction**	**Addition/ Subtraction**	**Measure –** Capacity and Mass	**Addition and Subtraction** *(context capacity and mass)*	
	Geometry – Position and direction	**Addition**	**Multiplication**	**Division**	**Fractions**		
Summer	**Measure –** Time	**Geometry –** Properties of shapes	**Number and place value**	**Measure –** Money	**Addition/ Subtraction**	**Addition/ Subtraction**	
	Addition/ Subtraction	**Addition/ Subtraction**	**Multiplication**	**Multiplication**	**Division**	**Fractions**	**Statistics**

MATHEMATICS YEARLY OVERVIEW 2014 - 2015
YEAR 2

Term	1	2	3	4	5	6	7
Autumn	**Number and place value**	**Number and place value**	**Addition/ Subtraction**	**Addition/ Subtraction**	**Addition/ Subtraction**	**Measure -** Time	
	Geometry – Properties of shapes	**Fractions**	**Division**	**Multiplication**	**Statistics**	**Measure –** Money	**Addition and Subtraction** *(context money)*
Spring	**Number and place value**	**Addition/ Subtraction**	**Addition/ Subtraction**	**Multiplication/ Division**	**Multiplication/ Division**	**Measure -** Time	
	Measure – Length	**Addition and Subtraction** *(context length)*	**Multiplication and Division** *(context length)*	**Fractions**	**Geometry –** Position and direction		
Summer	**Geometry –** Properties of Shapes	**Number and place value**	**Measure –** Capacity and Mass	**Addition and Subtraction** *(context capacity and mass)*	**Multiplication and Division** *(context capacity and mass)*	**Measure –** Time	
	Measure – Length	**Four Operations** *(context measure)*	**Four Operations** *(context money)*	**Fractions**	**Statistics**	**Geometry –** Position and direction	**Four Operations** *(context measure)*

MATHEMATICS YEARLY OVERVIEW 2014 - 2015
YEAR 3

Term	1	2	3	4	5	6	7
Autumn	**Number and place value**	**Addition and Subtraction**	**Addition and Subtraction**	**Multiplication and Division**	**Multiplication and Division**	**Measure –** Time	
	Fractions, Decimals and Percentages	**Fractions, Decimals and Percentages**	**Geometry** – Properties of shapes	**Statistics**	**Measure –** Volume and capacity	**Measure –** Length and mass	**Four Operations** *(context volume, capacity, length and mass)*
Spring	**Number and place value**	**Geometry** – Properties of shapes	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	**Addition and Subtraction**	**Multiplication and Division**	
	Statistics	**Measure –** Money	**Four Operations** *(context money)*	**Measure –** Time	**Four Operations**		
Summer	**Number and place value**	**Addition and Subtraction**	**Multiplication and Division**	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	
	Measure – Volume and capacity	**Four Operations** *(context volume and capacity)*	**Measure –** Length and mass	**Four Operations** *(context length and mass)*	**Geometry** – Properties of shapes	**Statistics**	**Measure –** Time

MATHEMATICS YEARLY OVERVIEW 2014 - 2015
YEAR 4

Term	1	2	3	4	5	6	7
Autumn	**Number and place value**	**Addition and Subtraction**	**Addition and Subtraction**	**Multiplication and Division**	**Multiplication and Division**	**Measurement –** Time	
	Fractions, Decimals and Percentages	**Fractions, Decimals and Percentages**	**Geometry –** Properties of shapes	**Statistics**	**Measurement –** Length and mass	**Measurement –** Volume and capacity	**Four Operations** *(context volume, capacity, length and mass)*
Spring	**Number and place value**	**Addition and Subtraction**	**Multiplication and Division**	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	**Geometry –** Position and direction	
	Statistics	**Measurement –** Money	**Four Operations** *(context money)*	**Measurement –** Time	**Geometry –** Properties of shapes		
Summer	**Number and place value**	**Addition and Subtraction**	**Multiplication and Division**	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	**Geometry –** Position and direction	
	Measurement – Volume and capacity	**Four Operations** *(context volume and capacity)*	**Measure –** Length and mass	**Four Operations** *(context length and mass)*	**Geometry –** Properties of shapes	**Statistics**	**Measurement –** Time

MATHEMATICS YEARLY OVERVIEW
YEAR 5

Term	1	2	3	4	5	6	7
Autumn	**Number and place value**	**Addition and Subtraction**	**Multiplication and Division**	**Multiplication and Division**	**Four Operations**	**Measurement –** Time	
	Fractions, Decimals and Percentages	**Fractions, Decimals and Percentages**	**Geometry –** Properties of shapes	**Measurement –** Length and mass	**Measurement –** Volume and capacity	**Four Operations** *(context volume, capacity, length and mass)*	**Statistics**
Spring	**Number and place value**	**Addition and Subtraction**	**Multiplication and Division**	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	
	Measurement – Money	**Four Operations** *(context money)*	**Measurement –** Time	**Geometry –** Position and direction	**Geometry –** Properties of shapes		
Summer	**Number and place value**	**Addition and Subtraction**	**Multiplication and Division**	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	
	Statistics	**Geometry –** Position and direction	**Geometry –** Properties of shapes	**Measurement –** Volume and capacity	**Measurement –** Length and mass	**Measurement –** Money	**Four Operations** *(measurement)*

MATHEMATICS YEARLY OVERVIEW
YEAR 6

Term	1	2	3	4	5	6	7
Autumn	**Number and place value**	**Addition and Subtraction**	**Multiplication and Division**	**Multiplication and Division**	**Four Operations**	**Measurement –** Time	
	Fractions, Decimals and Percentages	**Fractions, Decimals and Percentages**	**Geometry –** Properties of shapes	**Measurement –** Volume, capacity and mass	**Measurement –** Length and money	**Four Operations** *(measurement)*	**Statistics**
Spring	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	**Four Operations**	**Four Operations**	**Algebra**	**Assessment Week –** move accordingly	
	Four Operations *(context money)*	**Measurement –** Time	**Geometry –** Position and direction	**Geometry –** Properties of shapes	**Ratio and Proportion**		
Summer	**Number and place value**	**Four Operations** *(money)*	**Algebra**	**Fractions, Decimals and Percentages**	**Geometry –** Properties of shapes **Ratio and Proportion**	**Statistics**	
	Four Operations	**Geometry –** Position and direction	**Geometry –** Properties of shapes	**Measurement –** Volume, capacity and mass	**Measurement –** Length and money	**Four Operations** *(measurement)*	**Assessment Week –** move accordingly

National Curriculum Strands

PRIMARY ADVANTAGE
SCHOOLS ACHIEVING MORE TOGETHER

the learning trust
the future for education in Hackney

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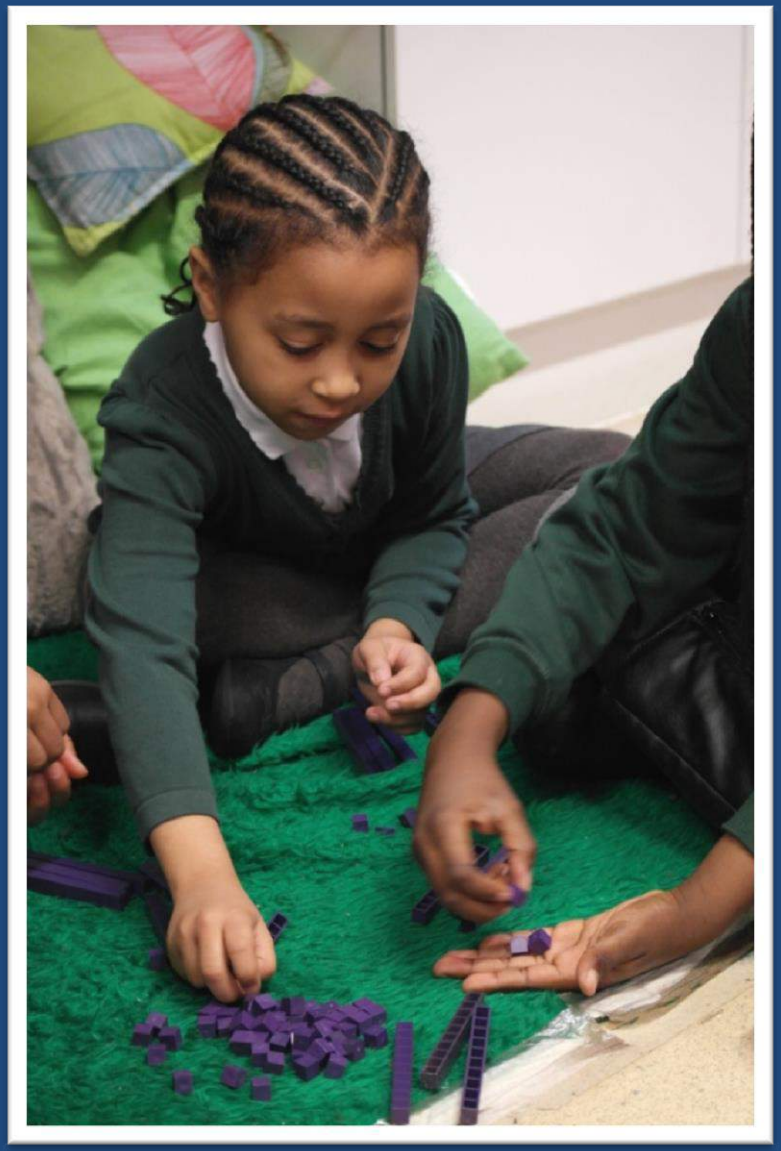
MAYOR OF LONDON
Department for Education



Chapter 1

Number and Place Value

EYFS 1 – Number and place value (When planning ensure you track forwards to year 1)

Early Learning Goal 11
Children count reliably with numbers from 1 to 20 and place them in order.

Key Vocab: number, zero, one, two, three etc, none, how many?, count, count(up) to, count on (from, to), count back (from, to), more, less, many, few, odd, even, every other, how many times?, pattern, pair, guess how many, estimate, nearly, close to, about the same as, greater, more, larger, bigger, less, fewer, smaller, compare, order, first, second, third, last, before, after, next, between.

Key concepts

When there are more objects the group gets bigger. When there are fewer (less) objects the group gets smaller.
When counting a group the last number represents the quantity.
Anything can be counted: claps, steps, jumps…
There are many numbers in the world around us.
When we are talking about objects we say 'more than' and 'fewer than'. When we are talking about numbers we say 'greater than' and 'smaller than'.

Potential barriers/misconceptions

Pupils show confusion in vocabulary- more / less.
Misconception can occur through the linking of words- the bigger a number (in size) the greater it's quantity. i.e.

3 is bigger than 7. (worth more than…)

Some pupils at this stage cannot differentiate between numbers and letters.
Pupils may be able to recite number words up to ten but do not count objects with 1 to 1 correspondence.
There may be little understanding of the value that each number holds
Pupils find it challenging to identify 'same' and 'different when working visually as they don't focus on the detail.

Learning objectives (see overleaf for exemplification)

To make comparisons between quantities.
To use language of quantities such as 'more' and 'a lot'.
To use the language of 'more' to compare sets of objects.
Recite number names in sequence 0-10.
Select a small number of objects from a group 'give me one, two etc'.
To compare two groups of objects (identifying 'the same').
To use number names and language.
To match numeral and quantity correctly.
To use one to one correspondence (touches each object and gives it a number).
To count objects in a line.
To know that numbers identify how many there are in a set (triad).
To create and experiment with symbols and marks representing ideas of numbers, then numerals.
To use more/most and less/least.

Example Questions

If we count around the circle starting with Gemma, who will say 5?
Look at the bowl of apples. Are there more green apples or red apples? How can you find out?
The birthday card has a 4 on it. Raza is four today. Put the right number of candles on his birthday cake.
How many counters are there?

Select the correct number card and match it with the counters. 1 2 3 4 5

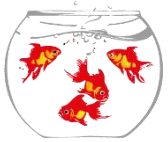
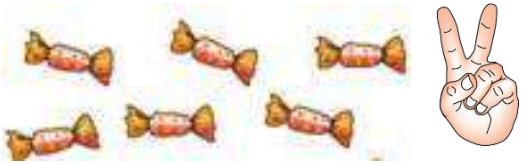
Count with me to 10. One, two, three….
Count these buttons. You can move them as you count them if you wish.
What number is the one before six?
Put three coats up on the pegs
Bring me five aprons. Can you put one back?
Are there more books on the top shelf or on the bottom shelf? How do you know?
Which set has more cubes? The set of red cubes or the set of green cubes? How do you know?
Show a card. 'Read me the number on this card'.
Look at this telephone. Can you press the number 4? Number 6?

Mental maths (can revisited throughout day once concept has explicitly shared)

One, two, three four five. Once I caught a fish alive…
One potato, two potatoes, three potatoes, four…
Higgledy, Piggledy, my fat hen…
This old man, he played one…
Recite sequence 1,2,3 up to 10
Count objects: tiny things in a matchbox, pieces of a jigsaw, letters in your name etc.
Count in 2s: pairs of socks, pairs of animals
Count in 1s (say aloud every other number)

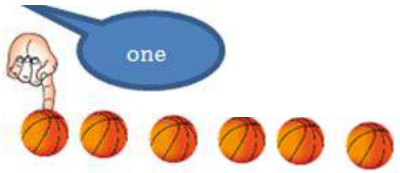
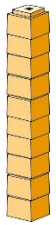
Recognise recitation errors: (could use a puppet)
One, two, four, five (word omitted)
One, two, four, three, five (words in the wrong order)
One, two, three, three, four (repeating a word)

Start from a given number name and stop at another (start with three, hold it in your head, count to six)
Recite the number names in order to 5 then 10
To count backwards from 10
To count backwards to zero (none) from any number
Estimate (guess) how many marbles in the jar, coins in a purse etc

EYFS 1 – Number and place value	Progression (a combination of these models and images can be used for every objective)		
To make comparisons between quantities.	**To use language of quantities such as 'more' and 'a lot'.**	**To use the language of more to compare sets of objects.**	
Which plate would you like? Why?  	 **I need more cars** **I have a lot of cars** 	  **Which bowl has more fish in?**	
Recite number names in sequence 0-10.	**Select a small number of objects from a group 'give me one, two etc'.**	**To compare two groups of objects (identifying 'the same').**	**To use number names and language.**
0 1 2 3 4 5 6 7 8 9 10			zero – none one two three four five six seven eight nine ten more, less bigger, smaller
To match numeral and quantity correctly	**To use one to one correspondence (touch each object and give it a number)**	**To count objects in a line**	
		 There are five cars in the group.	
To know that numbers identify how many there are in a set. (triad- three key elements)	**To create and experiment with symbols and marks representing ideas of numbers then numeral.**	**To find more/ most and less/least**	
It looks like: 0 1 2 3 4 5 6 7 8 9 10 It sounds like: **SIX** You make it like this: 	Make own marks or tallies resulting from practical activities. Pictorial representations of groups. When beginning to record numbers: Trace with a finger cut out numerals Sandpaper Rough fabric Tin foil Corrugated card  Make numbers in sand, foam, using plasticine.	**Isaac's bears** **Hussein's bear**   Who has the most bears? Who has the least? Isaac has more bears than Hussain.	

EYFS 2 – Number and place value **(When planning ensure you track forwards to year 1)**

Early Learning Goal 11 **Children count reliably with numbers from 1 to 20 and place them in order.**	**KS1 ready: Count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number.** **Count, read and write numbers to 100 in numerals; count in multiples of 2s, 5s and 10s** **Given a number, identify one more and one less** **Identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least** **Read and write numbers from 1-20 in numerals and words**
Key vocab: number, zero, one, two, three etc, none, how many?, count, count(up) to, count on (from, to), count back (from, to), more, less, many, few, odd, even, every other, how many times?, pattern, pair, guess how many, estimate, nearly, close to, about the same as, greater, more, larger, bigger, less, fewer, smaller, compare, order, first, second, third, last, before, after, next, between. **Key Concepts**. When there are more objects the group gets bigger. When there are fewer objects the group gets smaller. When counting a group the last number represents the quantity. Anything can be counted: claps, steps, jumps… There are many numbers in the world around me. We can write number with words and squiggles. When we are talking about objects we say 'more than' and 'fewer than'. When we are talking about numbers we say 'greater than' and 'smaller than'.	**Learning objectives** (see overleaf for exemplification) To count reliably from 1-20 To use one to one correspondence (touch each object and give it a number) To count objects in a line- beginning to count beyond 10 To count actions or objects without physically touching them. To count objects in a group/ irregular arrangement. (using first same objects/ then different objects) To represent numbers using fingers, marks and pictures. To recognise numerals (0-5), (0-10) and (0-20). To order numbers from 0-20. To select the correct numeral to represent 1-5 then 1-10 objects. Write numbers to 20. To estimate how many objects can be seen and check by counting. To recognise numbers in a group without counting out (subitise). To make ten (feel the ten-ness of ten).
Potential barriers/misconceptions Pupils show confusion in vocabulary- more / less. Misconception can occur through the linking of words- the heavier object is the one that is 'higher' on the balance (When using balances to compare quantity). Pupils may be able to recite number words up to ten but do not count objects with 1 to 1 correspondence. There may be little understanding of the value that each number holds **Example Questions** One, two, buckle my shoe, three, four, knock at the door. Which two numbers come next? 10, 9, 8, 7, carry on counting backwards until 'blast off!' If we count round the circle starting at Lewis with 3, who will say 9? What number comes next after 12 when you count? Make a line of toy cars. Make the second car yellow and the fifth car red. Count on for me as far as you can go. One, two, three… What is the next number after four? I will say some numbers. I want you to count on the next three numbers. Four, five, six… Take this box of unifix. Count out nine of the unifix cubes and put them onto the table. Can you check? How many circles are there in the picture (show up to 5). Tell me without counting them. Now check by counting. Guess how may cars there are on the table. (Place up to 10). Now check by counting them. Which plate has fewer biscuits on it? How do you know? There are 8 cubes in this stick of cubes. There are five cubes in this stick of cubes. Which stick has more cubes? Ella has five apples, Dom has three apples. Who has fewer apples? Ella or Dom? Choose two cards from this set. Which of your two numbers is more? Which number is less?	**Mental maths** (can revisited throughout day once concept has explicitly shared) To count from 1-20 To count from non-zero starting point (up to 20). To recite the number names in order, continuing the count from a given number Recognise recitation errors: (could use puppet) Thirteen, fourteen, fiveteen (not changing the pattern) Eighteen, nineteen, tenteen (error by analogy) Thirty-nine, thirty-ten (error by analogy) Start from a given number name and stop at another. (start with 2, hold it in your head, count on to 8) Count on several numbers from a given number (using fingers to help: count on three numbers from 4) To use ordinal numbers in different contexts (Who is third in the line?) To say the number name that goes before a given number name. (What number comes before □?) Recite the number sequence consistently back to zero from any given number to 20 To count in 2s, 5s and 10s. To count on in tens from any given tens number. (Count on in tens from 30) To count back in tens from any given tens number. Estimate the number in a group (how many children in class today?) To know which number is worth more/less. To say the number that is one more/one less than the given number. Say a number lying between two given numbers. Begin to use and understand ordinal numbers in different contexts (first, second, third….. & last)

EYFS 2 – Number and place value	Progression (a combination of these models and images can be used for every objective)		
To count reliably from 1-20	**To use one to one correspondence (touch each object and give it a number)**	**To count objects in a line (beyond 10)**	**To count actions or objects without physically touching them.**
one eleven two twelve three thir**teen** four four**teen** five fif**teen** six six**teen** seven seven**teen** eight eigh**teen** nine nine**teen** ten twen**ty**		 **11** There are eleven cars in the group	Listening to the number of claps. The rings of a bell. Jumps in the air. Children at the front of the class. Windows on a building
To count objects in a group/ irregular arrangement	**To represent numbers using fingers, marks and pictures**	**To recognise numerals (0-5), (0-10) and (0-20)**	**To order numbers from 0-20**
 There are nine cars in the group	 	  'Put the silver car in space 8'	
To select the correct numeral to represent 1-5 then 1-10 objects	**Write numbers to 20**	**To estimate how many objects can be seen and check by counting**	**To recognise numbers in a group without counting out (subitise)**	**To make ten* (recognise the ten-ness of 10)** (*see addition for number bonds within 10)
	Trace. Write in the air. Paint. Model in play dough. Make number signs for classroom. 	 How many oranges do you think there are? Can you check?	  Can recognise iconic, regular images of small numbers (e.g. dice patterns). 	 Egg box ten  Unifix ten  bundles of ten straws

Year 1 – Number and place value **(When planning ensure you track back to Reception and forwards to year 2)**

National Curriculum

Count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number.
Count, read and write numbers to 100 in numerals; count in multiples of 2s, 5s and 10s
Given a number, identify one more and one less
Identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least
Read and write numbers from 1-20 in numerals and words

Notes and guidance (non statutory)

Pupils practice counting (1,2,3…) ordering (e.g. first, second, third) and to indicate a quantity (e.g. 3 apples, 2 centimeters), including solving simple concrete problems until they are fluent.
Pupils begin to recognize place value in numbers beyond 20 by reading, writing, counting and comparing numbers up to 100, supported by objects and pictorial representations.
They practice counting as reciting numbers and counting as enumerating objects, and counting in 2s, 5s, and 10s from different multiples to develop their recognition of patterns in the number system (e.g. odd/even) including varied practice through increasingly complex questions.
They recognize and create repeating patterns with objects and with shapes.

Key vocab: count, count(up) to, count on (from, to), count back (from, to), more, less, many, few, odd, even, every other, how many times?, pattern, pair, ones, tens, regroup, fair swap, digit, equal to, greater, more, larger, bigger, less, fewer, smaller, compare, order, first, second, third, last, before, after, next, between, half way.

Key Concepts

A group of objects can be partitioned in a number of ways- the total stays the same. (conservation)
Ordinal numbers are for describing the position in a group of objects.
When comparing we use the terms 'greater than' and 'smaller than' and 'more than' and 'fewer than.'
When we are talking about objects we say 'more than' and 'fewer than'. When we are talking about numbers we say 'greater than' and 'smaller than'.
Making ten first supports with number conservation. Children then count on from ten rather than starting at 1.

Potential barriers/misconceptions

Unable to recognise numbers.
Knowledge of saying numbers out loud with no concern for value or amount of objects.
No understanding of the value that each digit holds.
Understanding of number size – confusion over 3 is bigger than 1.
Does not count with 1-1 correspondence.
Able to count forwards but struggles to count backwards or find 'one less than…'
Counts all rather than counting 'on' (no conservation of number).
Sees a 'ten' as one rather than ten ones.
Confusion between 'teen' numbers and multiples of ten: 16, 60.
Reversal of digits.

Example Questions

What number comes after 22? Before 65?
Count back from 10 to six. How many did you count?
Which tens number comes after 50? Before 80?
What would be the best way to count marbles into the jar?
There is always 1 left over when an odd number is divided by 2. True or false? Can you prove it?
Draw a ring around the person who is 9th in the line.
Estimate the number of pencils.
Estimate how many pairs of socks you could make. (Show a picture of unpaired socks)
Look at these numbers: 34 12 45 60 72 28 Which of these numbers is the largest? Which of these numbers is between 10 and 20?
This sentence is correct: 8 is less than 10. Two of these sentences are correct. Tick them: 18 is more than 30, 26 is less than 60, 50 is more than 17, 47 is less than 21.
Fill in the blanks: 35 is more than □, 35 is between □and □, 35 has □ tens.
Write the number thirty-two.
Fill in the missing numbers: 18 is 1 less than □ , 18 is 10 less than □.

Learning objectives (see overleaf for exemplification)

To identify one more and one less.
To compare quantities (using equal to, more than, less than (fewer), most, least)
To match numbers and quantities.
To locate numbers on a number line.
To read & write numbers from 1-20 in numerals and words.
To identify odd and even numbers.
To understand ordinal numbers.
To compare numbers up to 20 (and beyond).
To describe and extend number sequences.
To make ten.
To regroup (carry out a fair swap).
To make ten and count on (in concrete).
To identify ten and count on (in pictorial).
To count out a 2 digit number to 20 and regroup in the 1s.
To partition and recombine numbers to 20 into 10s and 1s (teen numbers).
To partition and recombine any 2 digit number into 10s and 1s.

Mental maths

To count to and across 100
To count larger collections by grouping into tens, then fives or twos.
To count backwards in ones from any two digit number
To count on any given single digit number from any two digit number (count on seven from 22)
To count in multiples of 2, 5 and 10
To count on in tens from a tens number stopping at a given number. (count from 20 to 60)
To count back in tens from a tens number stopping at a given number (count from 80 back to 30)
To describe and extend number sequences: counting on or back in steps of ones or tens from any given number.
Count in 2s from 0-20, count in 2s from any given number
To identify one more and one less than any given number
Can say whether any number from 1-100 is odd or even and why.
Count in tens from zero… from 40… from 8
Count in 2s from zero, count from 1,3,5
To say what number comes next in a given pattern. (16,14,12, □,□)
To recall number bonds (see addition strand for exemplification)
To know number bonds of all numbers within 10 (6 = 1+5, 5+1, 4+2, 2+4 etc)
To know number bonds to 10, To know number bonds within 20
To make a reasonable estimate (then count to check)
To state the value of the digits in a two digit number (14 is one ten and four ones)

Year 1 – Number and place value | **Progression (a combination of these models and images can be used for every objective)**

To find one more/less than a given number

To compare quantities

'More than' to compare objects.
'There are more green apples than red apples'
'There are fewer/less red apples than green apples'

'Greater than' to compare numbers.
5 is greater than 3
3 is smaller than 5

Count and compare

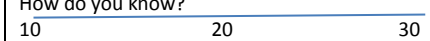
To match numbers and quantities.

9
Nine

Can you make this amount using unifix cubes?

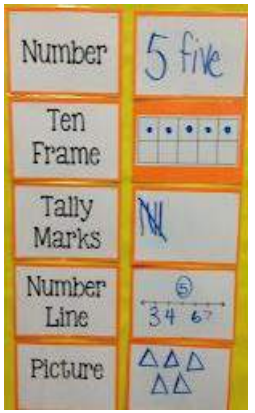
To locate numbers on a number line.

Where would 15 be on the number line?
How do you know?

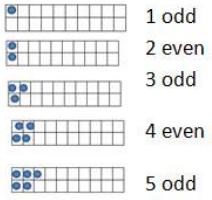


On a bead string? On a counting stick?

To read and write numbers to 20

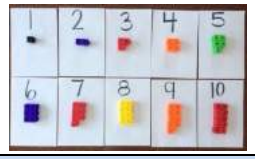


To identify odd and even numbers



Represent up to 9 using ten grid.

Use 'pairs of' to represent even



To understand ordinal numbers

Circle the fourth elephant

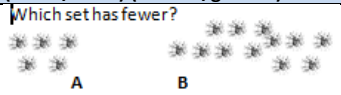
1st

Circle the ninth spider

1st

To compare numbers up to 20. (fewer/more) (smaller/greater)

Which set has fewer?



Set A has 5 spiders and set B has 12 spiders.
Set A has 7 spiders fewer than set B.

17
13
17 is greater than 13.

To describe and extend number sequences

How many stars in the next pattern?

Find the missing numbers:
15, 14, 13, □, □, □

To make ten

To use bundles of straws for children to feel the 'ten-ness' of ten.

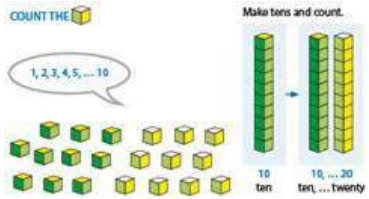
One ten and 2 ones = **12**

Tens	Ones
1	2

To regroup (carry out a fair swap)

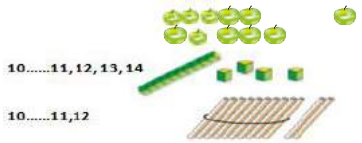
For children to use Dienes to create a 'fair swap' (regrouping of ten ones for one ten)

Fair swap



To make ten and count on (concrete)

Use objects, dienes or bundles of straws to 'make 10' then count on:



11
14
12

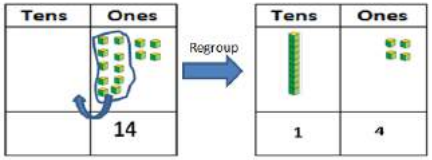
To make ten and count on (pictorial)

Draw around ten and then count on.

Ten and eight is eighteen.

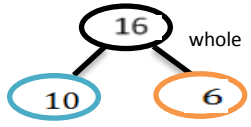
10..... 11,12,13,14,15,16,17,18

To count out a 2 digit number to 20 and regroup in the 1s



To partition and recombine numbers to 20 into 10s and 1s. (teen numbers then beyond 20)

'16 is 1 ten and 6 ones'



Year 2 – Number and place value **(When planning ensure you track back to year 1 and forwards to year 3)**

National Curriculum	Notes and guidance (non statutory)
Count in steps of 2,3, and 5 from 0 and in tens from any number, forward and backward Recognise the place value of each digit in a two-digit number (tens, ones) Identify, represent and estimate numbers using different representations including the number line. Compare and order numbers from 0 up to 100; use <, > and = signs Read and write numbers to at least 100 in numerals and in words Use place value and number facts to solve problems.	Using materials and a range of representations, pupils practice counting, reading, writing and comparing numbers to at least 100 and solving a variety of related problems to develop fluency. They count in multiples of three to support their later understanding of a third. As they become more confident with numbers up to 100, pupils are introduced to larger numbers to develop further their recognition of patterns within the number system and represent them in different ways, including spatial representations. Pupils should partition numbers in different ways (for example, 23 = 20 + 3 and 23 = 10 + 13) to support subtraction. They become fluent and apply their knowledge of numbers to reason with, discuss and solve problems that emphasise the value of each digit in two-digit numbers. They begin to understand zero as a place holder.

Key Vocab: count, count(up) to, count on (from, to), count back (from, to), more, less, many, few, odd, even, every other, how many times?, pattern, pair, ones, tens, regroup, fair swap, digit, equal to, greater, more, larger, bigger, less, fewer, smaller, compare, order, first, second, third, last, before, after, next, between, half way, place, place value, stands for, represents, round, nearest, estimate.

Key Concepts

Numbers can be partitioned in many ways into part, part, whole. (Unique partitioning is when numbers are broken up in the standard representation i.e. 63 is 6 tens and 3 ones. Multiple partitioning is the ability to also see: 63 = 5 tens and 13 ones or 2 tens and 43 ones- this is an important tool for mental strategies)

We can regroup ten ones for one ten. Ten tens is one hundred.

In a two digit whole number the digit indicating the multiple of 10 is written on the left, and that to distinguish between, say 20 and 2, a zero is put in the space on the right as a place holder. Zero is a place holder and means 'no ones, no tens, no hundreds etc.'

Numbers can be compared using the terms 'greater than' and 'smaller than' with and without concrete representation.

Potential barriers/misconceptions

Reversal of digits 03 for 30 and 31 for 13. This can create problems when ordering numbers.

Confusion about the place value of numbers. Difficulties especially apparent when ordering numbers such as 212 and 221.

Failure to understand that the position of the numeral gives it the value.

Pupils not always sure what makes a 'sensible' answer (not estimating).

Learning objectives (see overleaf for exemplification)

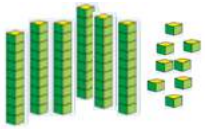
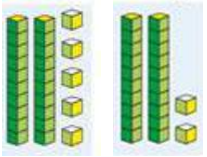
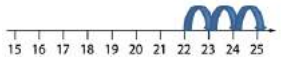
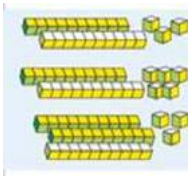
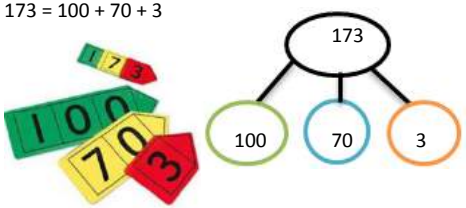
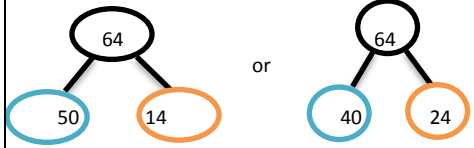
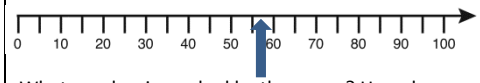
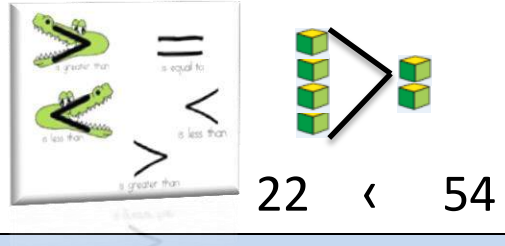
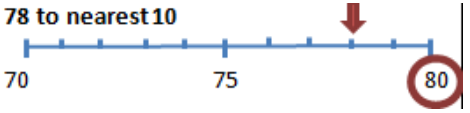
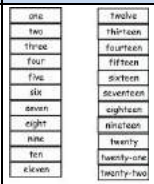
To represent 2 digit numbers (concrete)
To count within 100 by making tens first.
To recognise the place value of each digit in a 2 digit number.
To compare numbers from 0 – 100.
To order numbers from 0-100.
To partition and recombine 2 digit numbers into 10s and 1s.
To partition and recombine 3 digit numbers into 100s, 10s and 1s.
To partition numbers in different ways.
Identify numbers on a number line.
To use the greater than, less than and equals signs (<,>, =)
To begin to round numbers less than 100 to the nearest 10.
Read and write numbers in numerals and words.

Example Questions

Here are some numbers: 44 87 62 28 51. Write them in order; the first one is done for you: 28 □ □ □ □ .
Here are two signs: '< ' ' > '. Use the signs to make the following correct: 54 □16, 19 □ 94, 51□ 35
Ben puts 12 coins on a table. He hides some of them under his hand. How many coins is Ben hiding?
Tim bought two pieces of fruit. He spent thirty pence altogether. He bought an orange for 12 pence. What did he pay for the other piece of fruit?
Circle two numbers that add to make a multiple of 10: 11 12 13 14 15 16 17 18 19
Fill in the blank to make this correct: 40 – 30 = 10 + □.
Write the two missing numbers in this sequence: □ 45 47 49 51 □ 55 57
Charlie is making 3-digit numbers with these cards. He can make this number: 7 2 4. Write all the other 3-digit numbers he can make.
Write the missing digits to make this correct: □ 0 + 3□ = 43
Write an odd number between 34 and 44.
Write the missing numbers in this sequence: 47 42 37 □ □ 22 17 12
Write a number in the space to make this correct: 867 = □+ 60 + 7
Sarah has 60 sweets. She puts 6 sweets in each party bag. How many bags does she put sweets in?

Mental maths

To count in steps of 2, 3, 5 and 10 (forwards and backwards from any given number).
To count on in tens from any given number (with and without a hundred square).
To count to and across 100 from any given number (forwards and backwards).
To have rapid recall of the x2, x3, x5 and x10 tables.
To count up in threes from any given number (forwards and backwards).
To find ten more than a multiple of ten (ten more than 40).
To identify 1, 10, or 100 more/less than any given number.
To accurately say the sequence of odd numbers from 1-19.
To say whether any number is odd or even.
To recognise multiples of 10, 5 and 2 and say how they know.
To know the value of each digit (what is the number equivalent to 6 tens and 5 ones?).
To know number bonds within 10 (for number bond exemplification see addition strand).
To know number bonds to 10.
To know number bonds within 20.
To use the language of ordinality up to twentieth.

Year 2 – Number and place value	Progression (a combination of these models and images can be used for every objective)		
To represent 2 digit numbers (concrete) (for regrouping and making 10 see year 1)	**To count within 100 by counting tens first.**	**To represent numbers as tens and ones in a place value table.**	**To compare numbers**
Count 'bundles' of 10:  5 tens = 50 Make 2 digit numbers with dienes:  4 tens + 4 ones = 44	 10, 20, 30, 40, 50, 60…… 61, 62, 63, 64, 65, 66, 67, **68** There are 68 in the group. 60 and 8 make 68.	68= □ tens + □ ones 68 = 60 + 8  What are the missing numbers? Can you make these numbers? 97, 62, 33, 48…	Which number is greatest? How much greater is it?  If the tens are equal then we compare the **ones.**  25 is 3 more than 22 22 is 3 less than 25
To order numbers from 0-100	**To partition and recombine 2 digit numbers into 10s and 1s. (unique partitioning)**	**To partition and recombine 3 digit numbers into 100s, 10s and 1s. (unique partitioning)**	**To partition numbers in different ways. (multiple partitioning)**
 **Tens ones** 2 3 2 5 3 3 Abstract: pupils to place range of numbers in order.	57 = 50 + 7 	173 = 100 + 70 + 3 	64 can be partitioned as = 6 tens and 4 ones (60+4) 64 can also be partitioned as: 
Identify numbers on a number line.	**To use the greater than>, less than< and equals= signs (<,>, =)**	**To begin to round numbers less than 100 to the nearest 10.**	**Read and write numbers in numerals and words. (to 100 and beyond)**
Locate numbers on a number line  What number is marked by the arrow? How do you know? Can you locate: 22, 39, 94…..on the number line? On an unmarked number line?	 22 ‹ 54	 Look at the ones 5 or above? – round on to the next tens number Less than 5? – round back to the previous tens number	 Note that when recording numbers over 20: **22** twenty-two (this is written with a hyphen between the tens & ones)

Key steps in developing understanding of place value

1) Ten- 1 ten has a different value to 1 one
2) Tens and ones- Tens and ones can exist side by side
3) Number names- Instead of naming a number 1 ten and 4 ones we give it a name- fourteen
4) Hundreds, tens and ones- Children should be aware that when they have ten tens they must 'regroup' to make 1 hundred
5) Numbers can be broken into different parts. The number 36 can be 3 tens and 6 ones. It is also 2 tens and 16 ones

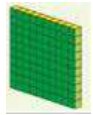
Year 3 – Number and place value **(When planning ensure you track back to year 2 and forwards to year 4)**

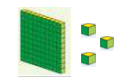
National Curriculum	Notes and guidance (non- statutory)
Pupils should be taught to : Count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number. Recognise the place value of each digit in a three-digit number (hundreds, tens, ones) Compare and order numbers up to 1000 Identify, represent and estimate numbers up to 1000 in numerals and words. Solve number problems and practical problems involving these ideas.	Pupils now use multiples of 2,3,4,5,8,10, 50 and 100 They use larger numbers to at least 1000, applying partitioning related to place value using varied and increasingly complex problems, building on work in year 2 (for example, 146=100+ 40 and 6, 146=130+16) Using a variety of representations, including those related to measure, pupils continue to count in ones, tens and hundreds, so that they become fluent in the order and place value of numbers to 1000.
Key vocab: count, count(up) to, count on (from, to), count back (from, to), more, less, many, few, odd, even, every other, how many times?, pattern, pair, ones, tens, hundreds, thousands, regroup, digit, equal to, greater, more, larger, bigger, less, fewer, smaller, compare, order, last, before, after, next, between, half way. **Key concepts** Number lines, hundred squares and Dienes all help to understand counting, ordering and place value. In a three digit whole number, if a digit is to represent hundreds it must be written in the place on the left and where necessary the middle and right hand spaces filled with zeros as place holders. Each column to the left of another is ten times greater; each column to the right is ten times smaller. Numbers can be partitioned in many ways into part, part, and whole. (Unique partitioning is when numbers are broken up in the standard representation i.e. 63 is 6 tens and 3 ones. Multiple partitioning is the ability to also see: 63 = 5 tens and 13 ones or 2 tens and 43 ones- this is an important tool for mental strategies) Numbers can be compared using the terms 'greater than' and 'smaller than' .	**Learning objectives** (see overleaf for exemplification) To represent 3 digit numbers (concrete) To find 1, 10 or 100 more than a given number (concrete). To recognise the place value of each digit in a three digit number. To use part, part whole to partition numbers in different ways. To compare numbers up to 1000 To order numbers up to 1000 Identify, represent and estimate numbers up to 1000 in numerals and words. To recognise the place value of different measures. To use dienes and coins to understand place value.
Potential barriers/misconceptions Ordering numbers is challenging if pupils don't have a strong understanding of place value. Trouble giving values to each of the digits. In 23 the value of the first number is not 2 it is 2 tens or 20. Confusion around zero as a place holder. Pupils move into abstract too quickly and although they can recognise and read numbers up to 1000 they are unfamiliar with the place value of each digit.	**Mental maths** Rapid recall of 2, 3, 4,5,8,10,50 and 100 tables. To count in multiples of 4 and 8. (x2 table double to get x4, double again to get x8) To count in multiples of 50 and 100. To find 10 or 100 more than a given number. To find 10 or 100 less than a given number. To count on and back in tens crossing hundreds from any given three digit number. Count on in tens from one given number to another. How many tens did you count? (336 to 416) Count back in tens from one given number to another. How many tens did you count? (202 to 182) Count on or back 400 in hundreds from any tens number. (from 500, from 520, from 570) Count back in hundreds round a circle of children, starting with Lucy with 970. Who will say 370? To know the next number in a sequence. (256, 356, 456, 556…) To identify all odd and even numbers up to 1000. To know what odd number comes before/after a given number. After a given number. (what odd number comes before 301?) To know what happens when odd numbers are added together. To know what happens when even numbers are added together. Create sequences with a given constraint. i.e. make a sequence with 107 and 116 in it. To know that multiples of: 100 end in 00 50 end in 00 or 50 10 end in 0 5 end in 5 or 0 2 end in 0, 2, 4, 6, 8, To know what multiple of 10, 5, 100 etc lies before/ after a given number. (What is the multiple of 5 after 805?) To say what digits represent. (What does the digit 3 represent in 345? = 3 hundreds) To know which numbers lie between given numbers. (What even numbers lie between 415 and 420?) To round numbers to the nearest 10 or 100.
Example Questions Write these numbers in order of size, starting with the smallest: 903 1094 912 106 190 What number is ten less than four hundred and three? Fill in the blanks to make this correct: 360 + □ + □ = 540 Look at these digit cards: '6 '' 9 ' ' 7' . Use each card once to make the largest number. Use each card once to make the smallest even number. Write the missing number in the space: □+ 47 = 100 Circle three numbers that add to make 160: 10 30 50 70 60 Write 'always', 'sometimes or 'never' in each space to make the following sentences correct: Multiples of 2 _____ end in 3. Multiple of 5 _____ end in 5. Multiples of 10 _____ end in 0. Write a calculation that you could do to check the answer to: 150 + 350 = 500 3 5 4 6. Use each of these digits once to make a total that is a multiple of 5. □ + □ = Write these numbers in order: 164, 146, 106, 160, 140 (from smallest to largest). Here are the first five numbers in a sequence: 420, 400, 380, 360, and 340. The sequence continues in the same way. Write the number that will be 10th in the sequence. What number is ten less than 1002? Write in figures the number one thousand and thirty. Choose three of these number cards to make an even number that is greater than 400. 3 8 9 1 Write one thousand, three hundred and forty-seven to the nearest ten.	

Year 3 – Number and place value | **Progression (a combination of these models and images can be used for every objective)**

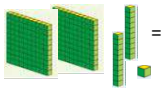
To represent 3 digit numbers (concrete)

10 tens = 1 hundred



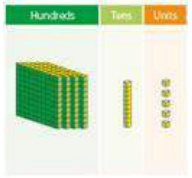


= one hundred and three = **103**



= two hundred and twenty-one = **221**

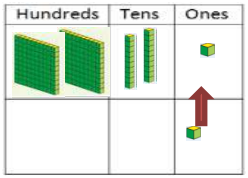
10, 20, 30, 40, 50, 60, 70, 80, 90, **100**.
(ten hundreds = 1 thousand)



4 **1** **5**

To find 1, 10 or 100 more than a given number (concrete).

Count on one

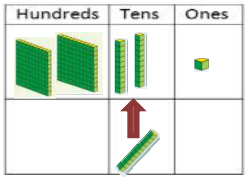


221 + 1 = one more than 221 is 222

+ 1

221, 222

Count on ten

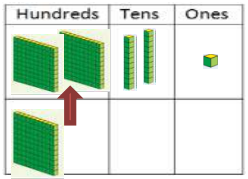


221 + 10 = ten more than 221 is 231

+ 10

221, 231

Count on one hundred

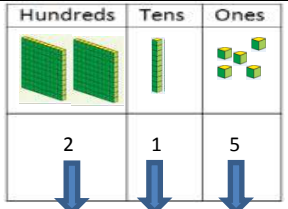


221 + 100 = one hundred more than 221 is 321

+ 100

221, 321

To recognise the place value of each digit in a three digit number.



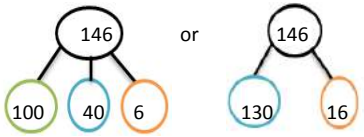
Stands for : 2 hundreds or 200 | 1 ten or 10 | 5 ones or 5

Pupils indicate:
the value of each digit
and
the place at which it is situated.

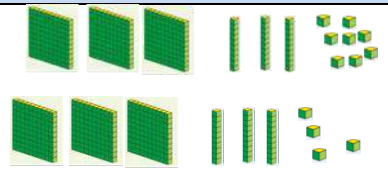
In 506 = 5 hundreds, 0 tens and 6 ones
506 = 500 + 0 + 6
In 506: The digit 6 is in the ones place, the digit 0 is in the tens place and the digit 5 is in the hundreds place.

To use part, part whole to partition numbers in different ways.

146 = 100+40+6 or 146 = 130 + 16



To compare numbers up to 1000



338

334

First compare the hundreds, then the tens, then the ones
338 is greater than 334 (334 is smaller than 338)

To order numbers up to 1000

Arrange these numbers in order. Begin with the smallest. 476, 259, 601

Hundreds	Tens	Ones
4	7	6
2	5	9
6	0	1

First compare the hundreds

601 is greater than 476 and 259.
476 is greater than 259.
In order from smallest: 259, 476, 601

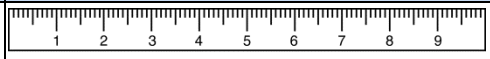
Identify, represent and estimate numbers up to 1000 in numerals and words.

500 **1000**

What number is here on the number line? How do you know?

Write numbers in words:
999 = nine hundred and ninety-nine
234 = two hundred and thirty-four
303 = three hundred and three

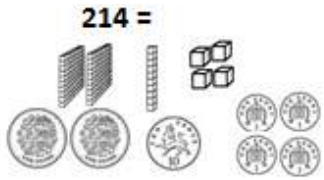
To recognise the place value of different measures.



139cm = 100 cm + 30 cm + 9 cm (100cm = 1 metre)

To use Dienes and coins to understand place value

The idea of 'regrouping' can be reinforced through the exchanging of coins.



Year 4 – Number and place value **(When planning ensure you track back to year 3 and forwards to year 5)**

National curriculum

Count in multiples of 6,7,9,25 and 1000
Find 1000 more or less than a given number
Count backwards through zero to include negative numbers
Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens and ones)
Order and compare numbers beyond 1000
Identify, represent and estimate numbers using different representations
Round any number to the nearest 10, 100, 1000
Solve number and practical problems that involve all of the above and with increasingly large positive numbers.
Read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value.

Notes and guidance (non-statutory)

Using a variety of representations, including measures, pupils become fluent in the order and place value of numbers beyond 1000, including measures, pupils become fluent in the order and place value of numbers beyond 1000, including counting in tens and hundreds, and maintaining fluency in other multiples through varied and frequent practice.
They begin to extend their knowledge of the number system to include the decimal numbers and fractions that they have met so far.
They connect estimation and rounding numbers to the use of measuring instruments.
Roman numerals should be put in their historical context so pupils understand that there have been different ways to write whole numbers and that the important concepts of zero and place value were introduced over a period of time.

Key vocab: numeral, place value, order, round, stands for, represents, regroup, >, greater than, <, less than, integer, positive, negative, above, below, zero, minus, next, consecutive, sort, classify, property.

Key concepts

In a two digit whole number the digit indicating the multiple of 10 is written on the left, and that to distinguish between, say 20 and 2, a zero is put in the space on the right as a place holder.
Zero is a place holder and means 'no ones, no tens, no hundreds etc.'
Negative numbers can be found on a temperature scale.
Negative numbers are below zero and the size of the negative number indicates the distance it is from 0.
< is less than and > is greater than. When we multiply by 10/100 the place value of the digits change.
Rounding numbers can alter a situation (i.e. results of a race etc.) A number line can be used to visualise decimals.

Potential barriers/misconceptions

Ordering numbers is challenging if pupils don't have a strong understanding of place value.
Trouble giving values to each of the digits. In 23 the value of the first number is not 2 it is 2 tens or 20.
Confusion around zero as a place holder.
Pupils move into abstract too quickly and although they can recognise and read numbers up to 1000 they are unfamiliar with the place value of each digit.
When multiplying by 10/100 children think they just 'add zero' without understanding that the place value of the digits have changed and zero then becomes a place holder.

Example Questions

The sum of two numbers is 100. Write the missing digits: 3 □ + □ 3 = 100
Each missing digit in this sum is a 9 or a 1. Write in the missing digits. □□ + □□+ □□= 201
Paul says, "Every multiple of 5 ends in 5". Is he correct? Explain how you know.
Write in figures the number five thousand and thirty-two
Jet has these numbers: 1330 1303 1033 1003 1030. She writes them in order from smallest to largest. What is the fourth number she writes?
The temperature in London is 3°C. Paris is 9 degrees colder than London. What is the temperature in Paris?
Circle the numbers nearest to 1000. 1050 1340 1046 1004 1040
Match 3500 to numbers with the same value: 35 hundreds 3500 ones 35 tens 350 tens 350 hundreds.
Write these prices in order from smallest to largest: 97p £11.50 £0.76 £8 £3.05
Write these amounts in order in the boxes: £60.06 £60.60 £6.60 £6.06
John makes a sequence of numbers. His rule is: "find half the last number, and then add 10". Write the next two numbers in his sequence: 36 28 24 _ _
Circle the number that is about the same as the correct answer to 49 + 48. 10 50 40 100 70 200

Learning objectives (see overleaf for exemplification).

To represent 4 digit numbers (concrete- place value counters).
To find 1, 10, 100 or 1000 more than a given number (concrete).
To recognise the place value of each digit in a four digit number.
Order numbers beyond 1000.
Compare numbers beyond 1000.
Round any number to the nearest 10, 100, 1000. (To round appropriately given context see division strand)
To identify and count in negative numbers.
To estimate and round numbers using measuring instruments.
To understand the history of different numeration systems.
To read and understand Roman numerals.
To understand the place value of decimals and fractions (see learning objectives in these strands).

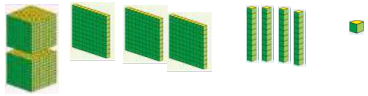
Mental maths

To count in multiples of 6,7 and 9
To count in multiples of 25 and 1000
To count backwards through zero to negative numbers.
To find 1,10, 100, 1000 more than any given number (with 4 or more digits)
To find 1,10, 100, 1000 less than any given number (with 4 or more digits)
To multiply by 10, 100 and 1000 (understanding that digits move to the left when multiplied by 10…).
To know what the value of each digit is up to 10,000.
To count on from any given number crossing boundaries (count on 7 in ones from 669, 70 in tens from 669, 700 in hundreds from 669, 7000 in thousands from 2669).
To round any two or three digit number to the nearest 10 or 100.
To round measurements in seconds, minutes, hours, metres, kilometres, litres to the nearest 10 or 100 units.
Estimate calculations by approximating. (608+297 = 610+300= approximately 910)
Approximate multiplications (19x16 = 20x16 = (2x16) x10= 320)
Extend and explain number sequences (48, 41, 34, 27…) continuing beyond zero.
To notice a pattern when counting from zero in 2s, 4s then 8s (4s are double 2s, 8s are double 4s)
To recognise odd and even numbers up to 10,000 and make general statements about them. (if you add odd numbers the answer is even. Check. Explain why?)

Year 4 – Number and place value

Progression (a combination of these models and images can be used for every objective)

To represent 4 digit numbers (concrete- place value counters)



= 2341

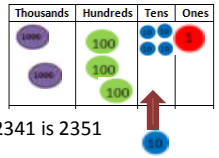
Using place value counters:

= 2341

(Pupils to regroup tens 1s for 1 ten counter etc.)

To find 1, 10, 100 or 1000 more than a given number (concrete).

Count on by tens

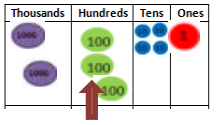


2341 + 10 = ten more than 2341 is 2351

+ 10

2341, 2351

Count on by hundreds

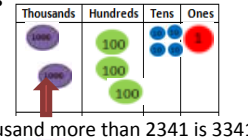


2341 + 100 = one hundred more than 2341 is 2441

+ 100

2341, 2441

Count on by thousands

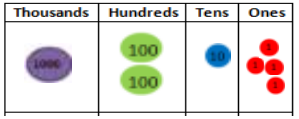


2341 + 1000 = one thousand more than 2341 is 3341

+ 1000

2341, 3341

To recognise the place value of each digit in a four digit number.



Stands for : 1 thousand or 1000 | 2 hundred or 200 | 1 ten or 10 | 5 ones or 5

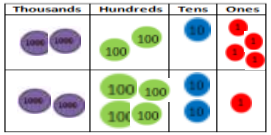
= 1000+200+10+5 = 1215

In 7506 = 7 thousands, 5 hundreds, 0 tens and 6 ones

7506 = 7000+ 500 + 0 + 6

In 7506: The digit 6 is in the ones place, the digit 0 is in the tens place, the digit 5 is in the hundreds place and the digit 7 is in the thousands place.

Compare numbers beyond 1000



Which is greater/smaller?

= 2214

=2421

If the thousands are same compare hundreds.

Order numbers beyond 1000

Arrange these numbers in order. Begin with the smallest.6476, 4259,4601

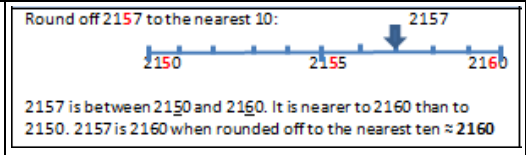
First compare the thousands

6476 is greater than 4259 & 4601

4601 is greater than 4259.

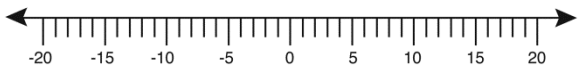
In order from smallest: 4259, 4601, 6476.

Round any number to the nearest 10, 100, 1000



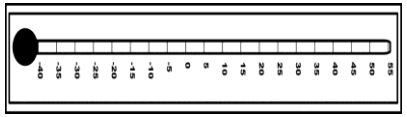
To identify and count in negative numbers.

Negative numbers are numbers that are less than zero.



Negative numbers < zero < positive numbers

Ensure thermometer is shown in both directions



To estimate and round numbers using measuring instruments

How much water is there? To the nearest 10ml

What time is it to the nearest 5 minutes?

How much does this weigh? To the nearest 100g?

How long is this? To the nearest cm?

To understand the history of different numeration systems (Derek Haylock, Mathematics explained for primary teachers, 2006)

Egyptian Hieroglyphics	Roman numerals	Hindu-Arabic
I	I	1
IIIII	V	5
∩	X	10
∞∞∞	L	50
𓆼	C	100
𓆼𓆼𓆼	D	500
𓆼𓆼𓆼 ∩∩∩∩∩ IIIII	CCCLXVI	366

Egyptian Hieroglyphic system (3000BC) had separate symbols for ten, hundred, thousand, ten thousand, a hundred thousand and a million. The Romans (3000 years later) based their numeration system on a similar system including additional symbols for 5, 50 and 500. The Hindu Arabic system (used today) uses fewer symbols and is based on the place value system.

We know the value of the digit 2 based on where it is written i.e. 200, 20 or 2. The Roman system would record this as CC, XX or II. In our Hindu-Arabic place value system, all numbers can be represented using a finite set of digits: 0,1,2,3,4,5,6,7,8,9.

The system uses ten as a base (ten fingers to count).

To read and understand Roman numerals.

1	4	7	10
ONE	FOUR	SEVEN	TEN
I	IIII	VII	X
2	5	8	11
TWO	FIVE	EIGHT	ELEVEN
II	V	VIII	XI
3	6	9	12
THREE	SIX	NINE	TWELVE
III	VI	IX	XII

L = 50

C = 100

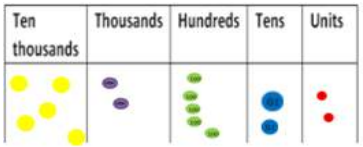
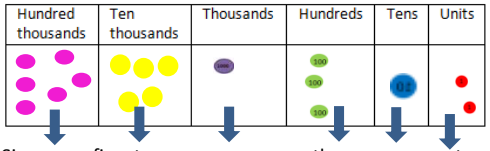
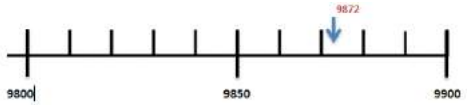
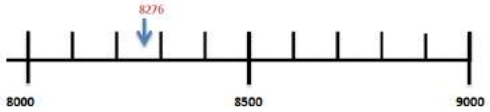
D= 500

M=1000

MMXV= 2015

Year 5 – Number and place value **(When planning ensure you track back to year 4 and forwards to year 6)**

National Curriculum	Notes and guidance (non-statutory)
Read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit. Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000. Interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers including through zero. Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000. Solve number problems and practical problems that involve all of the above. Read Roman numerals to 1000 (M) and recognise years written in Roman numerals.	Pupils identify the place value in large whole numbers They continue to use number in context, including measurement. Pupils extend and apply their understanding of the number system to the decimal numbers and fractions they have met so far. They should recognise and describe linear number sequences (e.g. 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$ …), including those involving fractions and decimals, and find the term-to-term rule in words (for example, add $\frac{1}{2}$)
Key vocab: numeral, place value, order, round, stands for, represents, regroup, >, greater than, <, less than, integer, positive, negative, above, below, zero, minus, next, consecutive, sort, classify, property, divisibility. **Key concepts** If we need to work in the concrete to consolidate our understanding we can use place value counters. We work from left to right when determining the place value. We increase the powers of ten as we move from right to left. 10 thousands = 1 ten thousands When counting in steps of powers of 10, we are multiplying by ten which changes the place value. The context for rounding is the most important element. Are pupils rounding up when buying packets of tiles for the floor (so as not be short of tiles) or are they rounding back to the nearest 5 minutes when catching a train (so as not to miss it). The number line and use of ordinal numbers are useful when introducing the concept of negative numbers. To associate positive and negative integers the number line can be shown both horizontally and vertically.	**Learning objectives** (see overleaf for exemplification) To represent 6 digit numbers (to 1 000 000) (concrete- place value counters). To recognise the place value of each digit in a six digit number. To compare & order numbers to at least 1 000 000 To recognise and describe linear number sequences. To find the term-to-term rule To interpret negative numbers To round numbers to the nearest 10, 100, 1000, 10 000 and 100 000 (To round appropriately in context see division strand) To count in steps of powers of 10 up to 1 000 000 Read Roman numerals (See progression year 4) To understand decimals and fractions (see strands on decimals and fractions).
Potential barriers/misconceptions As the numbers increase, pupils find it difficult to read numbers aloud. When counting in 1000s pupils sometimes unsure what comes after 9000 (10 000). Also what comes before and after this number (9999, 10 001) Pupils do not make the link between these numbers and real life contexts. (Could be linked to populations of countries, costs of items etc.). Pupils counting in powers of ten are often 'ill advisedly' told that they need to 'add a naught'. Children must observe the transformation of numbers as the digits move into new place on the place value grid. -5 can also be referred to as 'negative 5' as 'minus 5' can suggest the need for subtraction.	**Mental maths** To count in steps of powers of 10 up to 1 000 000: 10, 100, 1000 etc (see exemplification over page). Count forwards and backwards with positive and negative whole numbers including through zero. Know the value of every digit in six digit+ numbers. To compare two numbers (which is less 4 thousands or 41 hundreds). To make the biggest/ smallest integer possible with a range of digits (i.e. 8 3 0 7 6 0 2). To know 1000, 10,000, 100,000 more/less than any six digit number. To multiply any number by 10, 100 and 1000 (and explain how the place value changes). To divide any number by 10, 100 and 1000 (and explain how the place value changes). To multiply decimal numbers by 10, 100 and 1000 (and explain how the place value changes). To divide decimal numbers by 10, 100 and 1000 (and explain how the place value changes). To identify the number that sits halfway between two numbers. (i.e. 27,400 and 28,00) To place six digit numbers in ascending and descending order. To look at a quantity (i.e. coins in a jar, grapes in a bowl) and make a reasonable estimate. To round any two, three, four digit number to the nearest 10, 100, 1000. To round measures. (i.e. distance between cities to the nearest km) To identify the best approximation. To calculate the rise and fall in temperature using both positive and negative integers.
Example Questions □ and Δ each stand for a different number. What is their value? □ = 34 □ + □ = Δ + Δ + □ Here are four digit cards. '7' '5' '2' '1' Choose two cards each time to make the following two-digit numbers. The first one is done for you: An even number - 52. A multiple of 9 - _ _ a square number - _ _ a factor of 96 - _ _ In the number 15083, what does the 5 represent? Thousands, hundreds, tens, ones. A car costs more than £8400 but less than £9200. Tick the prices than the car could cost: £8397 £9190 £9230 £8999 Round each number to the nearest whole number: 5.01 8.51 6.65 James has 84 stamps. Emily has 57 stamps. Which of these is the BEST way to estimate how many stamps there are altogether: 90 + 60 = 150 80 + 60 = 140 80 + 50 = 130 What does the digit 3 in 305 642 represent?	

Year 5 – Number and place value	Progression (a combination of these models and images can be used for every objective)		
To represent 5 & 6 digit numbers (to 1 000 000) (concrete- place value counters).	**To recognise the place value of each digit in a six digit number.**	**To compare & order numbers to at least 1 000 000**	**To round numbers to the nearest 10, 100, 1000, 10 000 and 100 000**
 Fifty-two thousand, five hundred and twenty-two Children should be able to read numbers when given as digits and write numbers when given as words.	 In 651,312= six hundred and fifty-one thousand, three hundred and twelve. 651,312 = 600,000 + 50,000 + 1000 + 300 + 10 + 2	When comparing numbers look at the value of each of the digits starting from the left. Arrange these numbers in order. Begin with the smallest.16476, 14259,14601 **First compare the Ten thousands** 16476 is greater than 14259 & 14601. 14601 is greater than 14259. **In order from smallest: 14259, 14601, 16476.**	We round off numbers so that we are able to estimate. We use the approximation sign ≈ to stand for **approximately equal to**. It shows rounding off of the numbers. **Round off to the nearest 100**  What is 9872 rounded off to the nearest hundred? 9872 is between **9800** and **9900** 9872 is nearer to **9900** than **9800** 9872 is **9900** when rounded off to the nearest hundred. 9872 ≈ 9900
To recognise and describe linear number sequences.	**To find the term-to-term rule**	**To interpret negative numbers**	
Use subtraction to identify if sequence is: Increase or decrease **755,482** **705,482** **655, 482** **605,482** ☐ **70**5,482 is **50,000** less than **75**5,482 **65**5, 482 is **50,000** less than **70**5,482 **60**5,482 is **50,000** less than **65**5, 482 50,000 less than **60**5,482 is **55**0,482 The next number is **550,482**	'To explain rule for a sequence and work out what comes next.' | Term	Term-to-term rule	Sequence | |---|---|---| | a) 1st term = 3 | Add 7 | 3, 10, 17, 24, 31 | | b) 3rd term = 16 | Subtract 9 | -2, 7, 16, 25, 34 | | c) 5th term = 3 | Divide by 2 | 48, 24, 12, 6, 3 | | d) 1st term = 5 | Multiply by 2, add 1 | 5, 11, 23, 47, 95 | | e) 2nd term = 2 | Multiply by 4, divide by 2 | 1, 2, 4, 8, 16 | | f) 4th term = 202 | Multiply by 3, add 1 | 7, 22, 67, 202, 607 |	Familiar contexts such as temperatures, multi-story buildings, heights above and below sea level and bank balances can give meaning to positive and negative integers.  	**Round off to the nearest 100**  What is 8276 rounded off to the nearest thousand? 8276 is between **8000** and **9000** 8276 is nearer to **8000** than **9000** 8276 is **8000** when rounded off to the nearest thousand. 8276 ≈ 8000

To count in steps of powers of 10 up to 1 000 000

Larger whole numbers than 9 are constructed using powers of the base: ten, a hundred, a thousand etc.

Name of power	Numerals	Constructed from tens	Expressed as powers of ten in symbols	Expressed as powers of ten in words
A million	1000000	10x10x10x10x10x10 =	10^6	Ten to the power six
A hundred thousand	100000	10x10x10x10x10=	10^5	Ten to the power five
Ten thousand	10000	10x10x10x10x=	10^4	Ten to the power four
A thousand	1000	x10x10x10=	10^3	Ten to the power three
A hundred	100	10x10=	10^2	Ten to the power two
Ten	10	10=	10^1	Ten to the power one

Year 6 – Number and place value **(When planning ensure you track back to year 5 for progression)**

National Curriculum	Notes and guidance (non-statutory)
Read, write, order and compare numbers up to 10 000 000 and determine the value of each digit. Round any whole number to a required degree of accuracy Use negative numbers in context, and calculate intervals across zero. Solve number and practical problems that involve all of the above.	Pupils use the whole number system, including saying, reading and writing numbers accurately.
Key vocab: numeral, place value, order, round, stands for, represents, regroup, >, greater than, <, less than, integer, positive, negative, above, below, zero, minus, next, consecutive, sort, classify, property, factor, factorise, square, prime **Key concepts** The decimal point is used as the separator in the contexts of money and measurement. When counting in steps of powers of 10, we are multiplying by ten which changes the place value. When rounding numbers we talk about 'rounding on' and 'rounding back' to link to the number line/counting stick rather than rounding up and down. The context for rounding is the most important element. Are pupils rounding up when buying packets of tiles for the floor (so as not be short of tiles) or are they rounding back to the nearest 5 minutes when catching a train (so as not to miss it)?	**Learning objectives** (see overleaf for exemplification) To consolidate learning objectives from year 5 Then: To understand the place value of 7 digit numbers To identify negative integers. To calculate intervals across zero. To find the term-to-term rule To order and compare numbers up to 10 000 000 To round any whole number (To round appropriately given context see division strand) To extend place value to decimals To identify decimal numbers on a number line
Potential barriers/misconceptions Pupils counting in powers of ten are often 'ill advisedly' told that they need to 'add a naught'. Children must observe the transformation of numbers as the digits move into new place on the place value grid. Some pupils may still write three hundred and forty seven as 30047, showing a lack of understanding around zero acting as a place holder. When looking at the number 300, the position of the three is what makes is 300, rather than the zeros. The function of the zero is to make this position clear and to signify no tens and no ones. Pupils may hear 'tens' and 'hundreds' if the wording is not articulated clearly when saying 'tenths' and 'hundredths'.	
Example Questions Imagine you have 25 beads. You have to make a three-digit number on an abacus. You must use all 25 beads for each number you make. How many different three-digit numbers can you make? Write them in order. Here are some digit cards: '2' '4' '6' '6' Write all the three digit numbers, greater than 500, that can be made using these cards. Tariq makes a sequence of 5 numbers. The first number is 2. The last number is 18. His rule is to add the same amount each time. Write the missing numbers: 2 _ _ _ 18 Sarah is working with whole numbers. She says: "If you add 2 two-digit numbers you cannot get a four-digit number". Is she correct? Explain why. The temperatures were: Inside: -2°C Outside: - 10°C What is the difference between these two temperatures? The temperature inside an aeroplane is 20°C. The temperature outside is - 30°C. What is the difference between these temperatures? Round each decimal to the nearest whole number: 5.01 8.51 7.75 Write half a million in figures. Write 2 and a half million in figures. Which two of these numbers when multiplied together have the answer closest to 70? 7.4 8.1 9.4 10 Write a decimal which is greater than 0.7 and less than 0.71 Write these numbers in order of size. Starting with the smallest: 1.01 1.001 1.101 0.11 Write down a multiple of 4 that is greater than one thousand.	**Mental maths** To count in multiples of any number up to x12 forwards and backwards from any given number. To count in steps of powers of 10 up to 1 000 000 (see exemplification year 5) To count in 11s, 15s, 19s, 21s, 25s then back. Can you go past zero? To count in steps of 0.1, 0.5, 0.25 to 10 then back. To multiply and divide whole numbers by 10, 100, 1000 To multiply and divide decimal numbers by 10, 100 and 1000 Count forwards and backwards with positive and negative whole numbers including through zero. Know the value of every digit in six digit+ numbers. To compare two numbers (which is less 4 thousands or 41 hundreds?). To make the biggest/ smallest integer possible with a range of digits (i.e. 8 3 0 7 6 0 2). To know 1000, 10,000, 100,000 more/less than any six digit number. To round any whole number to the nearest multiple of 10, 100 or 1000 To put integers in order from smallest to largest crossing zero. (-37, 4, 29, -4, -28) To make statements about identification of odd and even numbers. To find all the prime factors of any number to 1000 (the prime factors of 60 are 2,2,3 and 5, since 60= 2x30 = 2x2x15 = 2x2x3x5.) Use factors for finding products mentally (32x24 = 32 x 3 x 8 = 96 x 8 = 800 – (4 x 8) = 768 Identify numbers with an odd number of factors (squares) Identify two digit numbers with only two factors (primes) Recognise prime numbers.

Year 6 – Number and place value — **Progression (a combination of these models and images can be used for every objective)**

To understand the place value of 7 digit numbers.

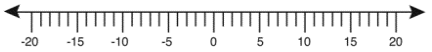
Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
1	6	4	9	0	0	0

One million, six hundred and forty-nine thousand.

In **1,649,000**
The digit 1 stands for 1,000,000. The value of the digit 1 is one million
The digit 6 stands for 600,000. The value of the digit 6 is six hundred thousand
The digit 4 is in the ten thousands place. Forty thousand. (40,000)
The digit 9 is in the thousands place. Nine thousand (9,000)

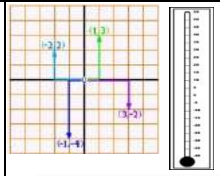
To identify negative integers.

Concrete apparatus cannot be used to teach negative integers.



Positive and negative integers are seen as reflections in zero.
+3 to be referred to as 'positive three'.
-3 to be referred to as 'negative three'.

To calculate intervals across zero.



Visual representations can be used to calculate the difference between positive and negative integers.

To find the term-to-term rule

Term	Term-to-term rule	Sequence
a) 1st term = 3	Add 7	3, 10, 17, 24, 31
b) 3rd term = 16	Subtract 9	-2, 7, 16, 25, 34
c) 5th term = 3	Divide by 2	48, 24, 12, 6, 3
d) 1st term = 5	Multiply by 2, add 1	5, 11, 23, 47, 95
e) 2nd term = 2	Multiply by 4, divide by 2	1, 2, 4, 8, 16
f) 4th term = 202	Multiply by 3, add 1	7, 22, 67, 202, 607

To order and compare numbers up to 10 000 000

When we compare numbers, we look at the value of each digit starting from the left.

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
1	7	1	2	9	3	5
1	7	1	2	8	4	6

9 hundreds is greater than 8 hundreds.
So 1,712,935 is greater than 1,712,846

To round any whole number

We round off numbers so that we are able to **estimate**.
We use the approximation sign ≈ to stand for **approximately equal to**. It shows rounding off of numbers.

Estimate the value of 6327 x 7 (round off the 4 digit number to the nearest thousand first)
6327 x 7 ≈ 6000 x 7
= 42000

Estimate the value of 6742 ÷ 8 (6742 ÷ 8 → 6400 ÷ 8, 7200 ÷ 8 using knowledge of x8 tables)

6742 is nearer to 6400 than to 7200. So 6742 ÷ 8 ≈ 6400 ÷ 8 = 800.

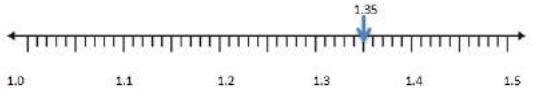
To extend place value to decimals

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
1000	100	10	1	0.1	0.01	0.001
10 x 10 x 10	10 x 10	10	1	$\frac{1}{10}$	$\frac{1}{10x10}$	$\frac{1}{10x10x10}$
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

To identify decimal numbers on a number line

1.35 can be explained in the context of length as 1 metre + 3 tenths of a metre + 5 centimetres

On the number line it lies between 1 and 2
It lies between 1.3 and 1.4
It lies between 1.34 and 1.36.



Chapter 2

Addition

EYFS 1 – Addition (When planning ensure you track forwards to year 1)

Early Learning Goal 11
Children count reliably with numbers from 1 to 20 place them in order (see number and place value)
Say which number is one more or one less than a given number
Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.

Key vocab: add, more, and, make, sum, total, altogether, score, double, one more, two more, how many more?

Key concepts

Adding is the combining of a set of objects.
Know that a group of things changes in quantity when something is added (or taken away).
More than means 'added on to'.
The number of objects does not change even if moved around (Conservation).
We know that when counting a group, the last number represents the total quantity.

Potential barriers/ misconceptions

Unable to recite numbers in the correct order.
Not associating number names with objects in group.
Unable to count without putting in line or touching.
Not being able to 'hold' the number they started with when adding the second group.
Not knowing the number order when counting on from any given number.

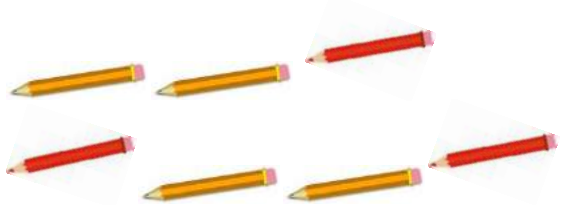
Learning objectives (turn over for exemplification)

To make comparisons between quantities.
To use language of quantities such as 'more' and 'a lot'.
To use the language of more to compare sets of objects.
To separate a group of 3 or 4 objects in different ways (total still same).
To respond to (and use) addition vocabulary in rhymes and games.
To find the total number of items in two groups by counting all of them.
To know that when counting a group the last number represents the total amount.
To find one more than a given number.

Example Questions

I am going to add one more button to this set of four buttons. How many buttons will there be then?

Show me five fingers using both your hands. Show me another way.
There are four balls in the bag. Put two more balls in the bag. How many balls all together are in the bag?
One, two, three... what goes next?
Hop two spaces on this number track. Now hop three more. Where are you now?
I have two carrots in a bowl. I add two more carrots to the bowl. How many carrots are in the bowl now?
Find all the dominoes that have a total of five spots.
Show me three fingers on one hand. Show me five fingers on the other hand. How many fingers altogether?
I have hidden two cups in this box. There are three cups on the table. How many cups are there altogether?
How many different ways can we put five sweets on two plates?
How many yellow pencils are there? How many red pencils are there? How many pencils are there altogether?



Mental Maths (can revisited throughout day once concept has explicitly shared)

Use of counting stick to count to ten

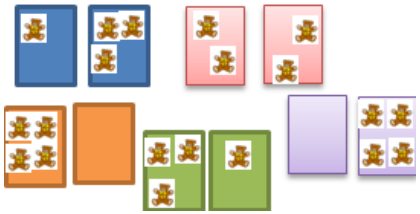
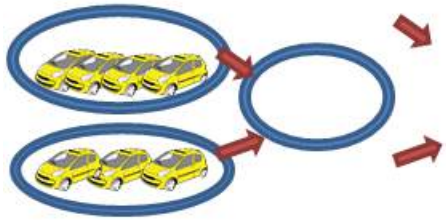
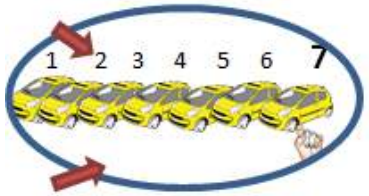
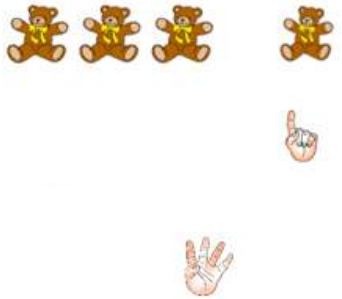
Use of counting stick to count on from any one digit number

Counting backwards from 10 along the counting stick

Use of songs/rhymes to add

Mathematical experiences should be based on real life experiences (who has more marbles?)

Dice games

EYFS 1 – Addition Progression (a combination of these models and images can be used for every objective)			
To make comparisons between quantities	**To use language of quantities such as 'more' and 'a lot'.**	**To use the language of 'more' to compare sets of objects.**	**To separate a group of 3 or 4 objects in different ways (total still same).**
Which group of sweets would you like? Why? 	Please may I have some more milk?  I have a lot of conkers. 	Isaac has more blocks than me.  My blocks Isaac's blocks	How many different ways can we put four teddies in two beds? 
To respond to (and use) addition vocabulary in rhymes and games	**To find the total number of items in two groups by counting all of them.**	**To know that when counting a group the last number represents the total amount.**	**To find one more than a given number**
Elephant song One elephant came out to play, Upon a spider's web one day, He found it such enormous fun, That he called another elephant to come  Two elephants went out to play.... etc  **Play** Can you put one more fish in the water tray? How many are there now? 	 Three paper plates can be used to represent part, part whole. Children move the cars together into one group to find the total amount. (starting with 0-5)	 There are seven in the group.	One more than three is four 

EYFS 2 – **Addition** (When planning ensure you track forwards to year 1)

Early Learning Goal 11

Children count reliably with numbers from 1 to 20, place them in order
Say which number is one more or one less than a given number
Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.

KS1 ready:

Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.
Represent and use number bonds and related subtraction facts within 10
Add and subtract one-digit and two-digit numbers to 20 including zero
Solve one step problems that involve addition and subtraction, using concrete objects and pictorial representations , and missing number problems such as 7 = ☐ - 9

Key vocab: add, more, and, make, sum, total, altogether, score, double, one more, two more, how many more?

Key concepts

Adding is the combining of a set of objects
Adding is the combining of two numbers
When we add numbers we can break them into 'bits' to make it easier. Part, part, whole: 5 + 1 = 6
Know that a group of things changes in quantity when something is added (or taken away).
We count on from the bigger number.
More than means 'added on to'
The number of objects does not change even if moved around (Conservation)
We know that when counting a group, the last number represents the total quantity. (After = sign)

Potential barriers/misconceptions

Unable to recite numbers in the correct order.
Not associating number names with objects in group
Unable to count without putting in line or touching.
Not being able to 'hold' the number they started with when adding the second group.
Not knowing the number order when counting on from any given number.

Example Questions

Flora has 4 stickers and Romeo has 2 stickers. How many do they have altogether? (combining 2 groups)
Sidney has three stickers and he is given two more. How many stickers does he have now? (Augmenting)
There are five children in the home corner. One more joins. How many are there now?
Jordine has no carrots on her plate. Marvin puts four carrots on her plate. How many carrots does Jordine have now?
What number is one more than five (use the objects to help).
Take away one egg from the set of seven below. How many are there now?

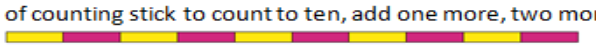
[Make a set of cards with jackets on. Arrange them randomly on a table.] Find two jackets that have four buttons altogether. Are there any other possibilities?
[Put four balls into a bag and then close it. Show two more balls in your hand]. How many balls are there altogether?

Learning objectives (turn over for exemplification)

To say the number that is one more than a given number.
To combine two or more quantities to find the total (combining)
To find one more than a group of up to five, then ten objects.
Uses quantities and objects to add two single-digit numbers and count on to find the answer:
To count on when adding to a group. (holding first number in head)
To add two sets of objects that are the same (cars+ cars)
To add two sets of objects that are different (apples + bananas)
Increase one quantity by a given amount to find the total (augmentation)
Records using marks they can interpret and explain
To recognise and name + and = signs
To read an addition number sentence
To solve an addition number sentence
To arrange an addition number sentence
To know doubles to 10.
To know number bonds to 5, 6 and 10

Mental Maths (can revisited throughout day once concept has explicitly shared)

Use of counting stick to count to ten, add one more, two more, etc



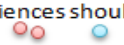
Use of counting stick to count within 20 from any given number.

8 9

Rapid recall of doubles

Use of songs/rhymes to calculate one more/less (speckled frogs)

Mathematical experiences should be based on real life experiences (who has more marbles?)



Dice games

EYFS 2 – Addition

Progression (a combination of these models and images can be used for every objective)

To say the number that is one more than a given number.

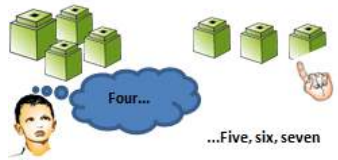
Combine two or more quantities to find the total (combining)

Maria has five sweets and Ismail has 3 sweets. How many sweets do they have altogether? (grouped)

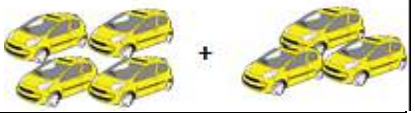
To find one more than a group of up to five, then ten objects.

One more than five is six

To count on when adding to a group. (holding first number in head)



To add two sets of objects that are the same (cars+ cars)



To add two sets of objects that are different (apples + bananas)

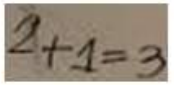
Increase one quantity by a given amount to find the total (augmentation) (where one quantity is increased by some amount)

Maria has five sweets and she is given 3 more.
How many does she have in total? (increase)

Children count 5 into bag. (hidden)
Then count on from 5. (6,7,**8**)

There are 8 sweets in the bag.

Records using marks they can interpret and explain



To recognise and name + and = signs

Where is the addition sign?

+

How do we know to add?

+

Where is the total?

=

What does this sign mean?

=

To read an addition number sentence

To read aloud:

4 + 3 = 7 Knowing that + is add

= is equals

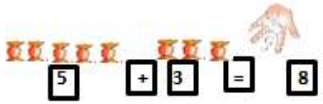
To solve an addition number sentence

To use objects to lay out & solve:

4 + 3 = 7

To arrange an addition number sentence

To match number cards to objects to make number sentence:



To know doubles to ten

1 + 1 = 2

2 + 2 = 4

3 + 3 = 6

4 + 4 = 8

5 + 5 = 10

To know number bonds to 5, 6 and 10

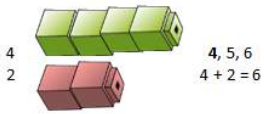
Part, part whole: number bonds to 5.

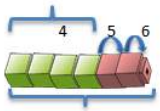
Progression towards bar model

Children start by adding objects to a group

What is two more than 4?

Children can then use unifix cubes, counting on from the greater number, to find the total number of cubes.





Year 1 – Addition (When planning ensure you track back to Reception and forwards to year 2)

National Curriculum

Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.
Represent and use number bonds and related subtraction facts within 20.
Add and subtract one-digit and two-digit numbers to 20 including zero
Solve one step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = □ - 9

Notes and guidance (non-statutory)

Pupils memorise and reason with number bonds to 10 and 20 in several forms (for example, 9+7=16; 16-7=9, 7=16-9)
They should realise the effect of adding or subtracting zero. This establishes addition and subtraction as related operations. Pupils combine and increase numbers, counting forwards and backwards.
They discuss and solve problems in familiar contexts, including using quantities. Problems should include the terms: put together, add, altogether, total, take away, distance between, more than and less than so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.

Key vocab: add, more, plus, make, sum, total, altogether, score, double, near double, one more, two more, ten more, how many more to make…?, how many more is… than…? How much more is…?

Key concepts

Relate addition to counting on
Adding two or more numbers gives another number.
A group of things changes in quantity when something is added.
When adding a 1 digit number to a 2 digit number- make ten i.e. 8+4 = 8+2+2= 12
When I add two objects to a group, and then take two objects away the total is unchanged.

Potential barriers/misconceptions

- Learn the pairs of numbers to 10 and 20 but not the pairs that total each number up to 20.
- Don't understand the commutativity of 3+7 = 7+ 3
- Don't associate number facts e.g 13+4=17 and 17-4=13 as they don't see + and – as inverse.
- Only able to complete empty box questions when on right hand side (answer) rather than any posn. 3+ □ = 8
-When counting on from a given number, include the start number in their counting. (6+3 = 6,7,8 = 8) rather than 6+3= 7,8,9 = 9)
- count on and back in 10s and 1s not combining i.e. when add 9, add 10 and subtract 1. (adjust)

Learning objectives (turn over for exemplification)

To add with number bonds within 10
To know all number bonds to 10
To add with number bonds to 20
To investigate all possible sets of two numbers to make a given number.
To partition numbers into part, part, whole.
To use a number line to count on.
To use inverse (write corresponding subtraction facts to given addition facts- number families).
To add two 1 digit numbers using 'make 10' strategy
To add a 1 digit and a 2 digit number using the 'regrouping into tens and ones' strategy.
To solve one step word problems using 'part-whole' or adding on concept.

Example Questions

What numbers go in the boxes: 12 + 5 =□ 12+ □ =15
□ + 5 =17 □ +□ = 15
I think of a number. I add 5 the answer is 9. What's my number?
What is 18 plus 5? What is the sum of 16 and 8? What is 15 more than 4?
How many different ways can I put 8 fish into 2 fish bowls?
On a number line I show 7 + 5. I start at 7 and do 5 jumps. What if I started at 5? How many jumps would I need to do? Would it be the same answer? Why?
How can you use the part, part whole model to make 12? How many ways can you make 12?

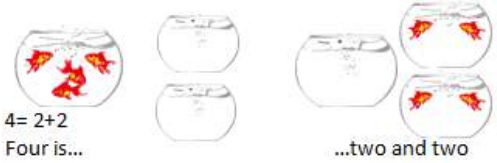
Mental Maths

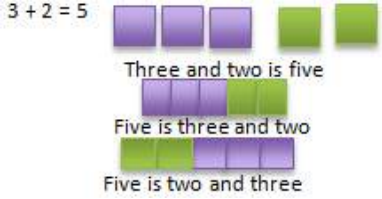
To add multiples of ten
To add ten to any two digit number by counting in 10s
Bridge through ten (and 20 etc) when adding a single digit number. (Making ten). i.e. 8 + 6 = 8 + 2 + 4 = 14
Count on from the largest number
Rapid recall of number bonds
Use of near doubles to add (6 + 7 = 6 + 6+ 1 = 13)
Add 9 to a single digit number by adding 10 and subtracting 1 (adjust)

Year 1 – Addition

Progression (a combination of these models and images can be used for every objective)

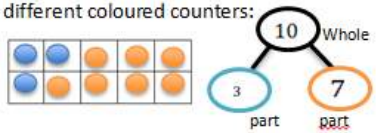
To add with number bonds within 10





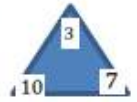
To know all number bonds to 10

Use of ten frame with different coloured counters:



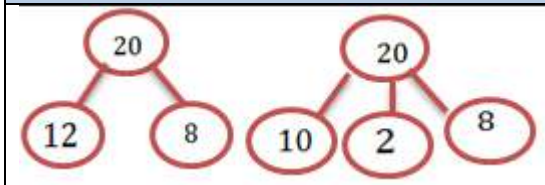
To use the inverse

3 + 7 = 10
7 + 3 = 10
10-7=3
10-3=7

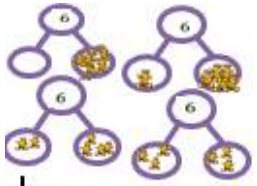


Number families

To add with number bonds to 20



To investigate all possible sets of two numbers to make a given number



0 + 6 = 6
1 + 5 = 6
2 + 4 = 6
3 + 3 = 6

(and understand commutative law i.e. 1+5=6, 5+1 = 6)

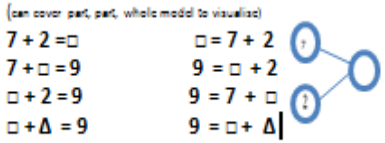
Solve missing number problems

+ and = signs and missing numbers:

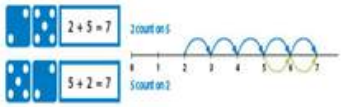
(can cover part, part, whole model to visualise)

7 + 2 = □
7 + □ = 9
□ + 2 = 9
□ + Δ = 9

□ = 7 + 2
9 = □ + 2
9 = 7 + □
9 = □ + Δ

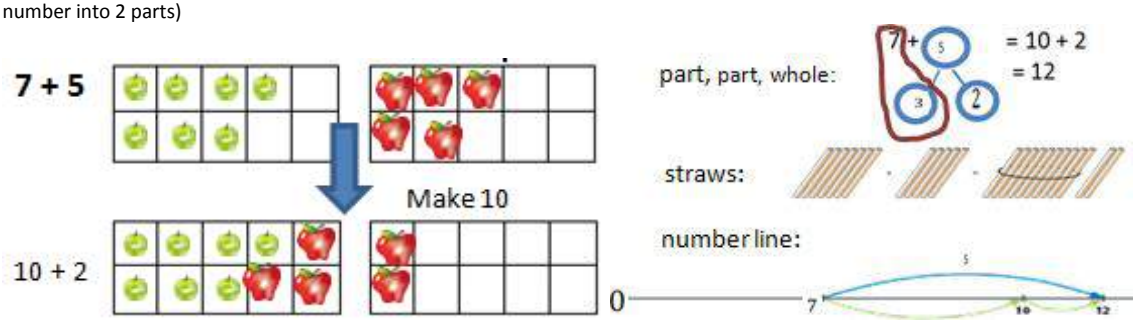


To use a number line to count on.



To add a 1 digit number using 'make 10' strategy (bridging through multiples of 10)

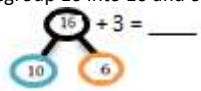
When performing addition within 20 children move towards number conservation. The total is unchanged when objects/counters are moved from the second frame to 'make ten'. Knowledge of number bonds to 10 is important. (this involves breaking the smaller number into 2 parts)



To add a 1 digit and a 2 digit number using the 'regrouping into tens and ones' strategy.

Pashur has made 16 muffins. His sister has made 3. How many altogether?

Regroup 16 into 10 and ones.

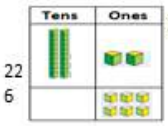


1)Regroup into tens and ones
2)Add the ones: 6+3 =9
3)Add the ten: 10 + 9 = 19 the same as 16 + 3 = 19
There are 19 muffins altogether

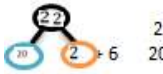
Vertical addition with no regrouping. (2 digit + 1 digit not bridging 10)

Using a place value chart & Dienes.

22 + 6 =



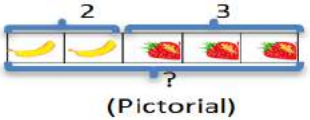
1)Add the ones
2 ones + 6 ones = 8 ones
2) Then add the tens
2 tens +0 tens = 2 tens



Tens	Ones
2	2
+	6
2 + 6 = 8	8
20 + 0 =20	2 0

20 + 8 = 28 **2 8**

Progression towards bar model



1) **Combining** (I have 2 bananas and 3 strawberries. How much fruit do I have?)
2) **Augmenting** (I have 2 bananas and I buy 3 strawberries. How much fruit do I have?)

Year 2 – Addition (When planning ensure you track back to Year 1 and forwards to year 3)

National Curriculum

Solve problems with addition and subtraction: -Using concrete and pictorial representations, including those involving numbers, quantities and measures. -Applying their increasing knowledge of mental and written methods.

Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100

Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:

- **A two digit number and ones**
- **A two digit number and tens**
- **Add two two-digit numbers**
- **Adding three one digit numbers**

Show that the addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot

Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

Notes and guidance (non-statutory)

Pupils extend their understanding of the language of addition and subtraction to include sum and difference.

Pupils practice addition and subtraction to 20 to become increasingly fluent in deriving facts such as using 3+7=10; 10-7=3 and 7=10-3 to calculate 30+70=100; 100-70=30 and 70=100-30. They check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (e.g. 5+2+1=1+5+2= 1+2+5) This establishes commutativity and associativity of addition.

Recording addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers.

Key vocab: add, addition more, plus, make, sum, total, altogether, score, double, near double, one more, two more, ten more, one hundred more, how many more to make…?, how many more is… than…? How much more is…?

Key concepts

Adding can be done in any order.
Counting on from the biggest number is not always most useful way: 8 + 5 + 5 (make 10 first).
To add 2 digit numbers we partition.
We add the ones first and then the tens.

Potential barriers/misconceptions

Pupils believe they have to add in the order that the question was asked (not understanding that addition can be done in any order to do mental calculations more efficiently).
Still don't have secure rapid recall of addition facts. i.e. struggle to identify all possible missing numbers in _ + _ = 7. (Number bonds).
Makes mistakes counting teen numbers or crossing boundaries.
Is insecure in making links between addition and subtraction and/or recognising inverse.
In vertical addition- placing the answer in the wrong column. i.e. 24 as 2 in the ones, 4 in tens.

Learning objectives (turn over for exemplification)

To use the counting on strategy (with number line, Dienes or mentally)
To use making ten strategy to add (see y1 progression)
To use partitioning to add
To add a two digit number and tens
To add a two digit number and ones without regrouping
To add 2 two-digit numbers without regrouping
To regroup and rename
To add three one-digit numbers
To add numbers with regrouping (in ones)
To add numbers with regrouping (in tens.)
Use the inverse to solve missing number problems
To solve one step word problems using 'part, whole' and adding on.

Example Questions

Addition questions phrased in a variety of ways:
64+ 10 Add 60 to 17 24 plus 36
What is the sum/ total of 18 and 7?
How many are 5 and 19 altogether?
Which two/three numbers could have a sum of 12?
What must I add to 26 to make 30?
I think of a number. I add 20. My answer is 50. What is my number?
Andre has 37 football cards. He buys 30 more. How many does he have now?

Mental Maths

Counting forwards/ backwards from any given number
Rapid recall of all addition facts to 20 & 100
Partitioning adding the ones and then the tens: 24+13 = 4+3+ 20+10=37
'Make ten' adding three one digit numbers: 6+7 = 6+4+3= 13
Compensating 24+9= 24+10-**1**=33 or 42+21= 42+20**+1**= 63 (adjust)
Near doubles: 30+29 = double 30-1 and 14+15 is double 14 + 1 or double 15-1

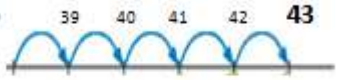
Year 2 – Addition

Progression (a combination of these models and images can be used for every objective)

To use the counting on strategy

Number line

38 + 5



38

Moving to jump of 2 then 3. (jump to next 10)

To make ten also shown in y1 progression

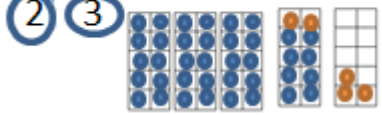
Dienes

38 +5

39,40,41,42,43

To partition

38 + 5 = 40 + 3 = 43



To add a two digit number & ten

Add tens using:

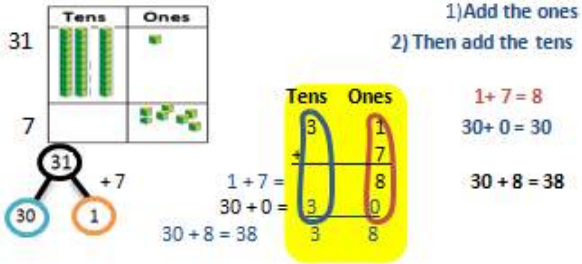
Hundred square (pattern)

Jumps of ten on a number line

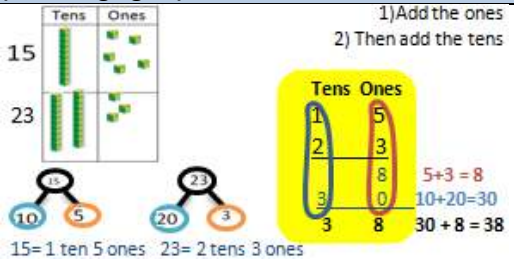
Counting stick add ten from any given number 23, 33, 43 etc.

Add 'ten' Dienes each time.

To add a two digit number and ones without regrouping. (not bridging 10) also shown in y1 progression



To add 2 two-digit numbers without regrouping (not bridging 10)



To regroup and rename

Concrete representation important.

(Could use Dienes.)

17 regrouped = **1 ten and 7 ones** (renamed)

To add three one-digit numbers

Use number bonds to make ten.

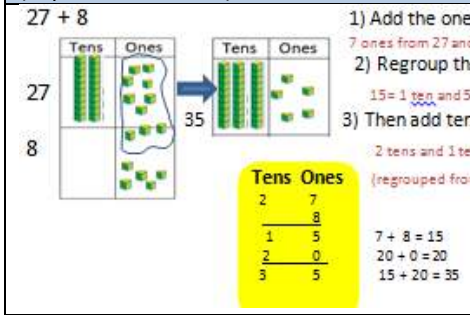
6 + 7 + 4 = 6 + 4 + 7 = 17

Or partition numbers:

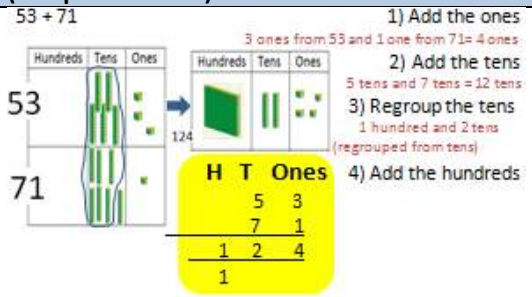
6 + 8 + 5 = 10 + 4 + 5 = 19

4 4

To add numbers regrouping in ones (bridging 10) (Expanded method)



To add numbers regrouping in tens. (bridging 100) (compact method)



Use the inverse to solve missing number problems

Understand that:

17 + 34 = 34 +17 but that

34-17 is not the same as 17 – 34

And

12 + 2 + 7 = (12 +2)+7 or 12 +(2+7)

Solve:

17 + □ = 30

□ + 13 = 30

30 - 17 = □

30 - 13= □

To solve one step word problems:

Part, Whole:

Laura has **25p** and finds another **30p**. How much does she now have?

Adding on:

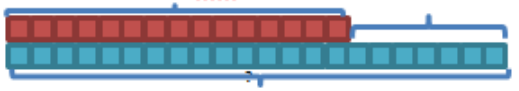
There are 68 sweets in the jar. How many more are needed to make 100?

68 + 2 + 30 = 100

Using the bar model: comparing (addition)

Sam has **10** red unifix cubes and **12** green unifix cubes. How many unifix cubes does he have altogether?

Lina has **15** unifix cubes and Nadir has **7** more. How many does Nadir have? (make with cubes)



T ones

1 5

7

2 2

1

Year 3 – Addition (When planning ensure you track back to year 2 and forwards to year 4)

National Curriculum : Add and subtract numbers mentally, including:

- **A three-digit number and ones**
- **A three- digit number and tens**
- **A three- digit number and hundreds**

Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

Estimate the answer to a calculation and use inverse operations to check answers.

Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction.

Notes and guidance (non-statutory)

Pupils practise solving varied addition a subtraction questions. For mental calculations with two-digit numbers, the answers could exceed 100.

Pupils use their understanding of place value and partitioning and practice using columnar addition and subtraction with increasingly large numbers up to three digits to become fluent.

Key vocab: add, addition more, plus, make, sum, total, altogether, score, double, near double, one more, two more, ten more, one hundred more, how many more to make…?, how many more is… than…? How much more is…? Tens boundary, hundreds boundary.

Key concepts

Addition can be done in any order
Addition is the same as: more, add, sum, total, altogether
Addition reverses subtraction (addition is the inverse of subtraction)
It is best to start adding from right to left. Add the ones, then the tens etc
We can move from the expanded method to the compact method for addition.

Potential barriers/misconceptions

Children may still not be secure with all addition facts for each number to 20.
Confused that addition is associative- 3+1 = 4 and 1+3=4.
Find it challenging to mentally add using 'near multiples of 10'. Not sure about which way to compensate: 26+19= 26 +20 – 1 often confused as 36 + 20 + 1.
Sometimes begin adding with the left hand column first
Not understand the concept of regrouping when the number totals more than ten, hundred etc.
Children find it difficult to add when there is a zero involved.
Children don't understand importance of zero as a placeholder.

Example Questions

Addition questions phrased in a variety of ways:
194 add 10, Add 60 to 280, 70 plus 50.
What is the sum/total of 26 and 39?
How many altogether are 121 and 345?
Increase 431 by 22.
Which two numbers could have a total of 102? Which three numbers?
There are 25 people standing on the bus and 62 sitting down. How many people in total?
Ali has 298 football stickers. He collects another 121. How many does he have now?
Adam has read 173 pages. He has 62 left to read. How many pages in total?

Learning objectives (turn over for exemplification)

To use partitioning to add
To use a number line for addition
To solve missing number problems
To add a three digit number and ones without regrouping (see progression year2)
To add a three digit number and tens without regrouping (see progression year2)
To add 2 three-digit numbers without regrouping
To add three-digit numbers with regrouping (revert to expanded method if tricky)
To add using place value counters
To develop and recognise patterns in addition
To estimate the answer to a calculation
To solve word problems

Mental Maths

Rapid recall of all addition facts up to and including 20
Derive quickly addition doubles from 1+1 to 20+20 e.g. 19+19=38
Doubles of multiples of 5 from 5+5 to 100+100 e.g. 95+95 = 190
Derive quickly pairs of multiples of 5 that total 100: e.g. 65 + 35
Know by heart all multiples of 100 that total 1000: e.g. 400 + 600 = 1000
Add several nos by: making ten & adjusting when adding 11 or 9 add 10 and +1/-1.
Partition and recombine: e.g. 24 + 35 = 20 + 30 + 4 + 5 = 59
Identify the corresponding subtraction facts. e.g. 22+57 = 79 and 79-57=22 etc.
Add a two-digit number to a multiple of 100.e.g. 200+64
Add a two-digit number to a multiple of 10 crossing 100. e.g. 80 + 34 = 114
Add 10 to any number crossing the hundreds boundary. e.g. 196 + 10
Add a pair of multiples of 10, crossing 100. e.g. 90 + □ = 130
Add pairs of multiples of 100 crossing 1000. e.g. 500 + 800
Add 100 to any 3 digit number, without crossing 1000. e.g. 347 + 100 =

Year 3 – Addition

Progression (a combination of these models and images can be used for every objective)

To use partitioning to add

325 + 247

	H	T	ones
(300 + 200)	5	0	0
(20 + 40)		6	0
(5 + 7)		1	2
	5	7	2

300 + 20 + 5 + 200 + 40 + 7

300 + 60 + 12 = 572

To use a number line for addition

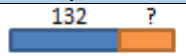
225 + 127 = 352

225 + 100 = 325; + 20 = 345; + 7 = 352

(or + 5 + 2)

To solve missing number problems

132 + □ = 200



132 + 8 = 140 + 60 = 200

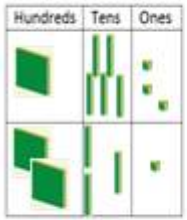
? = 68

1	3	2
	6	8
2	0	0
2	1	1

To add 2 three-digit numbers without regrouping

(see year 2 progression for further examples)

153 + 231 = 284

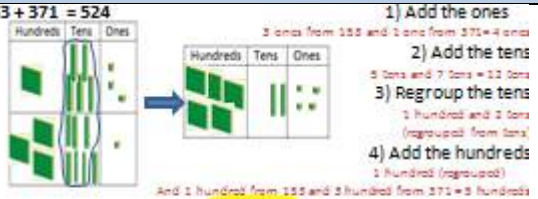


H	T	ones
1	5	3
2	3	1
3	8	4

1) Add the ones
2) Add the tens
3) Add the hundreds

To add three-digit numbers with regrouping

153 + 371 = 524



H	T	ones
1	5	3
3	7	1
5	2	4

Progression of regrouping:

Addition with no regrouping (not crossing, 10, 100 etc)

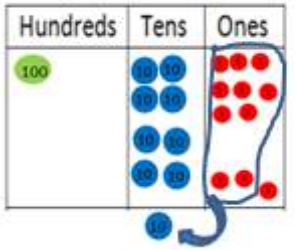
Addition regrouping in ones (crossing ten)

Addition regrouping in tens (crossing 100)

Addition regrouping in tens and ones.

To add using place value counters (moving on from Dienes)

148 + 43 = 191 (regrouping in the ones)



To develop and recognise patterns in addition

15 + 4 = 19

15 + 14 = 29

15 + 24 = 39

Deduce that:

15 + 84 = 99

5 + 8 = 13

50 + 80 = 130

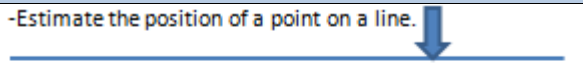
500 + 800 = 1300

Use pattern in addition table:

+	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

To estimate the answer to a calculation

-Estimate the position of a point on a line.

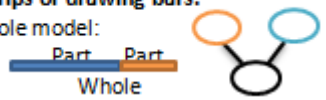


-Use rounding to estimate answer. 228 + 389 = 200 + 400= 600 (actual answer: 617)

To solve one step word problems

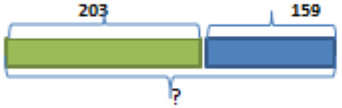
Starting with the use of unifix moving to paper strips or drawing bars.

Part, whole model:



Using the bar model: Combining sets of objects:

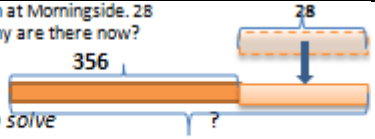
There are 203 children at Springfield and 159 at Holy Trinity. What is the sum of these numbers?



Use of column method to solve

Aggregation: joining one or more sets to another set:

There are 356 children at Morningside. 28 children join. How many are there now?



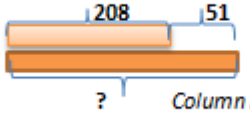
Column method to solve

Comparing two sets:

There are 208 children at St John and St James. St John the Baptist has 51 more children. How many children at St John the Baptist?

SJSJ

SJB



Column method to solve

Year 4 – Addition (When planning ensure you track back to Year 3 and forwards to year 5)

National Curriculum

Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate

Estimate and use inverse operations to check answers to a calculation

Solve addition and subtraction two step word problems in context, deciding which operations and methods to use and why.

Notes and guidance (non-statutory)

Pupils continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency.

Key vocab: add, addition more, plus, increase, make, sum, total, altogether, score, double, near double, one more, two more, ten more, one hundred more, how many more to make…?, how many more is… than…? How much more is…? Tens boundary, hundreds boundary, inverse.

Key concepts

We can use addition facts that we know to solve other additions.
We can use addition facts we know to solve additions with decimals.
Estimating can be used to predict the answer and the inverse can be used to check it.
We can use Dienes or place value counters to help solve addition problems.
Addition is:
Combining two or more quantities into one
The enlargement of a quantity, i.e. increasing the amount in the quantity.
Comparison of quantity with another: one quantity has a certain amount more than the other.
We can use a number line to round numbers to the nearest 10 or 100.

Learning objectives (turn over for exemplification)

To add four digit numbers (no regrouping)
To add with regrouping in the 100s
To add with regrouping in the 100s, 10s and 1s
To add with regrouping in the 1000s, 100s, 10s and 1s
To identify common misconceptions in column addition
To round off numbers to the nearest 10 / 100.
To estimate to check answers
To add decimals up to 2 decimal places
To solve two step word problems.

Potential barriers/misconceptions

Children sometimes begin adding with the left hand column first.
Pupils line up numbers from left to right rather than right to left. i.e. 3056 + 254:

Th	H	T	O
3	0	5	6
	2	5	4

Not understanding the concept of 'regroup' when a number totals more than ten, hundred etc
As numbers get larger pupils miscalculate because of lack of understanding of place value.
Some pupils will not remember to add the ten/hundred that they have regrouped.
Pupils don't use estimation skills to predict answer.
Lack of understanding around value of decimal numbers.
Forgetting to include or line up decimal point.

Example Questions

3964 add 30 add 500 to 9544
Which three numbers could have a total of 350?
□ + 88 = 120 7.6 + 5.8 = □
There are 654 girls. There are 276 more boys than girls. How many children altogether?
John, William and Oliver are saving their money to buy a computer game. John has £25. William has £10 more than John. Oliver has the same amount as John and William together. How much do they have?

Mental Maths

Rapid recall of all addition facts to 20. (e.g. all pairs of numbers to 15)
Derive quickly related facts: e.g. 9+6=15, 90+60=150, 900+600=1500
Derive quickly number pairs that make 100. 34 + □ = 100 , □ + 45=100
Derive pairs of multiples of 50 that total 1000: e.g. 250+750
Derive quickly addition doubles from: 1+1 to 50+50 e.g. Double 46
Multiples of 10 from 10+10 to 500+500: e.g. double 280
Multiples of 100 from 100+100 to 5000+5000: e.g. double 17000
Count on from any given number in repeated steps of 1,10,100,1000
Partition into hundreds, tens and ones to add mentally
Add or subtract the nearest multiple of 10, 100 or 1000 and adjust: add 9, 19, 29 or 11, 21, 31 to any number. e.g. 48+ 61 = 48+60+1
Identify addition and subtraction facts for any given algorithm.
Add three numbers mentally. (two digit and one digit)
Add three digit multiples of 10: e.g. 430+360 or 570+260
Find what to add to a three digit number to make the next higher multiple of 100. e.g. 246+ □ = 300
Add nos to 1 decimal place to make the next whole number. 3.4 +□ = 4.0

Year 4 – Addition — Progression (a combination of these models and images can be used for every objective)

To add four digit numbers (no regrouping)

To move from concrete representation or place value tables to support into:

4371 + 5208

	Th	H	T	Ones
	4	3	7	1
+	5	2	0	8
	9	5	7	9

Also find the sum of 4 digit + 3 digit.

To add with regrouping in the 100s

(for use of Dienes see year 3)

1700 + 2400 = 4100

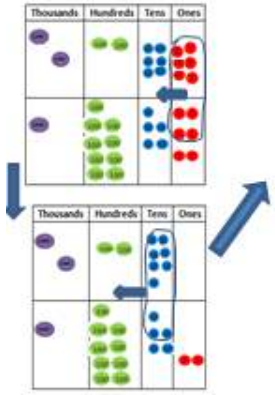
	Th	H	T	Ones
	1	7	0	0
+	2	4	0	0
	4	1	0	0
	1			

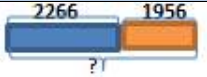
7 hundreds add 4 hundreds is 11 hundreds = 1 thousand and 1 hundred

Regrouped

To add with regrouping in the 100s, 10s and 1s (Same pattern for regrouping in 1000s etc.)

2266 + 1956 = 4222





	Th	H	T	O
	2	2	6	6
+	1	9	5	6
	4	2	2	2
	1	1	1	

Add numbers from right to left
Regroup and record below line
Understand place value of each digit

To identify common misconceptions in column addition

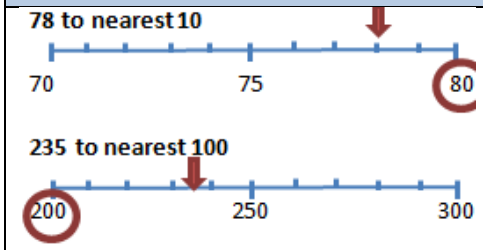
Pupils look at examples and identify mistakes.

Children articulate steps in procedure.

2266
+ 1956

1) Add 6 ones and 6 ones to get 12 ones.
2) Regroup 12 ones into 1 ten and 2 ones
3) Add 6 tens, 5 tens and 1 ten to get 12 tens.
4) Regroup 12 tens into 1 hundred and 2 tens
5) Add 2 hundreds, 9 hundreds and 1 hundred to get 12 hundreds.
6) Regroup to get 1 thousand and two hundreds.
7) Add 2 thousands 1 thousands & 1 thousand to get 4 thousands

To round off numbers to the nearest 10 / 100.



To estimate to check answers

Round numbers to the nearest 10 to check reasonable answers:

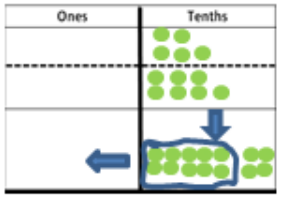
128 + 254 =
130 + 250= **380**
(Actual answer = 382)

To add decimals up to 2 decimal places

Progression:
1 decimal place no regrouping
1 decimal place regrouping
2 decimal places regrouping

0.5 + 0.7 =

0.5
+0.7



0.5
+ 0.7
1.2
1

Remember to align the decimal points!

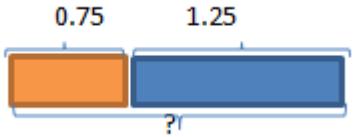
Using the bar model (to solve two step word problems)

A TV costs £1250. This TV costs £425 less than a computer. How much do both cost together?

The computer costs £1675
TV and computer cost:
£1250 +£ 1675= £2925

One step word problem with decimals:

Mark drank 0.75 litres of orange juice and 1.25 litres of water. How many litres did he drink altogether? (*Bar model to visualise and column method to solve).*



Year 5 – Addition (When planning ensure you track back to Year 4 and forwards to year 6)

National Curriculum

Add and subtract whole numbers with more than 4 digits, including formal written methods (columnar addition and subtraction)
Add and subtract numbers mentally with increasingly large numbers.
Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.
Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Notes and guidance (non-statutory)

Pupils practise using the formal written methods of columnar addition and subtraction with increasingly large numbers to aid fluency
They practise mental calculations with increasingly large numbers to aid fluency (e.g. 12,462+ 4200= 16,662)

Key vocab: add, addition more, plus, increase, make, sum, total, altogether, score, double, near double, one more, two more, ten more, one hundred more, how many more to make…?, how many more is… than…? How much more is…? Tens boundary, hundreds boundary, inverse.

Key concepts

We can use addition facts that we know to solve other additions.
We can use addition facts we know to solve additions with decimals.
Estimating can be used to predict the answer and the inverse can be used to check it.
We can use Dienes or place value counters to help solve addition problems.
Addition is:
Combining two or more quantities into one
The enlargement of a quantity, i.e. increasing the amount in the quantity.
Comparison of a quantity with another. i.e. one quantity has a certain amount more than the other.
We can use a number line to round numbers to the nearest 10 or 100.

Learning objectives (turn over for exemplification)

(for exemplification also look at year 4 progression)
To add four digit numbers **(**regrouping in the 1000s, 100s, 10s and 1s)
To identify common misconceptions in column addition
To round off numbers to the nearest 10.
To round off numbers to the nearest 100.
To add decimals up to 2 decimal places
To add money using the column method (regrouping)
To add measures using the column method (regrouping)
To find the missing value
To use part, part whole to add money
To solve two step word problems using the bar model.

Potential barriers/misconceptions

As numbers get larger, pupils miscalculate due to lack of understanding of place value.
Some pupils will not realise that they need to add the regrouped number.
Pupils sometimes forget to line up the decimal points when adding using the column method- particularly when adding mixed amounts. e.g. £4.50 + 72p

Example Questions

-Nadia is working with whole numbers. She says 'if you add a two digit number to a two digit number you cannot get a four digit number.' Is she correct? Explain why.
-□ and ○ each stand for a different number. If □ = 34 then what is the value of ○**?**
□ + □ = ○ + ○ + □
What is the sum/total of 753 and 227? How many altogether are 854 and 622?
Increase 250 by 420.
Find all the different totals you can make by using three of these 5 numbers:
14721, 76, 9534, 788, 6, 1.07, 0.3, 37.03, 17.73, 31.7

Mental Maths

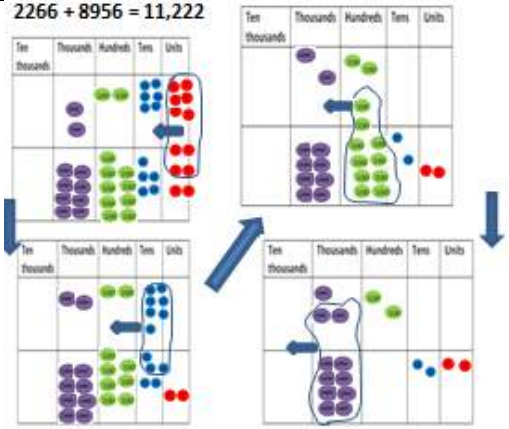
Add four digit multiples of 100 e.g. 3700 + 4500
Add three or more digit multiples of 100 e.g. 400 + 800 + 500
Add a single-digit multiple of 100 to a three or four-digit number crossing 1000 e.g. 300 + 876 =□ 300 + □ = 1176 □+ 876 = 1176
& 638 + 500=
Add a three digit multiple of 10 to a three digit number without crossing the hundreds boundary. e.g. 230+364 460+518
Find what to add to a three digit number to make the next higher multiple of 100. E.g. 651 + □ = 700
Find what to add to a decimal with units and tenths to make the next higher whole number e.g. 8.25 + □ = 9.0

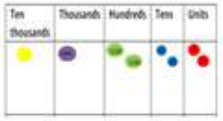
Year 5 – Addition

Progression (a combination of these models and images can be used for every objective)

To add four digit numbers (regrouping in the 1000s, 100s, 10s and 1s)

2266 + 8956 = 11,222





Use of place value counters to visualise

TTh	Th	H	T	O
	2	2	6	6
+	8	9	5	6
1	1,	2	2	2
1	1	1	1	.

Add numbers from right to left
Regroup and record below line
Understand place value of each digit

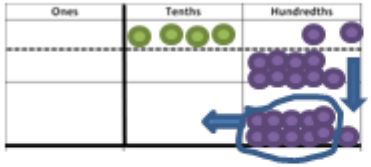
To identify common misconceptions in column addition

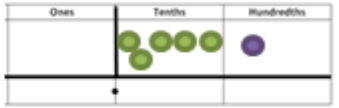
Pupils find the mistakes and work out the correct answers. Recording steps to solve can support this (see progression y4)

3566 +1932 4498	6429 + 3572 9991	6739 +4527 2212	
Hundreds not regrouped into thousands and hundreds.	Ones not regrouped into tens and ones.	Subtraction not addition	Can you create a mistake where you do not regroup when needed?

To add decimals up to 2 decimal places

0.42 + 0.09





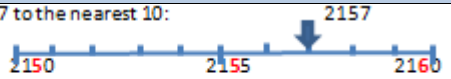
0 . 5 1

Add the hundredths
Add the tenths
Add the ones
Remember to regroup

0. 4 2
+0. 0 9
0. 5 1

To round off numbers to the nearest 10

Round off 2157 to the nearest 10:



2157 is between 2150 and 2160. It is nearer to 2160 than to 2150. 2157 is 2160 when rounded off to the nearest ten ≈ **2160**

To round off numbers to the nearest 100

To estimate: **1296 + 2508**
Children encouraged to articulate their thinking:
'3800 is my estimate because I rounded 1296 to 1300 and 2508 to 2500. 1300 +2500 = 3800'
(To check I did 1300 + 2508 = 3808 – 4 = 3804)

To add money using the column method (regrouping)

£23.48 + £15.99 + £2.24

= £41.71

£23.48
£15.99
+£ 2.24
£41.71
1 1 2

To add measures using the column method (regrouping)

1.72 metres + 750 cm + 1500 cm

= 3970 cm or 39.7 m

1720cm
750cm
+ 1500cm
3970cm
1

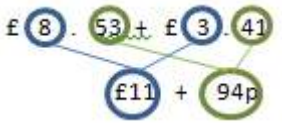
(convert so all the same unit of measurement)

To find the missing value.

3 4 □ 2
+ 1 3 2 9
4 7 9 1
1 1

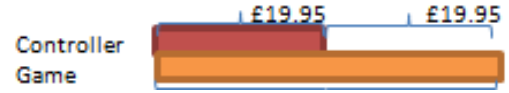
1 .□7
+ 0 . 9 1
2 .6 8

To use part, part whole to add money.



Using the bar model (to solve two step word problems)

Jordan bought a football game and a new controller. The controller cost £19.95 and the game was double. How much was the game? How much did he spend altogether?



1) Double £19.95 = Double £20.00 – 10p = The game costs £39.90
2) Total cost of the game and the controller= £39.90 + £19.95 = £40.00 + £20.00 = £60.00 -10p – 5p = **£59.85**

Most effective approach would also show step 2 in a bar model. What would this look like?

Year 6 – Addition (When planning ensure you track back to year 5 for progression)

National Curriculum

Perform mental calculations, including with mixed operations and large numbers.
Use their knowledge of the order of operations to carry out calculations involving the four operations
Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
Solve problems involving addition, subtraction, multiplication and division.
Use estimation to check answers to calculations and determine, the in context of a problem, an appropriate degree of accuracy.

Notes and guidance (non-statutory)

Pupils practise addition, subtraction etc. and use the formal written methods of columnar addition and subtraction.
They undertake mental calculations with increasingly large numbers and more complex calculations
Pupils round numbers to a specified degree of accuracy, for example to the nearest 10, 20, 50 etc., but not to a specified number of significant figures.
Pupils explore the order of operations using brackets; for example 2+1 x 3 = 5 and (2 + 1) x 3 = 9

Key vocab: add, addition more, plus, increase, make, sum, total, altogether, score, double, near double, one more, two more, ten more, one hundred more, how many more to make…?, how many more is… than…? How much more is…? Tens boundary, hundreds boundary, inverse.

Key concepts

We can use addition facts that we know to solve other additions.
We can use addition facts we know to solve additions with decimals.
Estimating can be used to predict the answer and the inverse can be used to check it.
Addition is:
Combining two or more quantities into one
The enlargement of a quantity, i.e. increasing the amount in the quantity.
Comparison of a quantity with another. i.e. one quantity has a certain amount more than the other.

Learning objectives (turn over for exemplification)
To solve any additions with numbers to 2 decimal places. (see y5 progression)
To carry out calculations involving the four operations.
To work systematically to solve a problem
To solve multi step word problems.
To use estimation to check answers to calculations.
To add negative numbers.
To understand the order of operations using brackets.

Potential barriers/misconceptions

Unless a pupil has a good understanding of place value they will continue to make mistakes with column addition. Such errors are often dismissed as common mistakes, when the pupil in fact has a fundamental weakness in their understanding. When adding the decimals such details are highlighted with the positioning of the decimal point.
Students get mixed up with operation and signs when there are subtraction and negative signs in a problem. i.e. important to refer to operation as subtract/ add and these signs as positive/negative.

Example Questions

Add 4250, 3536 and 242.
Which three numbers could have a total of 1? Are there any others?
□ + 4.47 = 6.38 91 + □ + 38 = 250
Find the mean of the following set of numbers….
What totals are possible with these three dice?
Is there a pattern when you add two consecutive numbers?

Mental Maths

(building on Mental Maths from y5)
Find the difference by counting up through the next multiple of 10, 100 or 1000: 7000-3675 is +5 + 20 + 300 + 3000= 3325
Identify near doubles: 421 + 387 = 808 (double 400 plus 21 minus 13)
Add or subtract the nearest multiple of 10, 100 or 1000 adjust: add 0.9, 1.9, 2.9 or 1.1, 2.1, 3.1 etc by adding 1,2,3 and adjusting by 0.1.
Add or subtract four digit multiples of 100
Find what to add to a decimal with units, 10^{th} and 100ths to make the next higher whole number or 10^{th}.
What must be added to 7.78 to make 8?
Add or subtract a pair of decimal fractions each less than 1 and with up to 2 decimal places.

Year 6 – Addition

Progression (a combination of these models and images can be used for every objective)

To work systematically to solve a problem

There are 20 cars and bicycles in a car park. The total number of wheels is 50. How many bicycles are there?

Cars	Bikes	No. wheels	50 wheels?
10	10	40+20=60	Too many
9	11	36+22=58	Too many
8	12	32+24 = 56	Too many
5	**15**	20 + 30 = 50	Yes

Use one set of digit tiles in the following task:

0 1 2 3 4 5 6 7 8 9

The sum of two three digit numbers is a 4 digit number. (No digit used more than once).

□ □ □
+ □ □ □
□ □ □ □

For a problem like this students need to know addition but also have a good number sense and use 'guess and check'.

Each symbol stands for a different digit.

□ Δ — What would each digit stand for?
□ Δ — What can you say about the digit ¤?
+ □ Δ — What can you say about the digit □?
○ ¤ — What can you say about the digit ○?

Try to work systematically, trying out different possible values for Δ.

To add negative numbers.

(-2) + (-3) = -5

3 + (– 2) = 1

To add negative numbers.

96 pupils in Key Stage 2. Last year 22 pupils left and 37 joined. How many children are there now in Key Stage 2?

96 – 22 + 37
74 + 37

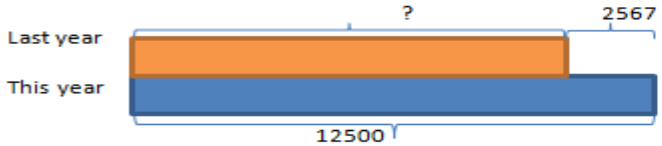
= **111** children are now in Key Stage 2.

Standard order of operations:

1) Do the operation in brackets
2) Do operation involving indices
3) Multiply and divide in order from left to right.
4) **Add and subtract in order from left to right.**

To solve multi step word problems (using the bar model)

12 500 people visited the skate park this year. This is 2567 more than last year.



How many people have visited in the last two years?

1) 12500-2567 = 9933 visited last year
2) 12500 + 9933 = 22,433 have visited in the last two years.

What is the average amount of visitors each year?
How many is that each week?

Most effective approach would also show step 2 in a bar model. What would this look like?

Chapter 3

Subtraction

EYFS 1 – Subtraction (When planning ensure you track forwards to EYFS 2 & year 1)

Early Learning Goal 11
Children count reliably with numbers from 1 to 20, place them in order (see number and place value)
And say which number is one more or one less than a given number
Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.

Key Vocab: take (away), leave, how many are left/left over? , how many have gone?, one less, two less, how many fewer is… than…?, difference between, is the same as.

Key concepts

Subtraction can be experienced by generating experiences that include change. Cakes eaten, balloons popped.
Subtraction is the taking away of one amount from another.
Concept of subtraction needs to be taught and experienced through play as they are the precursors to subtraction reasoning.

Potential barriers

Unable to recite numbers in the correct order
Not associating number names with objects in group
Unable to count without putting in line or touching.
Not being able to 'hold' the number they started with when taking away from the group.
Not knowing the number order when counting backwards.

Learning objectives (see overleaf for exemplification)

To make comparisons between quantities.
To use the language of fewer (less) to compare sets of objects.
To separate a group of 3 or 4 objects in different ways (total still same).
To know that a group of things changes in quantity when something is taken away.
To respond to (and use) subtraction vocabulary in rhymes and games.
To find the total number of items after some are taken away by counting all of them.
To know that when counting a group the last number represents the quantity.

Example Questions

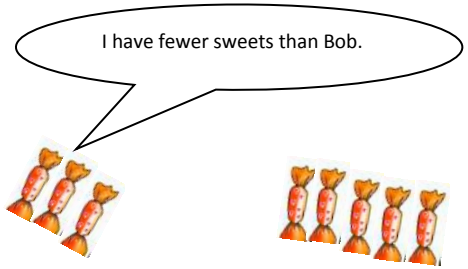
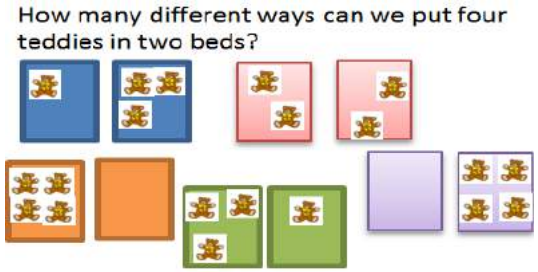
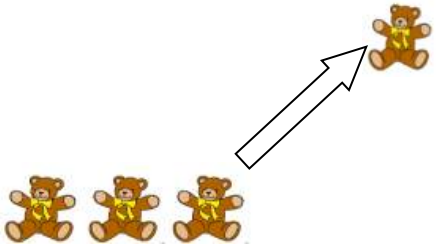
Using a play house- put three people in one room and four in another.
'Which room has more people in?'
'How do you know?'
Move some people from one room to another- 'What has happened in this room?'

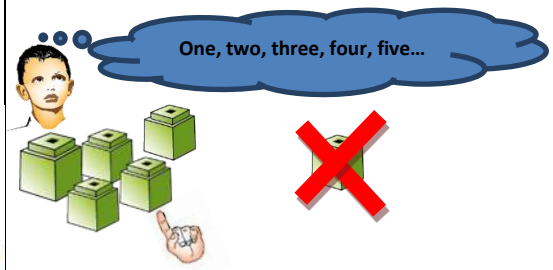
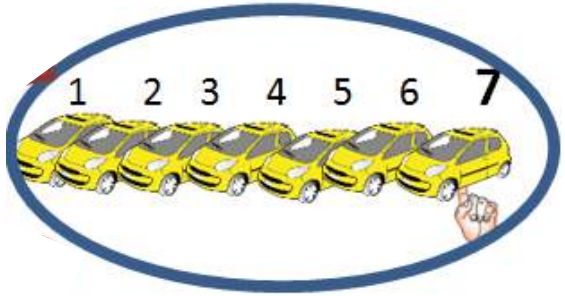
Which plate has fewer biscuits on?
Ellie has three apples, Diane has two apples. Who has fewer apples? Ellie or Diane? (use apples to show).
I am going to take away one of these five cubes. How many will be left?

Mental strategies (can revisited throughout day once concept has explicitly shared)

Join in rhymes and sing songs such as:
Five little ducks went swimming one day
Five little speckled frogs
Five little monkeys jumping on the bed
Five current buns in the baker's shop
Alice the camel has ten humps
Ten green bottles
One man went to mow...

Say the number name that goes before a given number.
Count forwards and backwards using a counting stick

EYFS 1 – Subtraction	**Progression** (a combination of these models and images can be used for every objective)		
To make comparisons between quantities.	**To use the language of fewer (less) to compare sets of objects.**	**To separate a group of 3 or 4 objects in different ways (total still same).**	**To know that a group of things changes in quantity when something is taken away.**
Which group of cars would you like to play with? Why? 			

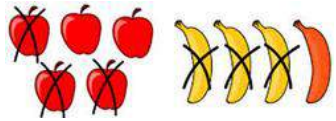
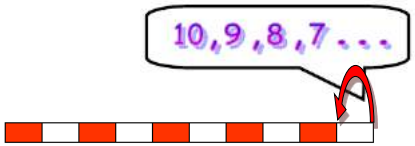
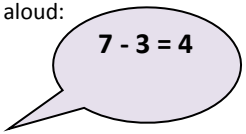
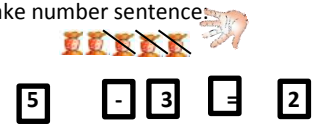
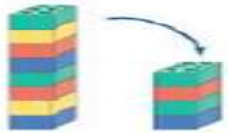
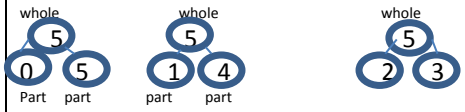
To respond to (and use) subtraction vocabulary in rhymes and games.	**To find the total number of items after some are taken away by counting all of them.**	**To know that when counting a group the last number represents the quantity.**
Five little ducks went swimming one day Five little speckled frogs Five little monkeys jumping on the bed Five current buns in the baker's shop Alice the camel has ten humps Ten green bottles One man went to mow... 		 There are seven in the group.

EYFS 2 – Subtraction

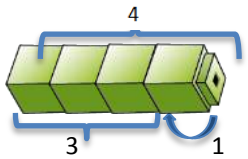
(When planning ensure you track back to EYFS 1 and forwards to year 1)

Early Learning Goal 11	KS1 ready:
Children count reliably with numbers from 1 to 20, place them in order (see number and place value) and say which number is one more or one less than a given number **Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.**	**Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.** **Represent and use number bonds and related subtraction facts within 10** **Add and subtract one-digit and two-digit numbers to 20 including zero** **Solve one step problems that involve addition and subtraction, using concrete objects and pictorial representations and missing number problems such as 7 = ☐ - 9**
Key Vocab: take (away), leave, how many are left/left over? , how many have gone?, one less, two less, how many fewer is… than…?, difference between, is the same as. **Key concepts** Subtraction is the taking away of one amount from another. Concept of subtraction needs to be taught and experienced through play as they are the precursors to subtraction reasoning. Concrete apparatus should be used prior to experimenting with recording. The last number in a number sentence/ number after the = sign is the total (not the answer). The subtraction situation of change is often referred to as 'take away'. To find the answer you need to count how many are left.	**Learning objectives** (see overleaf for exemplification) Relates subtraction to taking away. To find one **less** than a group of up to five, then ten objects. Records using marks they can interpret and explain. Uses quantities and objects to subtract two single-digit numbers and count on to find the answer. To count backwards on a number line or counting stick To recognise and name - and = signs To read a subtraction number sentence To solve a subtraction number sentence To arrange a subtraction number sentence To halve (an even group up to 12) To know number families to 5, 6 and 10 To begin to identify own mathematical problems based on own interests and fascinations.
Potential barriers Children unable to relate subtraction to taking away. Teach other phrases for 'taking away' e.g. 'How many less?' Misunderstanding of 'one less'; do not consistently identify the number before a given number. Children are unable to bridge from 10 to 11 and 20 to 21 as always 'stop' at 10 and 20 when counting.	
Example Questions There are five birds in the nest. One flew off. How many are there now? Look at this group of counters (eight counters positioned randomly) Now look at this group of cubes (five cubes positioned randomly). Are there fewer cubes or fewer counters? How do you know? Choose two number cards (from 1-5) Which of your two numbers is worth more? Which number is less? There are nine biscuits on this plate. Take three of the biscuits to eat. How many biscuits are left on the plate? We have four aprons. There are seven children who want to paint. How many more aprons do we need? John has four books. Lisa has one. How many more books does John have than Lisa?	**Mental strategies** (can revisited throughout day once concept has explicitly shared) Say the number name that goes before a given number (one less) Choose two groups of objects to make a given total. Six blocks. Four red, two green. Say how many are left when some are taken away by **counting how many are left**. We ate 2 of our six cakes. How many cakes are left? (Count 1, 2, 3, 4, 5, 6. Take away 1, 2… 1,2,3,4 left. Say together: 6 take away 2 is 4) Say how many are left when some are taken away, by **counting back from a number**. We made 6 mince pies. We ate 2 of them. How many pies are left? (Count back 2 from 6: 5, 4. Say together six take away two is four.) Find out how many have been removed by **counting up** to the larger number. There were 8 books on this shelf. There are only 5 books now. How many have gone? (count up from 5 to 8. 6, 7, 8… and say 3. Say together: 5 add 3 is 8. 8 take away 3 is 5).

EYFS 2 – Subtraction Progression (a combination of these models and images can be used for every objective)

Relates subtraction to taking away.	**To find one less than a group of up to five, then ten objects.**	**Records using marks they can interpret and explain.**	**Uses quantities and objects to subtract two single-digit numbers and count on to find the answer.**
	One less than six is five 		 5 – 3 = 2 4 – 3 = 1
To count backwards on a number line or counting stick	**To recognise and name - and = signs**	**To read a subtraction number sentence**	**To solve a subtraction number sentence**
	Where is the take away (subtraction) sign? - How do we know to take away? + Where is the total? = What does this sign mean?	To read aloud:  Knowing that - is subtract/ take away = is equals	To use objects to lay out & solve: **7 - 3 = 4** 
To arrange a subtraction number sentence	**To halve (an even group up to 12)**	**To know number bonds to 5, 6 and 10**	**To begin to identify own mathematical problems based on own interests and fascinations.**
To match number cards to objects to make number sentence: 	 Half of 8 is 4.	Part, part whole: number bonds to 5. 	

Progression towards bar model

Children start by subtracting objects from a group  **What is one less than 4?**	**Children then use unifix cubes, counting back from the greater number, to find the total number of cubes.**  4 -1 = 3 

Year 1 – Subtraction (When planning ensure you track back to Reception and forwards to year 2)

National Curriculum

Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.
Represent and use number bonds and related subtraction facts within 20.
Add and subtract one-digit and two-digit numbers to 20 including zero
Solve one step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = - 9

Notes and guidance (non-statutory)

Pupils memorise and reason with number bonds to 10 and 20 in several forms (for example, 9+7=16; 16-7=9, 7=16-9)
They should realise the effect of adding or subtracting zero. This establishes addition and subtraction as related operations.
Pupils combine and increase numbers, counting forwards and backwards.
They discuss and solve problems in familiar contexts, including using quantities.
Problems should include the terms: put together, add, altogether, total, take away, distance between, more than and less than so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.

Key vocab: - , subtract, take (away), minus, leave, how many are left/left over? , how many have gone?, one less, two less, ten less, how many fewer is…than…?, how much less is…? Difference between, half, halve.

Key concepts

When you subtract zero the total stays the same. (related to same concept of add zero)
Understand subtraction as 'take away' and 'find a difference' by counting up.
Subtraction is also used for 'how many more to make?' (complementary addition)
That the related vocabulary for subtraction is: take away, subtract, how many are left, how much less is…than…, difference between, how much more is…than…, how many more to make…
Number bonds help make the connection between addition and subtraction.
Subtraction is associated with the part- whole and the taking away concept.
A family of number sentences can be written from a set of three related numbers.
Two digit numbers can be regrouped into tens and ones.

Potential barriers

Lack of confidence in numbers bonds within ten, to ten and to twenty will prohibit children from fully understanding the rules of commutativity.
Children are confident with counting 'up' but have limited experience counting backwards from any given number.
Don't associate number facts e.g 13+4=17 and 17-4=13 as they don't see + and – as inverse.

Doesn't link the language of take away and find the difference.

Example Questions

Can we find the difference between two numbers by counting?
Using a number line show me two numbers that have a difference of two. How might you write that?
Which number comes before/after 17? Does 16 always come before 17?
How many are left?
What is the difference between these sets/numbers?
How can we subtract these things/numbers?
There are eight dogs in this kennel and 3 run out. How many dogs are left now?
There are 4 dogs in this kennel and 9 dogs in the other kennel. What is the difference between them?
Inverse: There are 3 cats on this chair. 2 more cats jump onto the chair. Now there are 5 cats. If 2 cats jump off, how many will be left on the chair?
If 14+5=19 what else do you know about these numbers?
How do you know you need to take away? What clues are there?
How many different ways can you show me that 12 subtract 4 is 8?
Can you make up a take away question and show me how to do it?
Which numbers in the sequence are missing? Explain how you know: ?, 9, ?, 11, 12

Learning objectives (see overleaf for exemplification)

To break numbers into parts
To subtract with number bonds
To subtract by taking away.
To subtract by counting on.
To subtract small numbers where sets are hidden. (counting on)
To subtract by counting backwards
To subtract within 20 by regrouping into tens and ones
To use a number line to count back.
To make a family of number sentences
To use inverse (write corresponding subtraction facts to given addition facts- number families).
To solve missing number problems
To solve one step word problems using part whole method

Mental strategies

Counting stick: counting forwards and backwards in steps (not only of ones) from any given number.
7-3 = count back in ones from 7
15-3= count back in ones from 15
18-6= count back in twos from 18

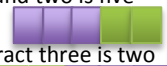
To use 'count back from' strategies. (8-6= 7,6,5,4,3,2… =2)
To use 'count back to' strategies. (8-6 = 7,6 = 2)
Find a small difference by counting up. (When two numbers are close together i.e. 15-12=3 counting up from 12 to 15 gives 3.)
Subtract ten from a teens number: 19-10= □ 19-□= 9 □-10= 9
Subtract ten from any two digit number, without crossing 100: 49-10 =□ ; 49 - □ = 10 ; □-10 = 39
Subtract a pair of multiples of ten without crossing 100: 50–20=□ ; 50 - □ = 30 ; □ – 20 = 30

Year 1 – Subtraction Progression (When planning ensure you track back to EYFS and forwards to year 2)

To break numbers into parts

5 - 2 = 3

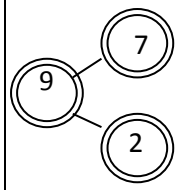
Three and two is five



Five subtract three is two

Five subtract two is three

To subtract with number bonds



9 is 7 and 2

So 9 – 7 = 2

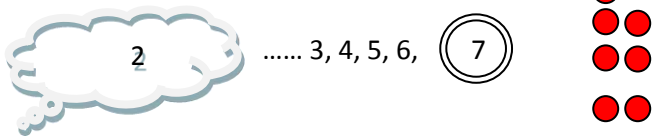
And 9 – 2 = 7

How else can we break 9?

To subtract by taking away

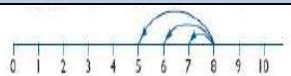
To subtract by counting on
To subtract small numbers where sets are hidden. (counting on)

7 – 2 = ?



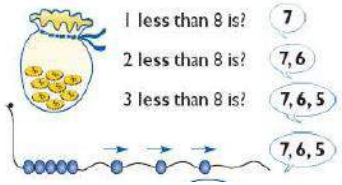
Counting up strategy using finger or counters.

To use a number line to count back.



count back one, two or three

To subtract by counting backwards



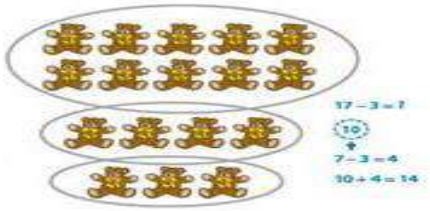
To subtract within 20 by regrouping into tens and ones

17 – 3 = 14

17 = 10 and 7

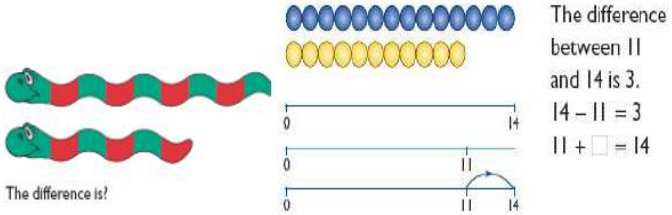
7 – 3 = 4

17 – 3 = 14



There are 14 bears left.

To find the difference by counting on



The difference between 11 and 14 is 3.

14 – 11 = 3

11 + □ = 14

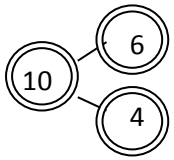
To make a family of number sentences

6 + ? = 10

10 – 6 = ?

? + 6 = 10

10 – 4 = 6



To solve missing number problems

- and = signs and missing numbers:

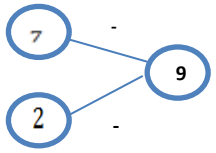
(can cover part, part, whole model to visualise)

9- 7 =□

9 - □ = 7

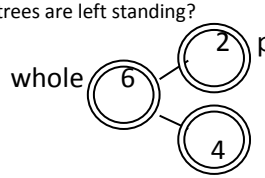
2 = 9 - □

7 = 9 - Δ

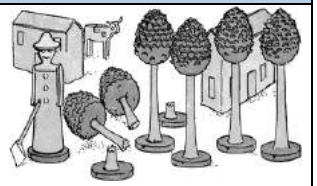


To solve one step word problems using part whole method

How many trees are left standing?

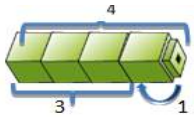


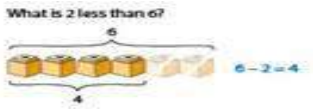
6 – 2 = 4

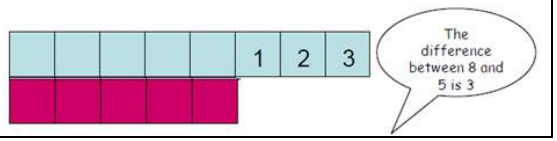


Progression towards bar model

4 -1 = 3







Year 2 – Subtraction (When planning ensure you track back to year 1 and forwards to year 3)

National Curriculum

Solve problems with addition and subtraction:
-Using concrete and pictorial representations, including those involving numbers, quantities and measures.
-Applying their increasing knowledge of mental and written methods.
Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:

- **A two digit number and ones**
- **A two digit number and tens**
- **Add two two-digit numbers**
- **Adding three one digit numbers**

Show that the addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

Notes and guidance (non-statutory)

Pupils extend their understanding of the language of addition and subtraction to include sum and difference.
Pupils practice addition and subtraction to 20 to become increasingly fluent in deriving facts such as using 3+7=10; 10-7=3 and 7=10-3 to calculate 30+70=100; 100-70=30 and 70=100-30.
They check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (e.g. 5+2+1=1+5+2= 1+2+5) This establishes commutativity and associativity of addition.
Recording addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers.

Key vocab: - , subtract, subtractions, take (away), minus, leave, how many are left/left over? , how many have gone?, one less, two less, ten less, one hundred less, how many fewer is…than…?, how much less is…? Difference between, half, halve, tens boundary, regroup.

Key concepts

Addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot.
Subtraction is taking away from a whole.
When we take away we can take away in bits. We subtract the units first, then the ten, then the hundreds.
A 2 digit number can be conceptualised as tens and ones.
Number lines can be used to support subtraction by counting backwards.
Number lines can be used for find the difference by counting on.
When using a hundred square to subtract 10, you can 'move up' on the grid.
Subtraction reverses addition (subtraction is the inverse of addition).

Learning objectives (see overleaf for exemplification)

To break numbers into parts
To use the number bond strategy to subtract
To subtract a one digit number from a two digit number without regrouping
To subtract 2 two-digit numbers without regrouping
To use the counting backwards strategy to subtract
Use the 'take away' strategy to subtract
To subtract a one digit number from a two digit number with regrouping
To subtract 2 two-digit numbers with regrouping
To solve one step word problems using 'part, whole'

Potential barriers

Avoid telling children 'you can't take a big number away from a smaller number' you can- this will then go into negative numbers. This could lead to misconceptions at a later point.
Children may not understand the commutative law and believe that it is possible to change any addition and subtraction around.
Children sometimes regroup but see the new number as one and not ten.
Pupils may struggle to see 'find the difference' as a form of subtraction. This can be linked to lack of consolidated skills in counting on and back.
Challenge in recalling addition and subtraction facts to 20
Difficulty using mental strategies to add and subtract two digit numbers.

Example Questions

Rapid recall of subtraction asked in a range of ways: 7 take away 3; Take 30 away from 70; 14 subtract 2; Subtract 30 from 70; 3 less than 7; What number must I take from 20 to leave 3?; What is the difference between 10 and 18? How many more is 11 than 3? How many less is 7 than 18?
When Matilda is 4 years old, Sadie is 9. When Matilda is 8, how old will Sadie be?
14 children are on a bus. 8 children get off the bus. 5 more children get off the bus. How many are left on bus?

Mental strategies

To know by heart all addition and subtraction facts for each number to 20
To use number bonds for mental subtraction. 9-4= □ (Think of addition: 4 and 5 make 9 therefor 9-4=5)
To subtract multiples of ten from any two digit number
To add and subtract mentally a 'near multiple of ten' to or from a two digit number. (15+39 = 1+39+10+4=54)
To find pairs of numbers with a difference of 10, a difference of 9 etc…
To find a small difference when counting up. 84-78 = 79,80,81,82,83,84 = 6
To mentally subtract 11 or 21 or 9 or 19 from any two digit number. 70-11=59 as it is the same as 70-10-1= 59
24-9=15 because it's the same as 24-10+1 = 15
Add or subtract any single digit from any two digit number without crossing the tens boundary (86-□=82)
Subtract multiples of ten without crossing 100. (90-40=□)
Subtract multiples of 100 without crossing 1000 (700 – 300 = □)
Use number bonds to find a small difference between a pair of numbers lying either side of a multiple of 10 (102-97 = 2+3 = 5)

Year 2 – Subtraction Progression (When planning ensure you track back to year 1 and forwards to year 3)

To break numbers into parts

12 = 5 + 7

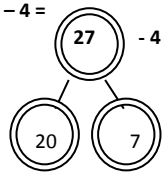
12- 7 = 5

12 – 5 = 7

How else could we break 12?

To use the number bond strategy to subtract

27 – 4 =



7 – 4 = 3

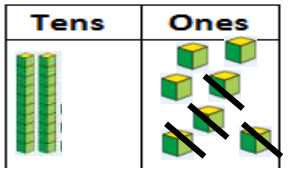
then add back on the 20:

20 + 3 = 23

So 27 – 4 = 23

To subtract a one digit number from a two digit number without regrouping (although shown visually as column for progression this could be solved in a number of ways)

27 – 4= 23



First subtract the ones

$$\begin{array}{r} 27 \\ -\ 4 \\ \hline 3 \end{array} \qquad \begin{array}{r} 27 \\ -\ 4 \\ \hline 23 \end{array}$$

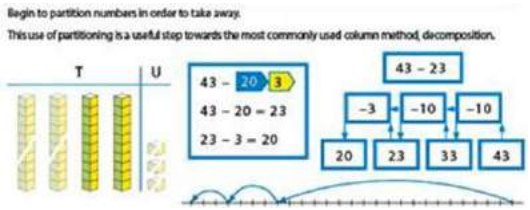
Seven ones subtract four ones = three ones

Then subtract the tens= 2 tens – 0 tens = 2 tens

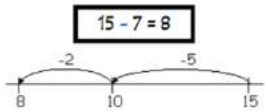
2 tens and 3 ones = **23**

(check by counting back 4 from 27)

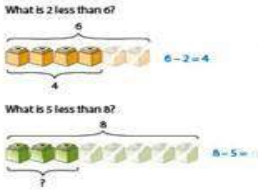
To subtract 2 two-digit numbers without regrouping



To use the counting backwards strategy to subtract



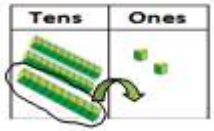
Use the 'take away' strategy to subtract



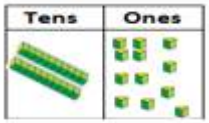
To subtract a one digit number from a two digit number with regrouping

32 – 9 =

We can't subtract 9 ones from the 2 ones…
So we regroup the tens and ones in 32.



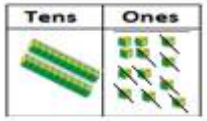
Regroup the tens in 32
3 tens = 2 tens and 10 ones



$$\begin{array}{r} T\ \ O \\ {}^{2}\not{3}\ {}^{1}2 \\ -\ \ 9 \\ \hline 3 \end{array}$$

First subtract the ones:

12 ones – 9 ones = 3 ones



Then subtract the tens.

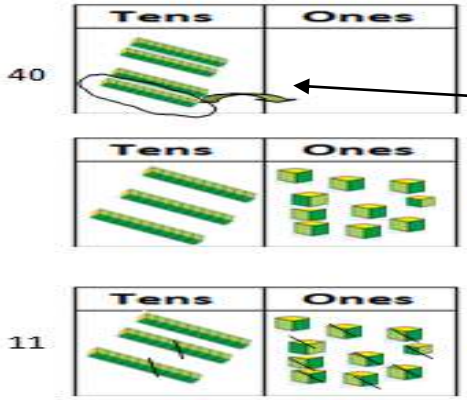
$$\begin{array}{r} {}^{2}\not{3}\ {}^{1}2 \\ -\ \ 9 \\ \hline 2\ 3 \end{array}$$

(2 tens – 0 tens = 2 tens)

So 32 – 9 = 23

To subtract 2 two-digit numbers with regrouping

40 – 29



First we subtract the ones- but we can't!
We regroup the tens and ones in 40

Regroup the tens in 40
4 tens = 3 tens and 10 ones

First subtract the ones:

$$\begin{array}{r} {}^{3}\not{4}\ {}^{1}0 \\ -\ 2\ 9 \\ \hline 1 \end{array}$$

10 ones subtract 9 ones = 1 one

Then subtract the tens:

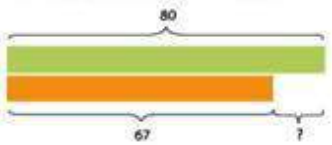
$$\begin{array}{r} {}^{3}\not{4}\ {}^{1}0 \\ -\ 2\ 9 \\ \hline 1\ \ 1 \end{array}$$

3 tens – 2 tens = 1 ten

So, 40 – 29 = 11

To solve one step word problems using 'part, whole' and the bar model

Ali has 80 points.
Michael has 67 points.
What is the difference?



Year 3 –Subtraction (When planning ensure you track back to year 2 and forwards to year4)

National Curriculum : Add and subtract numbers mentally, including:

- **A three-digit number and ones**
- **A three- digit number and tens**
- **A three- digit number and hundreds**

Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
Estimate the answer to a calculation and use inverse operations to check answers.
Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction.

Notes and guidance (non-statutory)
Pupils practise solving varied addition a subtraction questions.
For mental calculations with two-digit numbers, the answers could exceed 100.
Pupils use their understanding of place value and partitioning and practice using columnar addition and subtraction with increasingly large numbers up to three digits to become fluent.

Key vocab: - , subtract, subtractions, take (away), minus, leave, how many are left/left over? , how many have gone?, one less, two less, ten less, one hundred less, how many fewer is…than…?, how much less is…? Difference between, half, halve, tens boundary, hundreds boundary, regroup.

Key concepts

Subtraction is the inverse of addition
Children must move through the concrete, pictorial then onto the abstract (CPA) in order to fully develop understanding. (dienes and place value discs can be used to support this).
Regrouping can be used in subtraction
When using the column method we subtract from right to left
10 ones = 1 ten
The bar model can be used to find the 'whole' from two or more parts.

Potential barriers

Children sometimes begin subtracting with the left hand column first
In tens and units and other formal vertical calculations, children sometimes take the smaller unit number from the larger, regardless of whether it is part of the larger or the smaller number. e.g. 945-237=712
When the teacher uses the phrase 'near multiple of ten' for mental strategies children often get confused with needing to use multiplication as the operation.

Example Questions

15 take away 8, take 8 from 15, 63 subtract 40, subtract 8 from 15, subtract 40 from 95, ten less than 43, 110 less than 437, what must I take away from 14 to leave 6? What is the difference between ? and ?, 20 taken from a number is 35 what is the number?
Using only the numbers 15, 17, 32, 34, 49 write as many different number sentences as you can.
What is 100 subtract 24? Subtract 21 from 100.
John is 109cm tall. William is 136cm tall. How much taller is William than John?
Navneet had £10.00 she spent £2.45. How much money did she have left?
There are 265 children at Finching school. 103 have packed lunch, 26 go home for lunch. The rest have school dinners. How many children have school dinners? Show how you worked this out.
Chris had 50 books. He sold some and then had 20 left. Which of these is a number sentence that shows this?: □-20=50, 20 -□=50, □50=20, 50-□=20
There are 1000 pieces in a puzzle. 13 go missing. How many pieces are left?
Calculate 309-198=
There were 24 biscuits in a box. There are now only 18 left. How many have been eaten?

Learning objectives (see overleaf for exemplification)

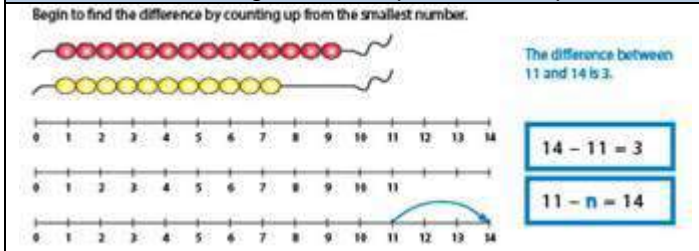
To find the difference using a number line (for near numbers)
To use number bonds to subtract mentally (see mental strategies below for progression and next page for exemplification)
To subtract without regrouping (see year 2)
To subtract with regrouping in tens and ones
To subtract a 3 digit number with regrouping in hundreds and tens
To subtract a 3 digit number with regrouping in hundreds, tens and ones
To count back to find the difference
To estimate the answer to a calculation
To use inverse operations to check answers
To subtract 'taking away' one set using the bar model
To subtract 'comparing two sets' using the bar model

Mental strategies (All calculations must also use missing number problems: □)

Use number bonds to mentally subtract a 1-digit number from:
-a 2-digit number within 100 with or without regrouping. (ten as the middle stage: 62-7 = 62-2-5= 60-5 = 55)
-a 3 digit number within 1000 with or without regrouping in tens and ones
- tens from a 3 digit number within 1000 with or without regrouping in hundreds into tens
- Hundreds from a 3 digit number without regrouping.
Subtract a single digit from a multiple of 100. (600-7=593) (600-□=593)
Subtract a pair of multiples of 10, crossing 100. (120-30= 90) (□ – 30 =90)
Subtract a multiple of 10 from a 2 digit number crossing 100 (112-30=82) (112-□=82)
Subtract a pair of multiples of 100 crossing 1000 (1500-800= 700) (1500-□=700)
Subtract 100 from any 3 digit number, without crossing 1000 (809-100= 709) (□-100=709)
Consolidate subtracting a single digit from a 'teens' number, crossing 10 (use two steps and cross ten as the middle stage: 15-8 = 7 I know this because 15-5-3 = 10-3= 7)
Find pairs of numbers with a difference of 29, 16…
Find the difference between two numbers that are close together by counting up. (504-498 = 2+4=6) (1003-992= 992+8+3=1003 = 11)
Mentally subtract 9,19,29… or 11,21,31 from any two digit number without crossing 100
Develop and recognise a pattern such as 68-5=63, 68-15=53, 68-25=43 therefor 68-45=23
Say the subtraction fact corresponding to a given addition fact: 56+27=83 therefor 83-27=56

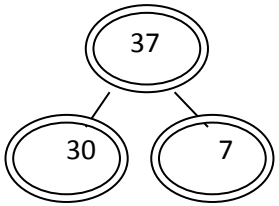
Year 3 – Subtraction **Progression** (When planning ensure you track back to year 2 and forwards to year 4)

To find the difference using a number line (for near numbers)



To use number bonds to subtract mentally

37 - 4



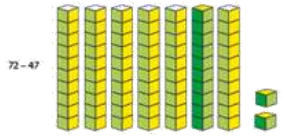
Regroup the 37 into tens and ones = 3 tens and 7 ones

First subtract the ones: 7 – 4 = 3

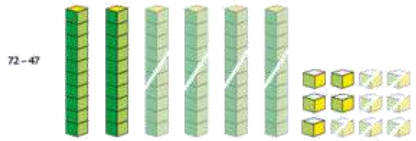
Then, add the result to the tens: 30 + 3 = 33

37 – 4 = 33

To subtract with regrouping
(for further exemplification of regrouping see year 2)



Regroup



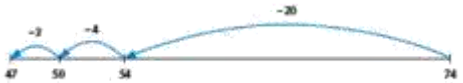
7 2 regrouped as 6 tens and 12 ones

60 + 12
- 40 + 7
20 + 5

To move on to subtraction of 3 digit number from a 3 digit number regrouping.

To count back to find the difference

74 – 47 = 27



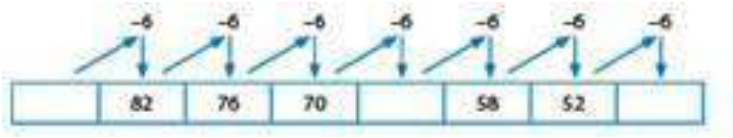
To estimate the answer to a calculation

Round numbers to the nearest 10 to check reasonable answers:

58 -- 31 =
60 - 30= **30**
(Actual answer = 27)

To solve missing numbers

Can you find the missing numbers in this sequence?



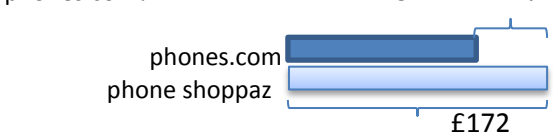
To subtract 'taking away' one set using the bar model

Dan has £98 that he has been saving
He spent £28 on a new game.
How much does he have now?

98 – 28 = 70
Dan has £30 now

To subtract 'comparing two sets' using the bar model

A mobile phone costs £172 in phone shoppaz and £125 with phones.com. How much cheaper is phones.com?



£ 172 -£ 125= **£47 cheaper at phone.com**

Year 4 – Subtraction (When planning ensure you track back to year 3 and forwards to year 5)

National Curriculum

Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate

Estimate and use inverse operations to check answers to a calculation

Solve addition and subtraction two step word problems in context, deciding which operations and methods to use & why.

Notes and guidance (non-statutory)

Pupils continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency. (build on year 3 mental strategies)

Key vocab: take away, subtract, how many are left, how much left, difference between, how much more, how many more to make, decrease, inverse and the minus sign.

Key concepts

Subtraction as: taking away, finding the difference between and complementary addition.
Subtraction is not commutative. i.e. 5-7 is not the same as 7-5
Subtracting a (positive) number makes a number less.
Subtracting zero leaves a number unchanged.
We need to regroup when carrying out a subtraction that crosses a tens, hundreds or thousands boundary.
We can use place value counters to support our conceptual understanding of subtraction
It is important to estimate first.
Subtraction should be carried out in a problem solving context.

Learning objectives (see overleaf for exemplification)

To subtract up to 4 digit numbers (no regrouping)
To subtract with regrouping in hundreds and thousands
To subtract with regrouping in hundreds, thousands, tens and ones
To subtract with numbers that have zeros
To identify common misconceptions in column subtraction
To round off numbers to the nearest 10 / 100.
To estimate and use the inverse to check
To subtract decimals up to 2 decimal places
To solve subtraction two step word problems
Use take away and comparing models to solve subtraction word problems.

Potential barriers

When using the column method pupils sometimes begin subtracting with the left hand column first.
In tens and units and other formal vertical calculations, children sometimes take the smaller unit number from the larger, regardless of whether it is part of the larger or the smaller number. e.g. $\begin{array}{r} 945 \\ -\ 237 \\ \hline 712 \end{array}$
Children may have been incorrectly told 'you can't take a big number away from a small number'. This will cause misconceptions when children start to work in negative numbers.
Pupils don't use estimation skills to predict answer.
Lack of understanding around value of decimal numbers.
Forgetting to include or line up decimal point.

Example Questions

Respond rapidly to oral or written questions explaining the strategy used: 93 take away 7, take 7 from 62, 63 subtract 46, subtract 120 from 215, 170 less than 250, 1000 less than 5437, what must I take from 84 to leave 26? What is the difference between 28 and 65? How many more than 234 is 249? How many less than 68 is 42? What must I add to 54 to make 93? Decrease 72 by 34. 28 add a number is 43, What is the number? Find pairs of numbers with a difference of 79.
1258- 576 = □, 1258- □= 682, Δ - □ = 682
Find the missing number in 91-□=48
Find all the different differences you can make by using two of these five numbers: 219, 193, 74, 156, 97
These are the prices in a shoe shop: Boots = £45.50, Sandals = £12.75 and trainers= £34.99 How much more do the boots cost than the trainers? Rosie buys a pair of trainers and a pair of sandals. How much change does she get from £50?

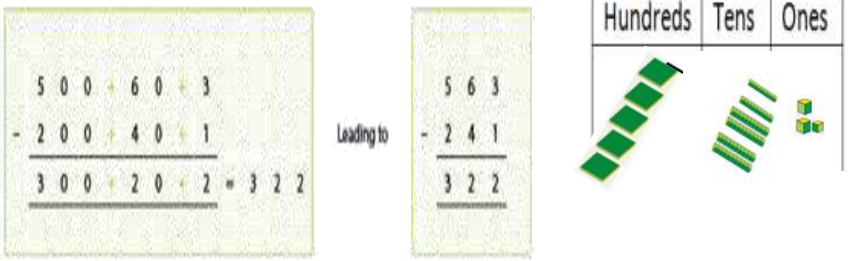
Mental strategies

Consolidate knowing by heart all addition and subtraction facts to 20. E.g all the pairs for 15: 10+5=15, 5+10=15, 9+6=15, 6+9=15, 8+7=15, 7+8=15 and 15-5=10, 15-10=5, 15-6=9, 15-9=6, 15-7=8, 15-8=7
Know how many steps are taken forwards (+) or backwards (-) when moving on a numberline. i.e. To get from 18 back to 6.
Derive quickly related facts: 160-90=70 therefor 1600-900=700 (1.6-0.9=0.7)
Find the difference by counting up through the next multiple of 10, 100 or 1000. i.e. count from smaller to larger number i.e. 483-386
Count back in repeated steps of 1, 10, 100, 1000 from any given number. i.e. 2003-8=1995 (counting back in 1s from 2003) or 387-50=337 (counting back in 10s from 387)
Partition into hundreds tens and ones: 98-43 = 98-40-3= 55
Subtract the nearest multiple of 10, 100 or 1000 and adjust. i.e. 9, 19, 29 or 11, 21, 31 etc (84-19= 65 because 84-20+1=65) (128-67=61 because it is 128-70+3=58+3=61)
Use the relationship between addition and subtraction (If I know 36+19=55 then I also know: 19+36=55, 55-36=19, 55-19=36).
Work out mentally one fact: (91-25=□) and then state the other three related facts.
Subtract 2 digit multiples of 10 (130-50=□)
Subtract a pair of multiples of 100, crossing 1000 (□-600=900)
Subtract a multiple of ten from a 2 or 3 digit number without crossing hundreds (76-□=36)
Subtract a single digit from a multiple of 10 or 100 (4000-3=□ or □-3=4997)
Subtract a single digit from a 3 or 4 digit number crossing tens (7003-6899=□ or 5952-□=5949)
Find a small difference between a pair of numbers lying either side of a multiple of 1000 (7003-6988=15 by counting up 2 from 6988 to 6990 then 10 to 7000, then 3 to 7003).

Year 4 – Subtraction **Progression** (When planning ensure you track back to year 3 and forwards to year 5)

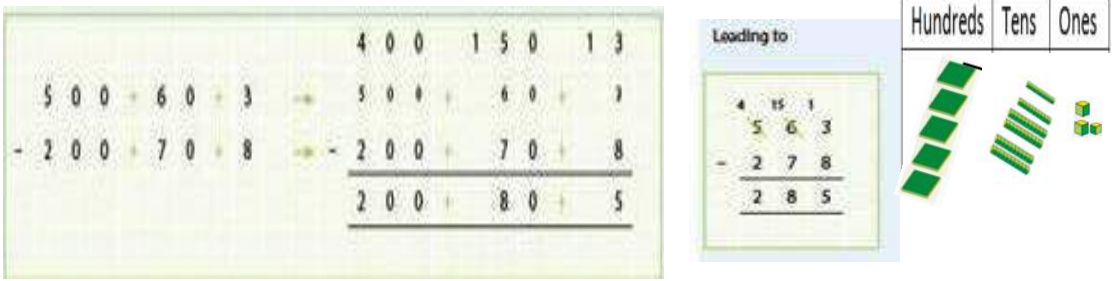
To subtract up to 4 digit numbers (no regrouping)

563 – 241 = 322



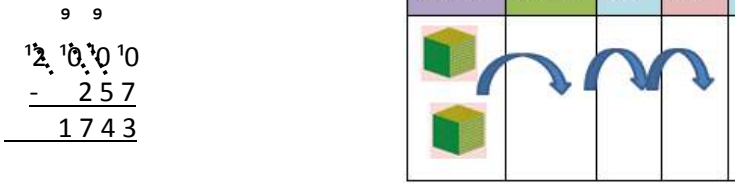
To subtract with regrouping (see year 3 for use of concrete)
To identify common misconceptions in column subtraction

563 – 278 = 285



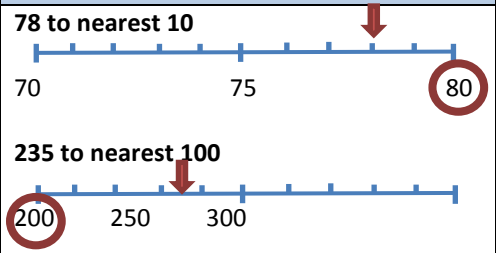
To subtract with numbers that have zero
(although shown visually in the column this could also be solved using the number line)

2000-257



2000 is rerouped by exchanging 1 thousand for 1 hundreds, 1 hundred for ten tens and ten for ten ones using concrete representation.

To round off numbers to the nearest 10 / 100.



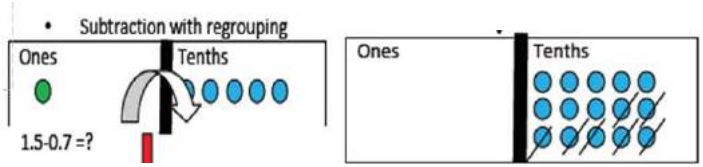
To estimate and use the inverse to check

-Estimate the position of a point on a line.

-Use rounding to estimate answer. 328 - 189 = 330 - 200= 130 (actual answer: 139)

To subtract decimals up to 2 decimal places

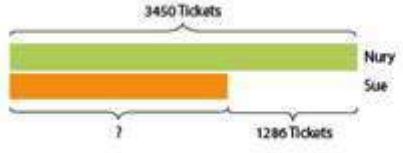
1.5 – 0.7 = 15 tenths – 7 tenths = 8 tenths = 0.8



To solve two step word problems

Nana and Pat were selling tickets for the Hackney festival.
Nana sold 3450 tickets and Pat sold 1286 tickets fewer than Nana.

How many tickets did Pat sell?
How many tickets did they sell altogether?



3450 – 1286 = 2164 Pat sold 2164 tickets 3450 + 2164 = 5614 Nana and Pat sold 5614 festival tickets altogether.

Year 5 – Subtraction (When planning ensure you track back to year 4 and forwards to year 6)

National Curriculum

Add and subtract whole numbers with more than 4 digits, including formal written methods (columnar addition and subtraction)
Add and subtract numbers mentally with increasingly large numbers.
Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.
Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Notes and guidance (non-statutory)

Pupils practise using the formal written methods of columnar addition and subtraction with increasingly large numbers to aid fluency
They practise mental calculations with increasingly large numbers to aid fluency (e.g. 12,462+ 4200= 16,662)

Key vocab: take away, subtract, how many are left, how much left, difference between, how much more, how many more to make, decrease, inverse and the minus sign.

Key concepts

Subtraction as: taking away, finding the difference between and complementary addition.
Subtraction is not commutative. i.e. 5-7 is not the same as 7-5
Subtracting a (positive) number makes a number less.
Subtracting zero leaves a number unchanged.
We need to regroup when carrying out a subtraction that crosses a tens, hundreds or thousands boundary.
We can use place value counters to support our conceptual understanding of subtraction
It is important to estimate first.
Subtraction should be carried out in a problem solving context.

Learning objectives (see overleaf for exemplification)
(for exemplification also look at year 4 progression)
To subtract four digit+ numbers **(**regrouping in the 1000s, 100s, 10s and 1s)
To identify common misconceptions in column subtraction
To round off numbers to the nearest 10.
To round off numbers to the nearest 100. (see exemplification year 4)
To subtract decimals up to 2 decimal places
To subtract money using the column method (regrouping)
To subtract measures using the column method (regrouping)
To find the missing value
To solve two step word problems using the bar model.

Potential barriers

Misconceptions can occur when decomposing from a 'high' number. e.g. 9000 -
3654
Some pupils will attempt subtraction calculations using the formal written method, failing to recognise that it would be more efficient to calculate the answer mentally.
Misconceptions occur when pupils (and teachers) use inaccurate language e.g. 2367-
1265
When talking about 2000 – 1000 they may refer to this as 2-1, unaware of the place value of each number.
Children can often misplace the decimal point when subtracting decimal numbers.

Example Questions

Respond rapidly to oral or written questions explaining the strategy used. For example: 127 take away 35, take 80 from 373, 678-105, subtract 50 from 225, 500 less than 720. What must I take from 220 to leave 55? What is the difference between 155 and 390? How many more than 952 is 1050? How many less than 305 is 94? What must I add to 720 to make 908? Decrease 92 by 78. 570 add a number is 620. What is the number? Find pairs of numbers with a difference of 599.
□-62=189 7.6-5.8=□ □-256=424 □-Δ= 1.2
141.36-32.58=□
Find the missing number in: 931-□=746
Tilda has read the first 85 pages in a book that is 125 pages long. Which number sentence could Tilda use to find the number of pages she must read to finish the book: 150+85 =□, □-85=150, 150-85=□
Scarves cost £7.95 and hats cost £4.50, £6.50 and £3.99. Chris buys one of the scarves and the £4.50 hat. How much change does he get from £20? Emily buys 2 scarves and a hat. What is the most she could pay?

Mental strategies

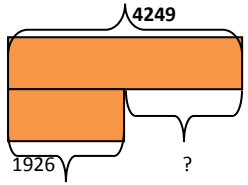
Derive quickly related facts such as: 150-80=70, 1500-800=700 and 1.5-0.8=0.7
Find a difference by counting up through the next multiple of 10,100 or 1000 (8006-2993=□ count up from the smaller to the larger number)
Subtract the nearest multiple of 10, 100 or 1000 and adjust (4005-1997= 2008 because it is 4005-2000+3=2008)
Recognise that knowing a fact such as 136+319=455 makes it possible to find 455-318 and 455-137
Work out mentally one fact such as 101-25 and be able to state the three other facts in the number family
Given the numbers 135, 228 and 363 say or write the four different sentences relating to these numbers
Subtract multiples of 10 and 100 (620-380=□ and 6200-3800=□)
Subtract a single digit multiple of 100 from a four digit number crossing 1000 (1263-400=□)
Subtract a three digit multiple of 10 from a three digit number without crossing the hundreds boundary (742-210=□, 742-□= 532, □-210=532)
To find what to add to a three digit number to make the next higher multiple of 100. (651+□=700)
Find what to add to a decimal with units, tenths and hundredths (5.71+ □ = 7)
Find the difference between a pair of numbers lying either side of a multiple of 1000 (8004- □=19)
Subtract a pair of decimal fractions each less than 1 and with up to two decimal places (0.7-0.26)

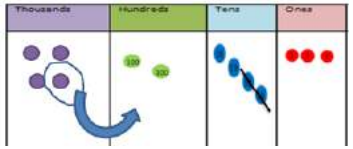
Year 5 – Subtraction

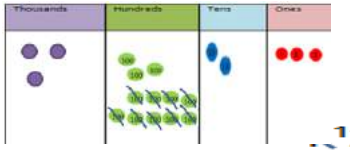
Progression (a combination of these models and images can be used for every objective)

To subtract four digit+ numbers (regrouping in the 1000s, 100s, 10s and 1s) & to identify common misconceptions in column addition

4249 – 1926 = 2323



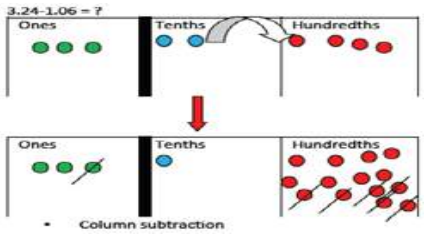




$$\begin{array}{r} {}^{1}2\,4\,9 \\ -\ 1\,9\,2\,6 \\ \hline 2\,3\,2\,3 \end{array}$$

When we subtract 1926 from 4249 we get 2323

To subtract decimals up to 2 decimal places



To subtract money and measures using the column method (regrouping)

0.38 – 5.5

£5.50 – 38 p

Regroup when 8 hundredths cannot be subtracted from 0 hundredths.

We can write 5.5 as 5.50

Remember to align the decimal point!

$$\begin{array}{r} 5.{}^{4}5{}^{1}0 \\ -\ 0.\,3\,8 \\ \hline 5.\,1\,2 \end{array}$$

5.5 – 0.38 = £5.12

To find the missing value

Use the inverse to find the value in the box:

$$\begin{array}{r} \square\,0\,0\,0 \\ -\ 2\,6\,4\,3 \\ \hline 3\,3\,5\,7 \end{array}$$

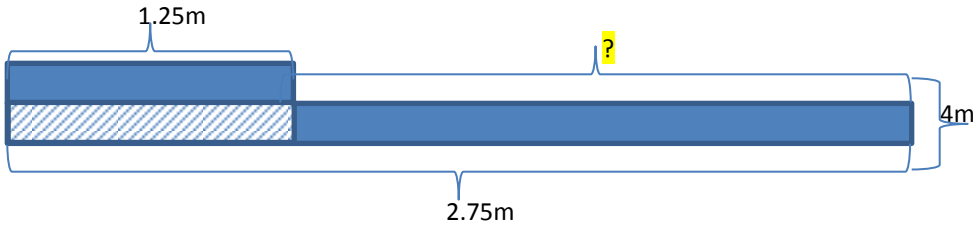
3357 +2643 = 6000 (the missing value is 6)

To solve two step word problems using the bar model.

A piece of cloth 4m long is cut into two pieces.
The first piece is 1.25m long
How much longer is the second piece of cloth?

4m – 1.25m = 2.75m
The second length is 2.75m

2.75m – 1.25 m = 1.5m
The second piece is 1.5m longer than the first piece.



Year 6 – Subtraction (When planning ensure you track back to year 5 for progression)

National Curriculum

Perform mental calculations, including with mixed operations and large numbers.
Use their knowledge of the order of operations to carry out calculations involving the four operations
Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
Solve problems involving addition, subtraction, multiplication and division.
Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Notes and guidance (non-statutory)

Pupils practise addition, subtraction etc. and use the formal written methods of columnar addition and subtraction.
They undertake mental calculations with increasingly large numbers and more complex calculations
Pupils round numbers to a specified degree of accuracy, for example to the nearest 10, 20, 50 etc., but not to a specified number of significant figures.
Pupils explore the order of operations using brackets; for example 2+1 x 3 = 5 and (2 + 1) x 3 = 9

Key vocab: - , subtract, subtractions, take (away), minus, leave, how many are left/left over? , how many have gone?, one less, two less, ten less, one hundred less, how many fewer is…than…?, how much less is…? Difference between, half, halve, tens boundary, hundreds boundary, regroup.

Key concepts

We can use addition facts that we know to solve other additions.
We can use addition facts we know to solve additions with decimals.
Estimating can be used to predict the answer and the inverse can be used to check it.
Key vocabulary: take away, subtract, how many are left, how much left, difference between, how much more, how many more to make, decrease, inverse and the minus sign.
Subtraction as: taking away, finding the difference between and complementary addition.

Potential barriers

Pupils without a strong foundation in place value will continue to make mistakes with column subtraction.
These are not 'careless mistakes' but fundamental misconceptions.
When subtracting with decimals such weaknesses are highlighted because of the decimal point.
Children are uncertain about the order of operations when carrying out calculations.
Pupils are unable to accurately estimate and use the inverse to check.

Learning objectives (see overleaf for exemplification)

To solve any subtraction with numbers to 2 decimal places. (see y5 progression)
To carry out calculations involving the four operations.
To work systematically to solve a problem
To use estimation to check answers to calculations.
To subtract negative numbers.
To understand the order of operations using brackets.
To round numbers accurately
To solve multi step word problems.

Example Questions

Respond rapidly to oral explaining strategy: 750-255, take 300 from 1240, 3500 subtract 2050, subtract 2250 from 8500, 1700 less than 2500, 3000 less than 10220, what must I take from 8.4 to leave 2.6? What is the difference between 2.2. and 6.5? How much more than 23.4 is 24.9? How much less than 6.8 is 4.2? What must I add to 5.4 to make 9.3? Decrease 5.6 by 1.9, 2.8 add a number is 4.3 what is the number? Find pairs of numbers with a difference of 13.5.
□-2.56=5.38, 7.65-6.85=□, □-1475=2924, □-Δ=0.03,
421.3- 82.57=□
Find the missing number in □-2485=4128
Vijay makes a sequence of numbers. He chooses a starting number and then subtracts equal amounts each time. The third number in his sequence is 45. The tenth number is -32. What is the first number in the sequence?
What number is 8 less than -4?
100-(22.75 + 19.08) = □

Mental strategies

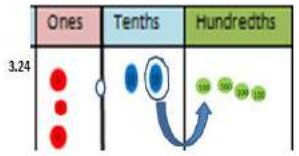
(building on mental strategies from y5)

To find the difference by counting up through the next multiple. (count up from the smaller to larger number mentally: 8000-2785 is 5 +10+ 200+ 5000 = 5215
Subtract 0.9, 1.9, 2.9 or 1.1, 2.1, 3.1 by subtracting 1,2,3 then adjusting by 0.1
Work out mentally one fact 4.97-1.58 and then state three other related facts
Subtract four digit+ multiples of 100 (570,000 + 250,000= □)
Find what to add to a decimal with units, 10ths and 100ths to make the next higher whole number or 10^{th}.
Subtract a pair of decimal fractions each less than 1 and with up to two decimal places.
Subtract numbers with different numbers of digits. Find the difference between 4387 and 782,175

Year 6 – Subtraction **Progression** (When planning ensure you track back to year 5)

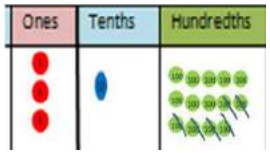
To solve any subtraction with numbers to 2 decimal places.

3.24 – 1.06= 2.18



3 ones 2 tenths 4 hundredths
3 ones 1 tenth 14 hundredths

First subtract the hundredths



3 . ¹2¹4
- 1. 0 6
8

14 hundredths – 6 hundredths = 8 hundredths
Then subtract the tenths:

3 . ¹2¹4
- 1. 0 6
. 1 8 (1 tenth – 0 tenths = 1 tenth)

Lastly subtract the ones:

3 . ¹2 ¹4
- 1 . 0 6
2 . 1 8

3 ones – 1 one = 2 ones

To carry out calculations involving the four operations.

9 - 4 + 3 =

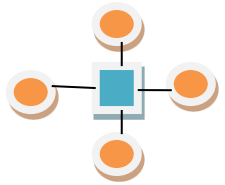
Calculating from left to right:

9 - 4 + 3 = 5 + 3 = 8

To work systematically to solve a problem

Arrange the numbers below in the circles so that the sum of the three numbers along each line is 4.5

1.2 1.8 1.4 1.6 1.5



To use estimation and rounding to check answers to calculations.

By rounding the actual values to more manageable numbers, you can estimate the answers to many problems:

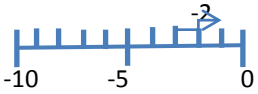
£2.99 + £3.10 + 99p ≈ £3 + £3 + £1 = £7

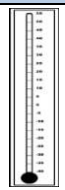
29 × 9 ≈ 30 × 10 = 300

61 ÷ 6 ≈ 60 ÷ 6 = 10

To subtract negative numbers.

(-3) - (-2) = -1





Subtracting a negative number is the same as adding:
eg (−5) − (−2) is the same as (−5) + 2 = −3

To understand the order of operations using brackets.

There were 94 players in Arsenal juniors. Last year 21 players left and 39 joined.
How many players are there now in Arsenal Juniors?

94 – 21 + 39
73 + 39

= **112** players are now in Arsenal juniors.

Standard order of operations:
1) Do the operation in brackets
2) Do operation involving indices
3) Multiply and divide in order from left to right.
4) **Add and subtract in order from left to right.**

How would you use a bar model to represent this word problem?

To solve multi step word problems

Using all four numbers exactly once and any of the operations (including one pair of brackets). Make a number sentence that has a value of 1.

Here is one solution: 6 + 5 - 9 - 1 = 11

Can you make number sentences that have values from 2 to 20?

Chapter 4

Multiplication

EYFS 2 – Multiplication (When planning ensure you track forwards to year 1)

Early Learning Goal 11
Using quantities and objects, they add two single-digit numbers and count on or back to find the answer. They solve problems, including sharing, doubling and halving.

KS1 ready:
Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Key vocab: count in, double, halve, lots of, groups of, times, group in pairs, equal groups of.

Key concepts

Multiplication begins with counting patterns and contexts involving equal groups.
Objects can be added over and over again to make 'more'.
Objects can be sorted into groups of the same number.
To get the total you count according to the number in the group.
All steps need to be taught through play as they are the precursors to multiplicative reasoning.
Children will be working in the concrete before moving towards the pictorial and abstract.

Learning objectives (see over page for exemplification)

To set out groups and find the total amount
To count patterns
To double numbers to 5, then 10
To double quantities of objects
To skip count in 2s
To skip count in 5s
To skip count in 10s
Begin to understand odd and even
Begin to relate the addition of doubles to counting on. (How many wheels on two cars? **4** (hold four in head) 5, 6, 7, **8** (count on). 4+4=8

Potential barriers/misconceptions

Children inaccurate when displaying arrays of cubes/objects and so pattern is not clear.
Link not clear between the array and the seemingly abstract number given as the answer.
Children unable to place objects in equal groups.
Not secure with one to one correspondence counting in ones, therefore will be unable to count pairs accurately
When counting orally in 10s: 60,70,80 follow a regular pattern which link to single digit numbers however 10, 20, 30 do not.
Conceptual understanding of 'same' and 'different' is not secure (both language and concept).

Example Questions

Give everyone two biscuits from the jar.
Can you count the spots on each side of your butterfly? Does it have an equal number of spots on each side?
I will clap where there is a number missing. 1, 2, 3, (clap) 5. Tell me the missing number
2, 4, 6, (clap), 10 Tell me the missing number
10, 20, 30, (clap), 40. Tell me the missing number
How many fingers are there on two hands?
How many eggs are there in the box? How are they arranged? (in 2s)
Count the pairs of animals on the ark.
Count these pairs of socks. How many pairs are there? How many socks are there altogether?
How many buttons are there on this coat? Count them in twos. Now count them in fives. (answer- 10)

Mental Maths (can revisited throughout day once concept has explicitly shared)

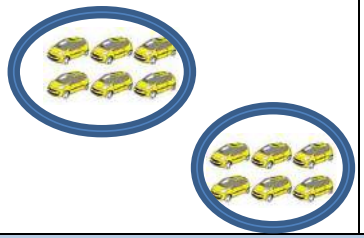
Count in tens (recite the sequence ten, twenty, thirty… one hundred.) Do the same backwards.
Count on and back in tens from a given tens number
Say the tens number that goes before or after a given tens number. (When you count in tens, what number comes before 60? 90?)
Count from a given tens number and stop at another. (count on in tens from 20 and stop at 70, count back in tens from 60 and stop at 30)
Count around in a circle of children, starting with Abdul on 20, who do you think will say 70?
Understand odd and even numbers linked to getting 'into pairs'.
Count pairs: children, socks, animals in the ark, eggs in an egg box

EYFS 2 – Multiplication Progression (a combination of these models and images can be used for every objective)

To set out groups and find the total amount

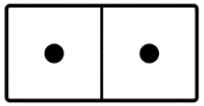
Can recognise when objects are put into groups of the same amount.

To find matching groups that are the same



To recognise when each person is given the same amount

To double numbers to 5, then 10

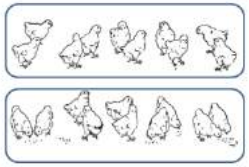


Double 1 is two

Double 4 is eight

To double quantities of objects

Using these toy chicks:

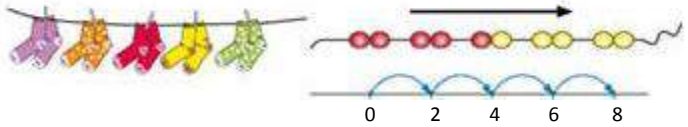


Can you double the chicks
In the first box?

How many chicks now?

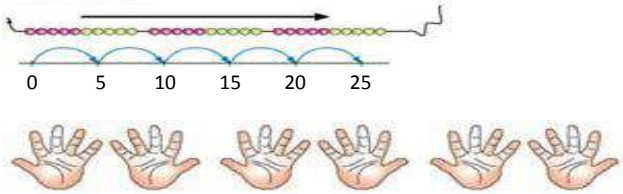
Can you double the cubes?

To skip count in 2s

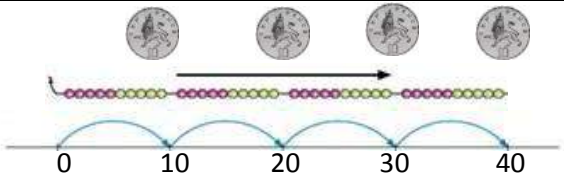


Count: Pairs of socks, pairs of shoes, pairs of gloves, pairs of animals, talk partners etc.

To skip count in 5s



To skip count in 10s



To place objects into arrays

Begin to relate the addition of doubles to counting on

How many wheels on two cars?

4 5, 6, 7, **and 8**.

4+4=8 **Double 4 = 8**

Begin to understand odd and even

Objects that can be counted in pairs are even

Objects that can't be counted in pairs are odd

Year 1 – Multiplication

(When planning ensure you track back to Reception and forwards to year 2)

National Curriculum	Notes and guidance (non-statutory)
Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities. They make connections between arrays, number patterns, and counting in twos, fives and tens.
Key vocab: count in, double, halve, lots of, groups of, times, group in pairs, equal groups of, x, times, multiplied by, multiple of, one, twice, three times, ten times, repeated addition, array, row, column. **Key concepts** Multiplication begins with counting patterns and equal groups in context. Multiplication is introduced when objects or numbers are combined. It is associated with the idea of repeated addition. Pupils will be able to conceptualise multiplication as groups of items.	**Learning objectives** (see over page for exemplification) To place objects into equal groups To double numbers To double two digit numbers To place objects into arrays To pictorially represent multiplication sentences To understand repeated addition Can describe an array in two ways To make multiplication stories To move towards the bar model to solve word problems
Potential barriers/misconceptions Still counts in ones to find how many there are in a collection of equal groups; does not understand vocabulary for example 'multiplied by'. When objects placed in arrays it may be done inaccurately therefor link between arrays and answers unclear. Pupils may not focus on 'rows of' or 'columns of' but only see arrays as a collection of ones. Don't understand how 'turning the grid around' shows that multiplication can be done in any order.	
Example Questions How many pencils do I need if everyone has to have 2? (There are eight people in the group) How many shoes do we need for these three dolls? John makes biscuits with 1 egg, 4 spoons of flour, 3 spoons of sugar, 5 spoons of milk. Lucy makes double the amount of biscuits. She will need: □ egg, □ spoons of flour, □ spoons of sugar, □ spoons of milk. Add more fives until the total is 25. 5 + 5 + ……. Write the answer: 6 x 2 =	**Mental Maths** To count in twos, fives and tens Count forwards and backwards in 2s from any given number. Count forwards and backwards in 5s from any given number. Count forwards and backwards in 10s from any given number. Recognition of all odd and even numbers Rapid recall of doubles to 10 (and corresponding halves) Rapid recall of doubles to 20

Year 1 – Multiplication

Progression (a combination of these models and images can be used for every objective)

To place objects into equal groups

How many pencils in each group?

How many altogether?

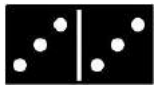
Can you count in 5s?

Adding the same number: How many groups of shells are ther
How many shells are there in each grou

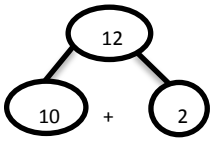
There are 4 groups
Each group has 3 shells in.
3 + 3 + 3 + 3 = 12
(4 threes = 6)
(3 groups of 4 = 12)

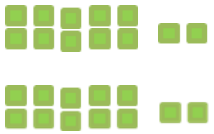
There are 12 shells altogether.

To double numbers



To double numbers (over 10)





Double 10 Double **2**

20 **+** **4** **= 24**

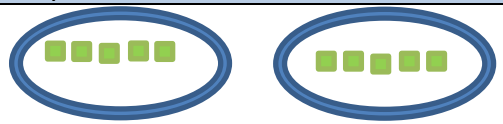
To place objects into arrays

Arrange your objects into ro
ı you arrange them in different ways?
Each row must have the same number.
Can you add the total amount?

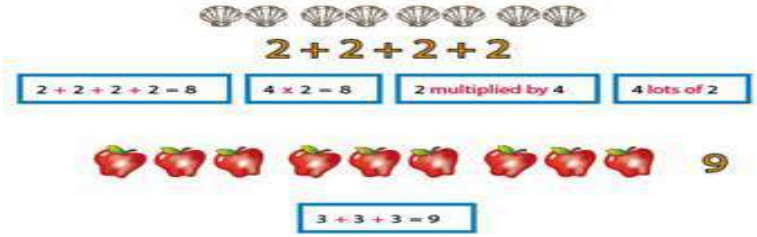
To pictorially represent multiplication sentences

5 x 2 = 10

5 + 5 = 10



To understand repeated addition



To describe an array in two ways

To make multiplication stories

Can you tell a multiplication story about these cakes?

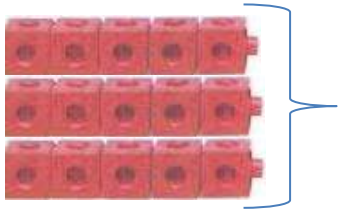
Take note of the number of groups first

Then the number of items in each group.

I see □ groups of cakes.
Each group has □ cakes. □ x □ = □ There are □ cakes altogether.

To move towards the bar model to solve word problems

There are three children. Each child has five sweets. How many sweets do they have altogether?



?

5 + 5 + 5 = 15

5 x 3 = 15

3 x 5 = 15

They have 15 sweets altogether

Year 2 – Multiplication (When planning ensure you track back to year 1 and forwards to year 3)

National Curriculum	**Notes and guidance (non-statutory)**
Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.	Pupils use a variety of language to describe multiplication and division. Pupils are introduced to the multiplication tables. They practise to become fluent in the 2, 5 and 10 multiplication tables and connect them to each other. They connect the 10 multiplication table to place value, and the 5 multiplication table to the divisions on the clock face. They begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations. Pupils work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition. They begin to relate these to fractions and measures (for example, 40 ÷ 2 = 20, 20 is a half of 40). They use commutativity and inverse relations to develop multiplicative reasoning (for example, 4 × 5 = 20 and 20 ÷ 5 = 4).
Key vocab: count in, double, halve, lots of, groups of, times, group in pairs, equal groups of, x, times, multiplied by, multiple of, one, twice, three times, ten times, times as (big, long, wide… and so on) repeated addition, array, row, column. **Key concepts** Key vocabulary: double, times, multiply, multiplied by, multiple of, lots of, groups of, times as (big, long, wide…) X Multiplication is introduced when objects or numbers are combined. It is associated with the idea of repeated addition. Pupils will be able to conceptualise multiplication as groups of items. Multiplication can be done in any order- this can be shown in the arrangement of arrays. Doubling is the inverse of halving	**Learning objectives** (see over page for exemplification) To identify odd and even numbers To understand multiplication as repeated addition To use arrays To know 2, 5, 10 times tables. To multiply using partitioning To understand the commutative property of multiplication. To interpret multiplication sentences (The first factor referring to the number of groups and the second factor as the number of items in each group.) To know all corresponding multiplication and division facts (i.e. 2x4= 8, 4x2= 8 and 8÷4 =2, 8÷2=4) To break a number into factors To connect the 10 times table with place value To use the bar model to represent word problems
Potential barriers/misconceptions Pupils may not focus on 'rows of' or 'columns of' but only see arrays as a collection of ones. Don't understand how 'turning the grid around' shows that multiplication can be done in any order. Not understanding that multiplication is repeated addition	
Example Questions Respond rapidly to oral and written questions such as: two fives, double 5, 6 times 2, 5 multiplied by 2, multiply 4 by 2. Two tens, double 2, 3 times 4, 9 multiplied by two, multiply 5 by 8. Is 20 a multiple of 5? 6x2=□ 9x□=18 □ x2=14 6x10=□ 2x□=20 □x10=40 How many wheels are there on three cars? Jo's plane is 6cm wide. Mo's box is twice as wide. How wide is Mo's box? (scaling) Ella's dad washes some cars. He uses 12 buckets of water. Each bucket has five litres of water. How many litres of water does he use altogether? Tara does not know how to work out 16 x 5. Can you show her how to do this? There are 15 apples in a tray. Ling has 4 trays of apples. How many apples does Ling have altogether? Show how you work it out.	**Mental Maths** Rapid recall of 2,5 and 10 times tables Count in 5s clockwise around a clock face/ anticlockwise around a clock face. Count forwards and backwards in 2s, 5s and 10s from any given number. Recognition of all odd and even numbers To recall related multiplication and division facts linked to other multiplication tables. (3x4=12, 4X3=12, 12÷4=3, 12÷3=4) Rapid recall of doubles and their corresponding halves. (double 12 is 24, half 24 is 12) Rapid recall of half of all 2 digit even numbers. (half of 12, 18, 42 etc) Recognise that multiples of 10 end in 0, 5 end in 5 and 0, 2 end in 0,2,4,6,8. Recognise two digit multiples of 10,5,2 (65 is a multiple of 5, 72 is a multiple of 2, 50 is a multiple of 5 and 10) Work out the four times table by doubling the two times table. Multiply a single digit by 1 or 10. (3x1 = 3, 7x10=70 etc) Multiply a single digit up to 5 by 2,3,4,5. (2x3=□ 4x4=□)

Year 2– Multiplication

Progression (a combination of these models and images can be used for every objective)

To identify odd and even numbers

All numbers that can be '**paired**' are even.
All numbers that are in the **x2 table** are even.
All numbers that end in **0, 2, 4, 6, 8** are even.
To establish if a number is even we look at the **ones**.

To understand multiplication as repeated addition

3 + 3 + 3 = 9
3 groups of 3
3 x 3 = 9

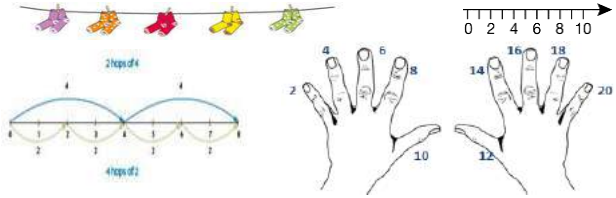
To use arrays

Three in a row
Four in a column
3+3+3+3 = 12

Four in a row
Three in a column
4 + 4 + 4 = 12

To know 2, 5, 10 times tables up to x12.

Use concrete objects grouped, number lines, skip counting etc



To multiply using partitioning

Use knowledge of the 2, 5 and 10 times table to work out multiples of 7.

7 x 3 = (5 + 2) x 3 = (5 x 3) + (2 x 3) = 15 + 6 = 21

To understand the commutative property of multiplication.

Using squared paper to support understanding
Each child has picked 3 flowers.
How many flowers will 5 children have altogether?

The fixed number is 3
It is being multiplied by 5.
3x 5 = 15 (5 children will have 15 flowers altogether)
If paper is turned it will show 5 x 3 = 15.

To interpret multiplication sentences (The first factor referring to the number of groups and the second factor as the number of items in each group.)

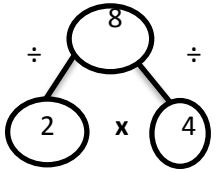
How many books are there?

There are two ways to find the numbers of books. (1 and 2)

1) First count the number of groups. (There are 3 groups)
Then count the number of books in each group. (5 books)
5 + 5 + 5 = 15
3 x 5 = 15 **There are 15 books altogether.**

2) First count the number of items in each group. (5 books)
Then, count the number of groups. (There are 3 groups)
The number 5 multiplies 3 times.
5 x 3 = 5 + 5 + 5 = 15 **There are 15 books altogether**

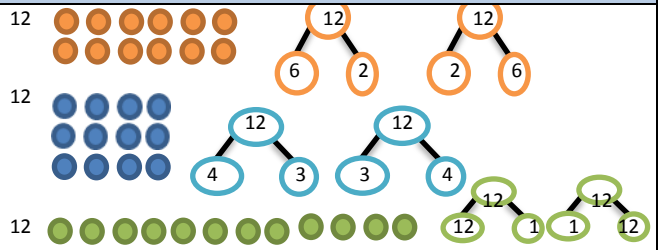
To know all corresponding multiplication /division facts



Part, part, whole.

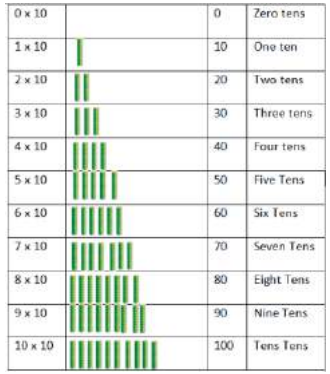
2 x 4 = 8
4 x 2 = 8
8 ÷ 2 = 4
8 ÷ 4 = 2

To break a number into factors



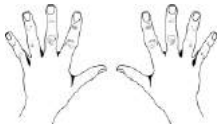
To connect the 10 times table with place value

Ten ones = One ten



0 x 10		0	Zero tens
1 x 10		10	One ten
2 x 10		20	Two tens
3 x 10		30	Three tens
4 x 10		40	Four tens
5 x 10		50	Five Tens
6 x 10		60	Six Tens
7 x 10		70	Seven Tens
8 x 10		80	Eight Tens
9 x 10		90	Nine Tens
10 x 10		100	Tens Tens

Count in tens using dienes:



To use the bar model to represent word problems

There are 5 biscuits in each packet.
There are 4 packets.
How many biscuits are there altogether?

5 + 5 + 5 + 5 = 20

5 x 4 = 20

There are 20 biscuits altogether.

Year 3 – Multiplication (When planning ensure you track back to year 2 and forwards to year4)

National Curriculum

Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables

Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Notes and guidance (non-statutory)

Pupils continue to practise their mental recall of multiplication tables when they are calculating mathematical statements in order to improve fluency. Through doubling, they connect the 2, 4 and 8 multiplication tables.

Pupils develop efficient mental methods, for example, using commutativity and associativity (for example, 4 × 12 × 5 = 4 × 5 × 12 = 20 × 12 = 240) and multiplication and division facts (for example, using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (for example, 30 × 2 = 60, 60 ÷ 3 = 20 and 20 = 60 ÷ 3).

Pupils develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division.

Pupils solve simple problems in contexts, deciding which of the four operations to use and why. These include measuring and scaling contexts, (for example, four times as high, eight times as long etc.) and correspondence problems in which m objects are connected to n objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).

Key vocab: double, times, multiply, multiplied by, multiple of, lots of, groups of, times as (big, long, wide…) product, lots of, groups of, repeated addition, array, row, column.

Key concepts

Multiplication as: repeated addition
describing an array
scaling (find a piece of string that is five times as long)

Multiplication can be done in any order (5x8 = 8x5) but to understand that division cannot. (16÷2 is not the same as 2÷16).

Multiplication reverses division (is the inverse of division)

When you multiply by ten the digits move one place to the left

Learning objectives (see over page for exemplification)

To use number bonds for factors and products
To understand how place value changes when multiplying by 10
To calculate two digit numbers multiplied by one digit numbers
To carry out short multiplication without regrouping
To carry out short multiplication with regrouping in ones, tens and hundreds
To understand measuring and scaling problems

Potential barriers/misconceptions

Children may need to go back to multiplication as an array, or repeated addition to gain security.

Some children struggle to apply partitioning and recombining when multiplying. E.g. 14 x3 is calculated as (10x3) + 4 = 34 Or 14 x 3 = 312 When they should do (10x3) + (3x4) = 30+12=42

Lack of confidence with place value sees confusion in the value of the two digits.

Children are incorrectly taught that x10 involves 'adding a zero' rather than developing understanding of place value. Also unable to see that x100 is the same as x10 and x10 again.

Example Questions

Respond rapidly to oral and written questions such as: three fives, double 11, 6 times 4, 5 multiplied by 8, multiply 4 by 3. Four tens, double 24, 3 times 5, 9 multiplied by four, multiply 5 by 8.

Is 20 a multiple of 5?

5x3=□ 8x□=40 □ x9=45

6x20=□ Δx□=60

In the shop there are 6 cans of beans in each of 4 rows. How many cans of beans are there?

Natalie has 10 football cards. Kojo has 3 football cards for every one of Natalie's. How many cards does Kojo have?

Finn has 4 stickers. Howard has 3 times as many stickers as Finn. How many stickers does Howard have?

Mental Maths

Rapid recall of 3, 4 and 8 times tables
Count forwards and backwards in 3s from any given number.
Count forwards and backwards in 4s from any given number.
Count forwards and backwards in 8s from any given number.
To use the 2,5 and 10 times table to derive other multiplication facts (if I know 2x5= 10 I also know 20x5=100)
To know doubles of all numbers up to 50
To know doubles of all multiples of 5 up to 100
Observe the effect of multiplying by 10
Multiply any single digit by 1, 10, 100 and 0
Multiply a two digit number by 2,3,4, or 5 without crossing the tens boundary. (11x5, 23x2)
Check halving with doubling
To multiply multiples of 10 with 1 digit number

Year 3– Multiplication

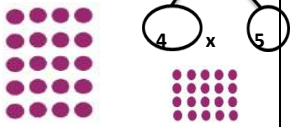
Progression (a combination of these models and images can be used for every objective)

To use number bonds for factors and products

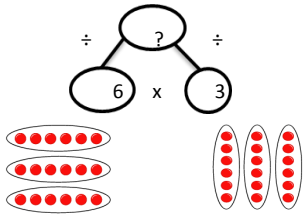
(factor ÷ factor = product)

5x 4 = 20

And 4 x 5 = 20

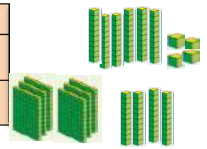


6 x 3= □



To understand how place value changes when multiplying by 10

Thousands	Hundreds	Tens	Ones
		6	4
	6	4	0



64 x 10 = 640

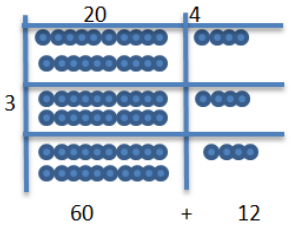
When multiplying by ten the original number moves one place to the left.

It gets ten times bigger(X10).

Zero acts as a place holder in the ones.

To calculate two digit numbers multiplied by one digit numbers

24 x 3=



20 X 3 = 60

4 X 3 = 12

60 + 12 = 72

Visual link to arrays

To carry out short multiplication without regrouping

12 x 3 =

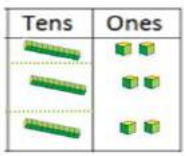
First multiply the ones by 3

1 2

X 3

6

2 ones x 3 = 6 ones



Then multiply the tens by 3

1 2

X 3

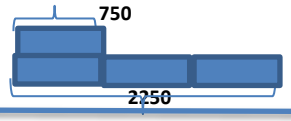
3 6

1 ten x 3 = 3 tens

When we multiply 12 by 3, we get the product of 12 and 3. (36 is the product of 12 and three)

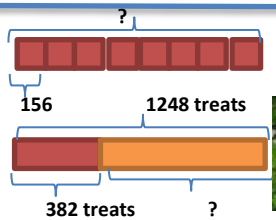
To understand measuring and scaling problems

Beccy read 750 words in one day. Izabela read three times as many words. How many pages did Izabela read?



750 x 3 = 2250 **Izabela read 2250 words**

Mr Miaow had 8 bags of cat treats. Each bag contained 156 cat treats. He fed 382 cat treats to his cats. How many cat treats had he left?

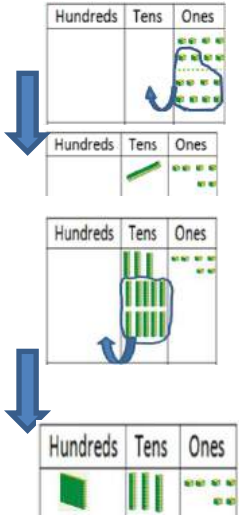


156 x 8 = 1248

1248 – 382 = 866 **Mr Miaow had 866 cat treats left.**

To carry out short multiplication with regrouping in ones, tens and hundreds

68 x 2=



First, multiply the ones by 2

6 8

X 2

1 6

8 ones x 2 = 16 ones

Regroup the ones.

16 ones = 1 ten and 6 ones

Then, multiply the tens by 2

6 8

X 2

1 3 6

6 tens x 2 = 12 tens

Add the tens:

12 tens + 1 ten = 13 tens

Regroup the tens:

13 tens = 1 hundred 3 tens

So…. 68 x 2 = 136

Year 4 – Multiplication (When planning ensure you track back to year 3 and forwards to year 5)

National Curriculum

Recall multiplication and division facts for multiplication tables up to 12 × 12
Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
Recognise and use factor pairs and commutativity in mental calculations
Multiply two-digit and three-digit numbers by a one-digit number using formal written layout
Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Notes and guidance (non-statutory)

Pupils continue to practise recalling and using multiplication tables and related division facts to aid fluency.
Pupils practise mental methods and extend this to three-digit numbers to derive facts, (for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6).
Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers.
Pupils write statements about the equality of expressions (for example, use the distributive law 39 × 7 = 30 × 7 + 9 × 7 and associative law (2 × 3) × 4 = 2 × (3 × 4)). They combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations for example, 2 x 6 x 5 = 10 x 6 = 60.
Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as the numbers of choices of a meal on a menu, or three cakes shared equally between 10 children.

Key vocab: double, times, multiply, multiplied by, multiple of, lots of, groups of, times as (big, long, wide…) product, lots of, groups of, repeated addition, array, row, column, factor, inverse.

Key concepts

When you multiply by ten the digits move one place to the right
That commutative law is: 8x15 = 15x8
Associative law is: 6x15 = 6 x (5x3) = (6 x 5) x 3 = 30 x 3 = 90
Distributive law is: 18x5 = (10+8) x 5 = (10x5) + (8x5) = 50+40= 90
56+56+56 is equivalent to 56x3 or 3x56
Multiplication by 1 leaves a number unchanged
Multiplication by zero results in zero
Multiplication is the inverse of division

Learning objectives (see over page for exemplification)

To multiply by ten using place value grids and dienes
To multiply two digit numbers by a one digit number (see year 3 exemplification)
To multiply three digit numbers by one digit number
To multiply two digit by two digit number
To use the distributive law: 32x3 = (30x3) + (2x3) = 90+6 = 96
To use associative law to multiply three numbers
To solve problems using scaling
To derive multiplication and division facts from three digit numbers
To solve two step word problems.
To recognise factors of a number
To multiply decimals

Potential barriers/misconceptions

Children unclear around language of multiplication: multiple, factor
Children not secure in their rapid recall of times tables facts and conceptual understanding
Some children not using known facts and commutative properties to solve sums such as 8x□=48
Children may not see the link between known facts and multiplying multiples by 10 and 100.
Children are incorrectly taught that x10 involves 'adding a zero' rather than developing understanding of place value. Also unable to see that x100 is the same as x10 and x10 again.

Example Questions

How many times larger is 250 than 25? Prove it.
How many £1 coins are there in £15, £150, £1500? How many 10p coins?
Tins of cat food are put in packs of 10. One tin costs 62p. How much does one pack cost? Ten packs?
Respond rapidly to oral questions explaining strategy: two elevens, double 16, seven times six, 9 multiplied by 3, multiply 15 by 7, by zero, by 1.
Is 40 a multiple of 5 how do you know? What else is it a multiple of? What is the product of 12 and 7?

Mental Maths

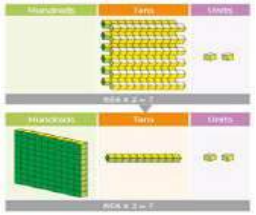
Rapid recall of all numbers multiplied by 10, 100, 1000
Rapid recall of all multiplication and division facts up to 12 x 12
To understand what happens when multiplying by 1 and 0
To multiply together three numbers
To know by heart all doubles and halves (double 34 is double 30 + double 4 = 60+8= 68)
To multiply by 4 (double and double again: 7x4 = double 7= 14. Double 14 = 28)
To multiply by 5 (multiply by 10 and halve: 5x9 = 10x9= 90 halved = 45)
To multiply by 20 (multiply by 10 and double)
Work out 8 times table by doubling four times table.
Use doubling to work out multiples of 15. (1x15= 15, 2x15= 30, 4x15=60, 8x15=120, 16x15=240) Use combinations of these facts to find e.g. 11x15 (8x15 + 2x15 + 1x15 = 120 +30+15 = 165)
Work out the six times table by adding 2 times table facts and 4 times table facts.
To multiply a number by 9 or 11, multiply it by 10 and add/subtract the number (14x9 = 140-14 = 126 and 14x11= 140+14= 154)
To know the three corresponding number facts when given a multiplication number sentence.

Year 4– Multiplication — Progression (a combination of these models and images can be used for every objective)

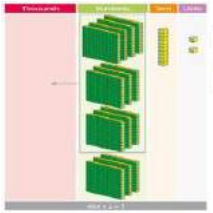
To multiply three digit numbers by one digit number

656 x 2 =

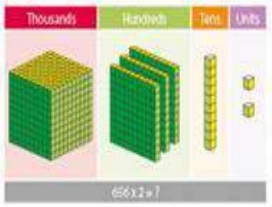
6 ones x 2 = 12 ones
12 ones = 1 Ten and 2 ones



5 tens x 2 = 10 tens
Add the tens: 10 tens + 1 ten = 11 tens
Regroup the tens: 11 tens = 1 hundred 1 ten



6 hundreds x 2 = 12 hundreds
Add the hundreds: 12 hundreds + 1 hundred = 13 hundreds



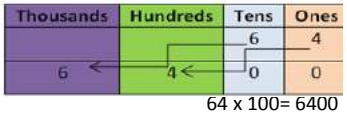
Regroup the hundreds:
13 hundreds = 1 thousand and 3 hundreds

```
  6 5 6
X     2
-------
  1 3 1 2
  1 1
```

To multiply by ten and one hundred

64 x 10= 640

64 x 100 = 6400

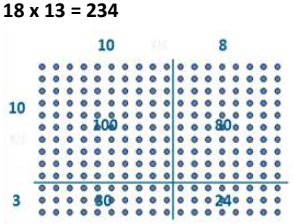


64 x 100= 6400

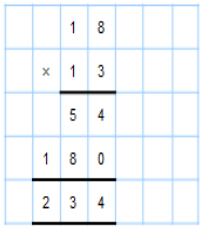
When multiplying by one hundred the original number moves one place to the left.
It gets one hundred times bigger(X100).
Zero acts as a place holder in the ones.

To multiply two digit by two digit

18 x 13 = 234



18 x 3 = 54
18 x 10=180



To multiply two digit by two digit

56 x 27= 1512

×	20	7	
50	1000	350	1350
6	120	42	162
			1512

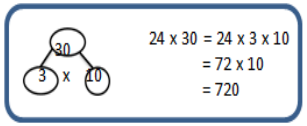
1

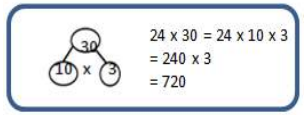
```
  56
x 27
----
1120
 392
----
1512
 1
```

56 × 20
56 × 7

To use the distributive law: 32x3 = (30x3) + (2x3) = 90+6 = 96

Find the product of 24 and 30





To use associative law to multiply three numbers

For any three numbers a, b and c:

(a x b) x c = a x (b x c)

(2 x 3) x 4 = 24
2 x 3 = 6
6 x 4 = 24

2 x (3 x 4) =
(3 x 4) = 12
2 x 12 = 24

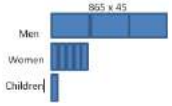
To solve problems using scaling

Scaling is the process modelled by multiplication in which a given quantity is increased by a scale factor. Doubling and trebling are examples of scaling by 2&3. Scaling by a factor of less than 1 (halving) reduces the size of the quantity.

John earns £125 a week- his brother earns three **times as** much = 125 x3
I purchased 25 tickets at £12 **per** ticket. = 25 x 12
A box of drinks has 24 cans. **Each** can costs 45p each. = 24 x 45

To solve two step word problems. (including inter scaling)

At the Olympic stadium the number of men is 3 times the number of women. The number of women is 5 times the number of children at the stadium.



1. How many times the number of children is the number of men?
3 x 5 = 15 The number of men is 15 times the no. of children.

2. If there are 730 children how many men are there?
730 x 15 = 10 950. There are 10 950 men

To recognise factors of a number

'If 24 is a multiple of 6 then 6 is a factor of 24'.
We colloquially say 6 'goes into' 24 four times'.
1 is a factor of all numbers & every number is a factor of itself.
Arrays support understanding of factor pairs: 1x24, 2x12, 3x8, 4x6

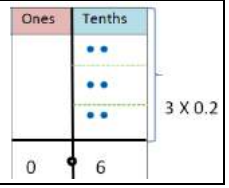
Factors of 24 are:
1, 2, 3, 4, 6, 8, 12 and 24

xxxx
xxxxxxxxxxxx xxxxxxxx xxxx
xxxxxxxxxxxxxxxxxxxxxxxx xxxxxxxxxxxx xxxxxxxx xxxx
xxxxxxxx xxxx

To multiply decimals

0.2 x 3 =
0.2 + 0.2+ 0.2 = 0.6
3 x 2 tenths = 6 tenths = 0.6

```
 0.2
X 3
---
 0.6
```



Year 5 – Multiplication (When planning ensure you track back to year 4 and forwards to year 6)

National Curriculum

Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers know and use the vocabulary of prime numbers, prime factors and composite (nonprime) numbers
Establish whether a number up to 100 is prime and recall prime numbers up to 19
Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
Multiply numbers mentally drawing upon known facts
Multiply whole numbers and those involving decimals by 10, 100 and 1000
Recognise and use square numbers and cube numbers, and the notation for squared(^{2}) and cubed (^{3})
Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Notes and guidance (non-statutory)

Pupils practise and extend their use of the formal written methods of short multiplication and short division. They apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations.
They use and understand the terms factor, multiple and prime, square and cube numbers.
Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (for example, $98 \div 4 = \frac{98}{4} = 24$ r 2 $= 24\frac{1}{2} = 24.5 \approx 25$).
Pupils use multiplication and division as inverses to support the introduction of ratio in year 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1000 in converting between units such as kilometres and metres.
Distributivity can be expressed as a(b + c) = ab + ac.
They understand the terms factor, multiple and prime, square and cube numbers and use them to construct equivalence statements (for example, 4 x 35 = 2 x 2 x 35; 3 x 270 = 3 x 3 x 9 x 10 = 9^2 x 10).
Pupils use and explain the equals sign to indicate equivalence, including in missing number problems (for example, 13 + 24 = 12 + 25; 33 = 5 x □).

Key vocab: times, multiply, multiplied by, product, multiple, inverse and x sign, double, times, multiply, multiplied by, multiple of, lots of, groups of, times as (big, long, wide…) product, lots of, groups of, array, row, column, factor.

Key concepts

When you multiply a number by 10/100, the digits move one/ two places to the left
Multiplying by 100 is equivalent to multiplying by 10 and 10 again
That commutative law is: 8x75 = 75x8
Associative law is: 18x11 = (2 x9)x 11 = 2 x (9x11) = 2 x 99 = 198
Distributive law is: 26x7 = (20+6) x 7 = (20x7) + (6x7) = 140 + 42= 182
With positive whole numbers, multiplying makes a number larger
Multiplication is the inverse of division and can be used to check results

Potential barriers/misconceptions

Children may struggle to partition a two digit number into tens and units correctly for whole numbers and tenths for decimals.
Children believe that multiplication always increases a number. This is only when a positive number is multiplied by a whole number greater than 1.

Example Questions

How many times larger is 2600 than 26? Prove it.
How many £10 notes are there in £120, £1200? How many £1 coins, 10p coins, 1p coins?
Tins of cat food are put in packs of 10. One tin costs 62p. How much does one pack cost? Ten packs?
What is 4 squared? What is the square of 6? What is 8^2? Which number multiplied by itself is 36?
Respond rapidly to : two twelves, double 32, 7 times 8, 9 multiplied by7, multiply 31 by 8, zero, 1.
Is 81 a multiple of 3? How do you know? What is the product of 25 and 4?
132x46=□ 6x□=4.8 □ x9=189
80x9=□ Δx□=162
I have 16 boxes each weighing 20.5kg. What is their total weight?
What is 72 multiplied by 38?
Calculate 349 x 6
What is the area of a field 27 metres by 37 metres?

Learning objectives (see over page for exemplification)

To identify common factors of two numbers.
To know prime numbers, prime factors and composite (non-prime) numbers
To solve problems involving multiplication.
To multiply numbers up to four digits by a one digit number
To multiply numbers up to four digits by a two digit number
To recognise and use squared and cubed numbers
To understand the law of distributivity
To multiply whole numbers& decimals by 10, 100, 1000

Mental Maths

To find all factor pairs of a number & find common factors of two numbers
To establish whether a number up to 100 is prime
To recall prime numbers up to 19
Recognise 1,4,9,16,25,36,49,64,81,100 as square numbers (relate to drawings of squares)
Find all the pairs of factors for any number to 100 (pairs of factors to 36 are 1&36, 2&18, 3&12, 4&9, 6&6)
Use factors for finding products mentally (16x12 = 16 x 3x2x2= 48 x2x2 = 96 x2 = 192)
To double using known facts (double 79 = double 70 + double 9 = 140+18= 158)
Double a number ending in 5 and halve the other number (16x5 is equivalent to 8x10=80)
To multiply by 50 (multiply by 100, then halve: 26 x 50 = 26 x 100 = 2600 halved = 1300)
Calculate 16 times table by doubling 8 times table facts
Calculate 25 times table by doubling: (1x25=25, 2x25=50, 4x25=100, 8x25=200, 16x25=400 use combinations of these facts to work out e.g. 25x25 = (16x25) + (8x25) + (1x25) = 625
Work out 12 times table by adding 2 times table and 10 times table
To multiply a number by 19 or 21, multiply it by 20 and add or subtract the number (13x21 = 13x20+13= 273)

Year 5– Multiplication

Progression (a combination of these models and images can be used for every objective)

To identify common factors of two numbers.

What are the common factors of **8** and **12**?

8 = 1 x 8 12 = 1 x 12
8 = 2 x 4 12 = 2 x 6
12 = 3 x 4

The factors of 8 are: **1, 2, 4**, and 8
The factors of 12 are: **1, 2,** 3, **4,** 6, and 12
The common factors of 8 & 12 are **1, 2 and 4.**

To know prime numbers,

Any number that has **two** factors and no more is a **prime number**. (it cannot be divided by any number except one and itself)

The first 20 prime numbers are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67 and 71.

(one is not prime as it has only 1 factor)

….prime factors

Every number can be written as a product of prime numbers:

40 = 2 x 2 x 2 x 5

126 = 2 x 3 x 3 x 7

28 = 2 x 2 x 7

….and composite (non-prime) numbers

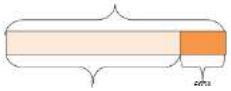
e.g. 10 has more than 2 factors: 1, 2, 5, and 10. It is called a composite number and can be arranged as a rectangular array with more than one row. (Prime numbers can only be arranged in a single row). xxxxx

Composite numbers are often called **rectangular numbers.** (links to arrays) xxxxx

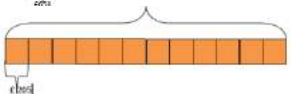
All non-prime numbers (except 1) are composite.

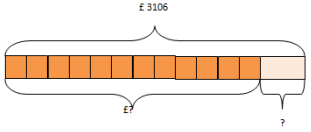
To solve problems involving multiplication.

Tom had £3756. He saved £650 and spent the rest on 12 pairs of shoes and some clothes. The pairs of shoes cost £205 each. How much did he spend on clothes? £3756



£3756- £650 = £3106
She spent £3106 altogether



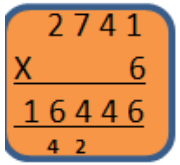


12 x £205 = £?
12 pairs of shoes cost £

£ 3106 - £ _____ = £_____
He spent £_____ on clothes.

To multiply numbers up to four digits by a one digit number (short multiplication)

2741 x 6 = 16446



To multiply numbers up to four digits by a two digit number (long multiplication)

286 x 29 = 8294

The same method would Be applied for four digit multiplication.

To recognise and use squared numbers

3^2 means '3 squared', or 3 x 3.
It tells us how many times we should multiply 3 by itself.

1^2 = 1 x 1 = 1
2^2 = 2 x 2 = 4
3^2 = 3 x 3 = 9
4^2 = 4 x 4 = 16
5^2 = 5 x 5 = 25 etc

The opposite of a square number is a square root.

We use the symbol $\sqrt{}$ to mean square root.

So we can say that $\sqrt{4}$ = 2 and $\sqrt{25}$ = 5.

1, 4, 9, 16, 25… are known as square numbers.

To recognise and use cubed numbers

2 x 2 x 2 means '2 cubed', and is written as 2^3

1^3 = 1 x 1 x 1 = 1
2^3 = 2 x 2 x 2 = 8
3^3 = 3 x 3 x 3 = 27
4^3 = 4 x 4 x 4 = 64
5^3 = 5 x 5 x 5 = 125…etc.

The opposite of a cube number is a cube root.

We use the symbol $\sqrt[3]{}$ to mean cube root.

So $\sqrt[3]{8}$ is 2 and $\sqrt[3]{27}$ is 3.

Each number only has one cube root.

1, 8, 27, 64, 125… are known as cube numbers.

To understand factor, multiple, prime, square and cube numbers and construct equivalence statements.

4 x 35 = 2 x 2 x 35

3 x 270 = 3 x 3 x 9 x 10 = 9^2 x 10

To understand the law of distributivity

Distributivity can be expressed as:

If: a=12, b=5, c=2

12 (5 + 2) = 12 x 7 = 84

=

(12 x 5) + (12 x 2) = 60 + 24 = 84

$a(b+c) = ab+ac$

To multiply whole numbers& decimals by 10, 100, 1000

5.928 x 10 = 59.28

X10

Tens	Ones	Tenths	Hundredths	Thousandths
	5	9	2	8
5	9	2	8	

Multiplying decimals by 100 and 1000:

0.3 x 100 = $\frac{3}{10}$ x 100 = 3 x 10 = 30

0.3 x 1000 = $\frac{3}{10}$ x 1000 = 3 x 100 = 300

Year 6 – Multiplication (When planning ensure you track back to year 5 for progression)

National Curriculum

Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
Perform mental calculations, including with mixed operations and large numbers
Identify common factors, common multiples and prime numbers
Use their knowledge of the order of operations to carry out calculations involving the four operations
Solve problems involving addition, subtraction, multiplication and division
Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Notes and guidance (non-statutory)

Pupils practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of columnar addition and subtraction, short and long multiplication, and short and long division.
They undertake mental calculations with increasingly large numbers and more complex calculations.
Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency.
Pupils round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures.
Pupils explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.
Common factors can be related to finding equivalent fractions.

Key vocab: times, multiply, multiplied by, product, multiple, inverse and x sign, double, multiple of, lots of, groups of, times as (big, long, wide…) product, lots of, groups of, repeated addition, array, row, column.

Key concepts

When you multiply by 10/100/1000 the digits move 1/2/3 places to the left and get 10x/100x/1000x bigger
That commutative law is: 95x78 = 78x95
Associative law is: 10.4x40 = 10.4 x (10x4) or (10.4x10) x 4
Distributive law is: 46x98 = 46 x (100-2) = (46x100) - (46x2) = 4600 - 92 = 4508
Multiplication is the inverse of division

Learning objectives (see over page for exemplification)

To multiply multi digit numbers up to 4 digits by a two digit whole number
To carry out operations involving the four operations
To multiply fractions
To multiply decimals
To understand the order of operations using brackets
To relate common factors to equivalent fractions

Potential barriers/misconceptions

Misconceptions can arise when multiplying decimals. Belief that if 1x1=1 then 0.1x0.1=0.1 (this is ⅟10 x⅟10 which is one tenth 'of' one tenth which =1/100= 0.01)

Interpreting a multiplication number sentence

2x6 is often interpreted as the same as 6x2.
2x6 is '2 multiplied by 6', 2 taken six times or 2+2+2+2+2+2. The first number is 'operated on'.
6x2 would be '6 multiplied by 2', 6 taken twice or 6+6.
'Everyday' interpretation (referred to throughout programme) can be different as 2x6 is referred to as 2 'times' 6 or 2 'groups of' 6. This is not a problem as it is commutative and both will give the answer 12.

Example Questions

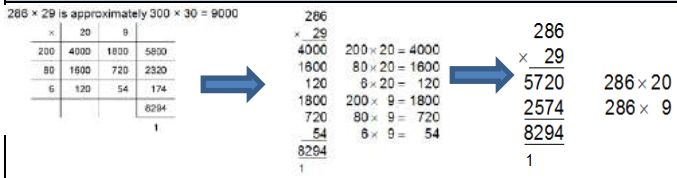
How many times larger is 26000 than 26? Prove it.
How many £100 notes are there in £1200, £12 000, £120,000? How many £10 notes, £1 coins, 10p coins, 1ps?
Tins of cat food cost 62p. They are put in packs of 10. Ten packs are put in a box. Ten boxes are put in a crate. How much does one crate cost? 10 crates? 100 crates?
Respond rapidly to : two nineteens, double 75, 11 times 8, 9 multiplied by8, multiply 25 by 8, zero, 1.
Is 210 a multiple of 6? How do you know? What is the product of 125 and 4?
132x46=□ 38x□=190 □ x9=18.9
4x0.9=□ Δx□=9506

Mental Maths

Rapid recall of all multiplication tables up to 12 x 12 (and derive corresponding division facts)
Identify common factors, multiples and prime numbers
Know the squares of all numbers from 1x1 to 12x12
Derive quickly squares of multiples of 10 to 100, such as 20^2, 80^2.
To double decimal numbers
Double all multiples of 10 up to 1000
Double all multiples of 100 up to 10,000
Use related facts to double (double 277 = 400 + 140 + 14 = 554)
Double a number ending in 5 and halve the other number (14x5 = 7x10= 70)
Halve/double one number in the calculation, find the product then double/halve it.
To multiply by 15 (multiply by 10, halve the result then add the two parts together: 22x15 = 22x10=220+110=330)
To multiply by 25 (multiply by 100 and then divide by 4.)
To know the 24 times table (six times table, double and double again – or double 12x)
To calculate 17 times table (add seven times table and ten times table)
To multiply a number by 49 or 51 (multiply it by 50 and add or subtract the number)
To multiply a number by 99 or 101 (multiply by 100 and add or subtract the number)

Year 6– Multiplication Progression (a combination of these models and images can be used for every objective)

To multiply multi digit numbers up to 4 digits by a two digit whole number



To multiply fractions

$\frac{3}{4} \times \frac{8}{9} = \frac{3 \times 8}{4 \times 9}$

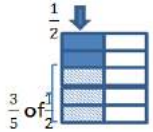
$= \frac{24}{36}$

product $= \frac{2}{3}$

$\frac{3}{5}$ of $\frac{1}{2} = \frac{3}{5} \times \frac{1}{2}$

$= \frac{3x1}{5x2}$

$= \frac{3}{10}$



To multiply decimals

1 x 10 = 10 0.1 x 10 = 1

11 x 10 = 110 0.11 x 10 = 1.1 (see below)

111 x 10 = 1110 0.111 x 10 = 1.11

Tens	Ones	Tenths	Hundredths
	0	1	1
	1	1	

To understand the commutative, distributive and associative laws of multiplication.

Commutative laws of multiplication: a x b = b x a

Associative law of multiplication: (a x b) x c = a x (b x c)

Distributive laws of multiplication: (a + b) x c = (a x c) + (b x c)

(a – b) x c = (a x c) – (b x c)

5 x 28 **5 x 28 = 28 x 5 (commutative law)**

28 x 5 = 28 as 14 x 2 Could be 2 x 5 (to get 10) then multiply this by 14 = 14 x 10 = 140 (associative law)

We are associating the 2 with the 5 rather than the 14 in order to make the calculation easier:

(14 x 2) x 5 = 14 x (2 x 5) (associative law)

Or (20 + 8) x 5 = (20 x 5) + (8 x 5)

Or we could choose to think of the 28 as 30-2 then distribute the multiplication by 5 across this subtraction:

30 x 5 = 150 and 2 x 5 = 10 therefore 150-10= 140 (distributive)

To relate common factors to equivalent fractions

$\frac{1}{2} + \frac{1}{3}$ = To add they need to be converted

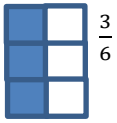
Multiples of denominators 2 & 3

Multiples of 2 : 2 4 6 8

Multiples of 3: 3 6 9 12

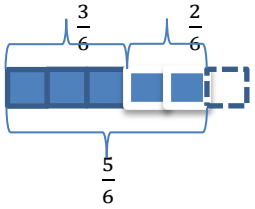
6 is the first common multiple of 2 and 3

$\frac{1}{2} = \frac{3}{6}$ (x3) $\frac{1}{2} = \frac{2}{6}$ (x2)



$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}$

$= \frac{5}{6}$



To understand the order of operations using brackets

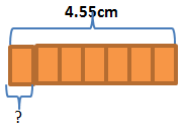
Brackets
Indices
Division
Multiplication
Addition
Subtraction

Calculate: 3 x (7 - 3)
In this question, we have a bracket, a subtraction and a multiplication.
BIDMAS tells us that brackets come first,
So we calculate:
3 x (7 - 3) = 3 x 4
= 12

To solve word problems involving all four operations

A pile of books is 4.55cm high. Find the height of 9 books. Round off the answer to 1 decimal place.

0.65 x 9 = 5.85 (rounded off is 5.9)



The height of nine books is 5.9 cm.

4.55 ÷ 7 = 0.65

The height of one book is 0.65cm

Chapter 5

Division

EYFS 1&2 – Division (When planning ensure you track forwards to year 1)

Early Learning Goal 11
Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.
They solve problems, including doubling, halving and sharing.

KS1 ready:
Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Key vocab: share, share equally, one each, two each, three each, group in pairs, threes…., equal groups of,

Key concepts
Division is an understanding that all groups must be equal.
Division is the sharing of items into equal groups.
Halving is the same as dividing by two.
Division must be taught with links made to real life.

Potential barriers/ misconceptions
Pupils do not have 1:1 correspondence and incorrectly count the small group of objects.
Pupils do not realise that the last number counted represents the total number of objects in the group.
When objects are shared between two and four, pupils do not share equally and understand that each group should have the same number of objects.
Conceptual understanding of 'same' and 'different' is not secure (both language and concept).

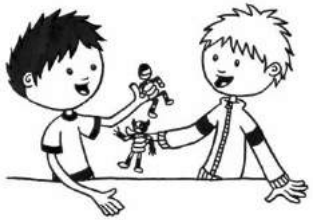
Learning objectives (see overleaf for exemplification)
To share objects equally
To group objects
To halve an even group up to 12
To share an even group of objects between 2
To share an even group of objects between 4
To identify odd and even
To count up to 20 (objects/ images arranged in an array)
To solve problems involving grouping and sharing

Example Questions
Look at all these cars. How could you sort them?
Share the biscuits out so that everyone has the same number.
Can you count the spots on each side of your ladybird? Does it have an equal number of spots on each side?
Share these pencils equally between Mustafa and Gracja. How many pencils will each of them get?
Show a chocolate bar with ten pieces of chocolate. How many children will have two pieces each?
Put half of these ten animals in the ark. How many animals are in the ark?
Put seven forks and six knives on a table. Here are some knives and forks. Find out if there are more knives or more forks, or the same number. How did you find out there are more forks?
Half of the cakes in this box of ten are gone. How many are left?

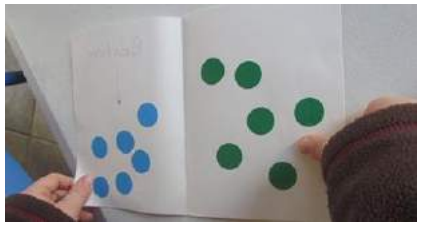
Mental Maths (can revisited throughout day once concept has explicitly shared)
To count forwards and backwards in 1s
To count forwards and backwards in 2s
To count forwards and backwards in 10s
To count forwards and backwards in 1s from any given number
To count in pairs (children, shoes, animals)
Put half of the: sheep in the field, cars in the garage, dinosaurs in the box
To quickly derive:
Doubles of numbers 1-10
Halves of even numbers to 20

EYFS 1&2 – Division **Progression (a combination of these models and images can be used for every objective)**

To share objects equally (with self + others)



To group objects

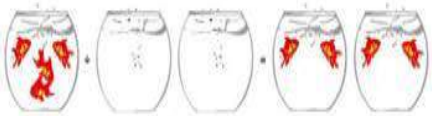


To halve an even group up to 12

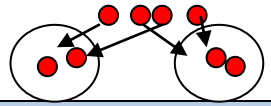
Half of 8 is 4

Half of 6 is 3

To share an even group of objects between 2

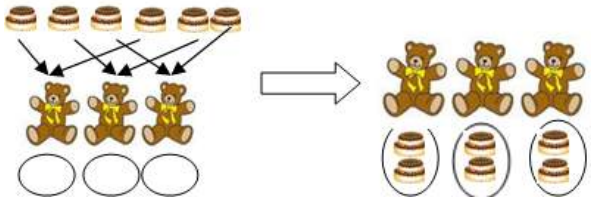


Four Fish divided into two bowl= 2 fish in each bowl.

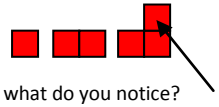


To share an even group of objects between 3 or 4

We have six cakes. Can you share them equally between the bears?



To identify odd and even



When making these numbers with unifix/numicon what do you notice?

<u>Make pairs</u>
There are 5 children.
Would each one have a partner?
No? Then this is an odd number.

To count up to 20 (objects/ images arranged in an array)

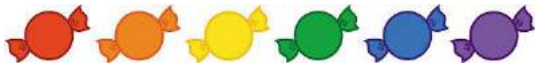
If there are 4 in this row. How many do you think there are in this row?
Can you check?

To solve problems involving grouping and sharing

How should we put the seeds in these four pots?
Is there a way so that we'll have the same? Are there any left over?

Can we share out these sweets fairly? How shall we do it?
Between 2 people? What would happen if it was between 3 people?

Count out these stickers round the circle of children.
How many times will they go around? Are there any left over?



Year 1 – Division (When planning ensure you track back to Reception and forwards to year 2)

Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities. They make connections between arrays, number patterns, and counting in twos, fives and tens.
Key vocab: share, share equally, one each, two each, three each, group in pairs, threes…., equal groups of, ÷, divide, divided by, divided into, left, left over., halve. **Key concepts** Children all come with an extensive variety of successful experiences of division situations. Division is an understanding of equal groups and sharing of items. Halving is the same as dividing by two. Division can be seen through two different lenses- when the number of groups is known and when the quantity in each group is known.	**Learning objectives** (see overleaf for exemplification) To solve division problems by sharing equally (up to 20 then beyond) To solve division problems by finding the number of groups (up to 20 then beyond) To relate grouping to repeated subtraction. Use arrays to help solve division problems To know the link between multiplication and division To solve one step word problems To use reasoning to explain (i.e. divisibility of multiples and odd numbers)
Potential barriers/ misconceptions Pupils confuse numbers when counting in twos; have difficulty understanding a pair consists of two objects. Pupils halve by sharing or forming pairs and counting but may not associate it with division by two or division between two. Pupils do not recognise that the division symbol means divide. Pupils are not given the opportunity to explore division through both sharing and grouping. When dividing by two or four, pupils do not share objects equally as they do not accurately count numbers in each group.	
Example Questions Forty children visit the zoo. Half visit the tigers. How many children visit the tigers? Some children share 12 apples (show 12 apples). Each child gets 3 apples. How many children are there? Here are 12 bananas (show them). How many is half the set? There are twenty children in a swimming pool. Half of them are boys. How many are girls? Count fives hops of two along this number line. Start at zero. What number will you reach? Charlie has 2 pots. She plants 7 seeds in each pot. How many seeds does she plant altogether? Write the missing number. The first is done for you. 5 → double and add three→ 13 8 → double and add 3 → _ Write the correct numbers in the blank spaces: Half of 12 is _ Double 12 is _ Imagine a number line. What number is halfway between 11 and 19. At the shop, all chocolate bars cost the same. Lucy buys two chocolate bars. She pays 40 pence. How much does one chocolate bar cost? There are ten pencils in each box (show five boxes). How many pencils are there altogether? Tara has 12 shells. She groups them into threes (show three shells together). How many threes can she make altogether? She puts the same number of shells into fours. How many fours can she make altogether? How many two pence coins make 20p?	**Mental Maths** To count forwards and backwards in 2s To count forwards and backwards in 5s To count forwards and backwards in 10s To count forwards and backwards in 2s, 5s and 10s from any given number To have rapid recall of numbers up to 20 divided by 2. To have rapid recall of numbers up to 100 divided by 10. To derive the corresponding division facts when given multiplication fact (number families) To quickly derive: doubles of numbers 1-15 doubles of 5,10, 15 to 50 halves of even numbers to 20 halves of even multiples of 10 (20, 40, 60, 80, 100) halves of multiples of 10 up to 100 To divide a two digit multiple of ten by 1 or 10 (20÷1 = 20 and 50÷10=5)

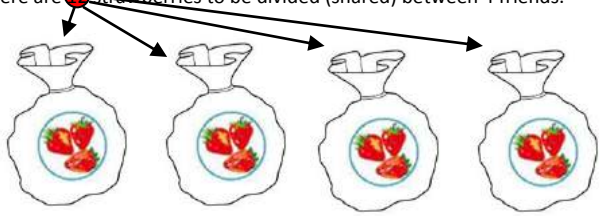
Year 1 – Division

Progression (a combination of these models and images can be used for every objective)

To solve division problems by sharing equally (up to 20 then numbers beyond 20 that can be shared equally)

There are 12 strawberries to be divided (shared) between 4 friends.

12 ÷ 4 = 3



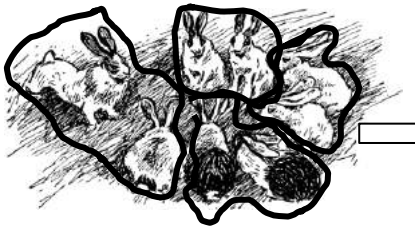
This can be done with counters and sharing bowls.

What if 2 more friends joined? How many would they each get now? More or less?

Share the strawberries between each of their lunch boxes.
Each friend has 3 strawberries.
Twelve strawberries shared between 4 friends means each friend gets 3 each.

To solve division problems by finding the number of groups (up to 20 then beyond)

There are eight rabbits. Each hutch fits two rabbits. How many cages do we need?



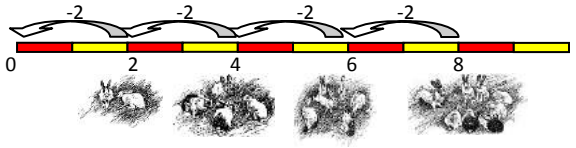
8 ÷ 2 = 4

Grouping

Four hutches are needed for the rabbits.

To relate grouping to repeated subtraction.

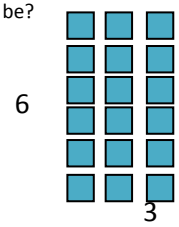
Eight rabbits. Two rabbits to be put in each hutch. How many hutches?
Use of counting stick/ number line to visualise



8- 2 – 2 – 2 - 2 (4 jumps of -2) = 4 cages
(children show actions 'putting' two rabbits away in a cage on each jump)

Use arrays to help solve division problems

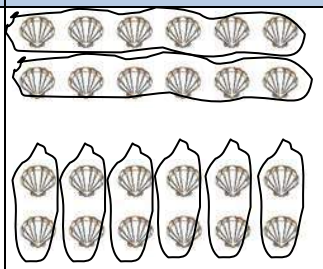
There are 18 cubes. You need to build tower of six cubes each. How many towers will there be?



There will be four towers.

18 ÷ 6 = 3

To know the link between multiplication and division

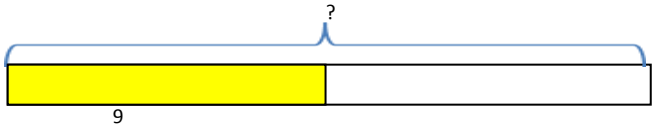


12 shells divided by 2 equals 6
12 ÷2 = 6
There are six shells in each group
(inverse:6x2=12)

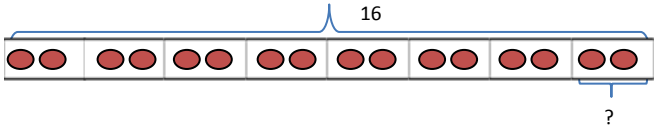
12 shells divided by 6 equals 2
12 ÷6 = 2
There are two shells in each group
(inverse: 2x6=12)

To solve one step word problems

I think of a number then halve it. The answer is 9. What was my number?



There are 16 plums. 8 children share them equally. How many plums does each child have?

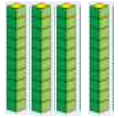


Share counters onto bar model

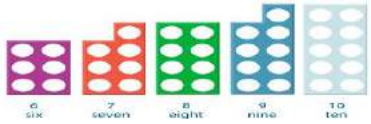
To use reasoning to explain (i.e. divisibility of multiples and odd numbers)

If a number ends in 0 then it divides exactly by 10

40 ÷ 10 = 4



Odd numbers have one left over when you divide them by two, but even numbers do not.
9÷2is 4 remainder 1



even odd even odd even

Year 2 – Division (When planning ensure you track back to year 1 and forwards to year 3)

National Curriculum

Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers

Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs

Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot

Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Notes and guidance (non-statutory)

Pupils use a variety of language to describe multiplication and division.

Pupils are introduced to the multiplication tables. They practise to become fluent in the 2, 5 and 10 multiplication tables and connect them to each other. They connect the 10 multiplication table to place value, and the 5 multiplication table to the divisions on the clock face. They begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations.

Pupils work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition. They begin to relate these to fractions and measures (for example, 40 ÷ 2 = 20, 20 is a half of 40). They use commutativity and inverse relations to develop multiplicative reasoning (for example, 4 × 5 = 20 and 20 ÷ 5 = 4).

Key vocab: share, share equally, one each, two each, three each, group in pairs, threes…., equal groups of, ÷, divide, divided by, divided into, left, left over, halve.

Key concepts

Division cannot be carried out in any order (whereas multiplication is commutative and can be).

We can use the inverse operation to find unknown values.

In the same way that multiplication is repeated addition, division is repeated subtraction.

Potential barriers/ misconceptions

Pupils confuse the words 'halving' and 'doubling'.

Pupils do not use knowledge of doubles to find half of a number; for example: continue to find half by sharing using a 'one for you' approach and cannot apply knowledge of doubles.

Pupils do not understand that 'sets of' or 'groups of' need to be subtracted to solve the problem.

Interpret 12 ÷ 3 as 12 shared between 3 and use objects or pictures to share out the 12 but lose track of their recording as numbers increase due to having no other strategy available such as counting in steps or groups.

Pupils may not be proficient in counting forwards and backwards in equal steps so make mistakes when carrying out repeated subtraction.

Learning objectives (see overleaf for exemplification)

To use number bonds for factor and products (using multiples of 2, 5 and 10)
To identify missing factors
To recognise odd and even numbers
To use concrete apparatus to solve division problems (sharing)
To use concrete apparatus to solve division problems (grouping)
To divide with remainders (in concrete)
To use pictorial representations to solve division problems (sharing)
To use pictorial representations to solve division problems (grouping)
To use the bar model to help solve division problems
To use arrays to help solve division problems
To create number families using multiplication and division facts. (4x5=20 and 20÷5=4)
To know whether to round up or down depending on context.

Example Questions

There are 35 candles. They are put into bags of 5. How many bags are there altogether?
Jo worked out the correct answer to 70 ÷ 5. His answer was 14. Show how he worked out his answer.
Tariq has a set of 22 marbles. How many is half the set?
At the shop, all chocolate bars cost the same. Hannah buys 2 chocolate bars. She pays 40 pence. How much does one bar cost?
When I doubled a number the answer was 18. Which number did I double?
Write the correct numbers in the blank spaces: Half of 12 is □ Double 12 is □
What is half of fourteen?
Nina has thirty-two stamps. She gives half to her sister. How many stickers does she give her?
Write the number which is half of 38.
What is half of this amount? Show as coins: 50p, £1, £1 and £1.
Write the missing number: □ ÷ 2 = 7 Write the answer: 45 ÷ 5 = □
Write a calculation that you could do to check the answer to: 24 ÷ 2 = 12
23 Children are coming to Michelle's party. Each child will get one ice cream. There are 10 ice creams in a box. How many boxes does Michelle need to buy?
Emma needs 18 apples. The shop sells apples in bags of 5. How many bags does she need to buy?
: + - ÷ x write the correct sign in each box. One is done for you: 3 + 3 = 6, 3 □3 = 1, 3 □3 = 9
Write the missing number in the blank space: □÷ 2 = 7

Mental Maths

To count in 2s, 5s and 10s forwards and backwards from any given number.
To have rapid recall of 2,5 and 10 times tables.
To connect the ten times table to place value.
To count around the clock face using the five times table.
To know multiplication facts and corresponding division facts. (2x3=6 therefor 3x2= 6 and 6÷2=3 and 6÷3=2)
To halve two digit numbers
To identify half past the hour using an analogue clock. (knowing that 30 is half of 60)
Respond rapidly to oral questions phrased in a variety of ways (share 18 between 2, divide 6 by 3, how many tens make 80? How many £2 coins do you get for £20? How many 2cm lengths can you cut from 10cm ribbon?)
Use known facts to derive quickly:
doubles of numbers 1-20
doubles of 5,10, 15 to 100
halves of even numbers to 20
halves of multiples of 10 up to 200
To know that to find a quarter you must halve and halve again. (one quarter of 20 is 5, half of 20 is 10 and half of 10 is 5)
To divide a two digit multiple of ten by 1, 10 or zero(divide 30 by 1, divide 50 by 10, divide 70 by zero)
To halve any multiple of ten to 100

Year 2 – Division

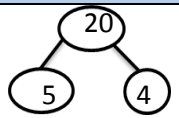
Progression (a combination of these models and images can be used for every objective)

To use number bonds for dividend, divisor and quotient (using multiples of 2, 5 and 10)

(Dividend ÷ divisor = quotient)

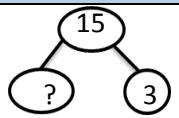
20÷ 5 = 4

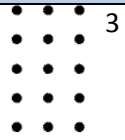
And 20 ÷ 4 = 5



To identify missing factors

15 ÷ □ = 3





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15 ÷ 5 = 3

To recognise odd and even numbers

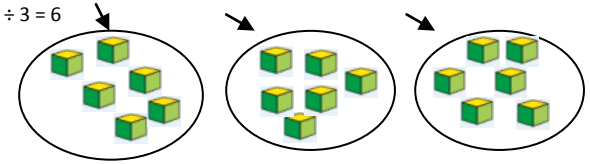
Even numbers always have a 'partner'.

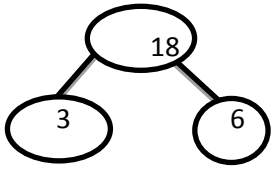
To use concrete apparatus to solve division problems (sharing equally)

I share my 18 sweets equally between 3 friends. How many sweets do they get each? (Sharing as number of groups known (3) but quantity not).

Could do, 'one for you' etc. or visually distribute i.e. 4 each and adjust.

18 ÷ 3 = 6



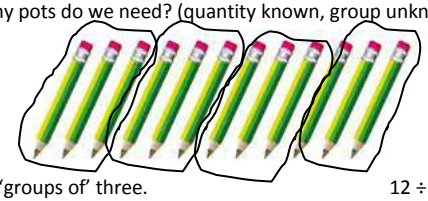


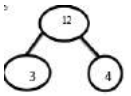
There are six sweets in each group. Eighteen divided by three equals six. (and 18÷ 6 =3)

To use concrete apparatus to solve division problems (grouping)

There are 12 pencils in total. Each pot needs three pencils in.

How many pots do we need? (quantity known, group unknown)

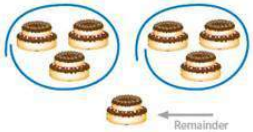




Count in 'groups of' three. 12 ÷ 3 =4

There are four groups of pencils. Twelve divided by 3 is 4. (and 12÷ 4 =3)

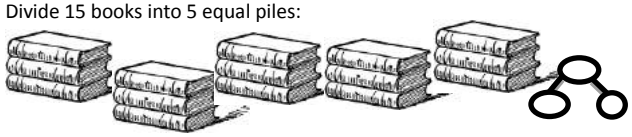
To divide with remainders (in concrete).



7 ÷ 2 = 3 R 1

To use pictorial representations to solve division problems (sharing)

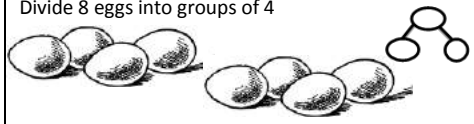
Divide 15 books into 5 equal piles:



15 ÷ 5 = □ There are 3 books in each pile

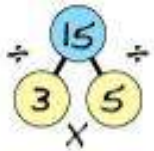
To use pictorial representations to solve division problems (grouping)

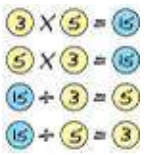
Divide 8 eggs into groups of 4



8 ÷ 4 = □ There are two groups of eggs

To create number families using multiplication and division facts.





To know whether to round up or down depending on context.

I have £10. Tickets cost £3. How many tickets can I buy?

ROUND DOWN: I can buy 3 tickets (£1 left over)

There are 10 people. A taxi holds 4 people. How many taxis do we need for the journey?

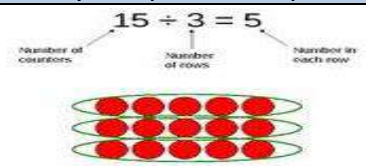
ROUND UP: We will need three taxis.

To use the bar model to help solve division problems

Hannah buys 12 cakes. She puts an equal number of cakes into 3 boxes. How many cakes are there in each box?

12÷ 3 = 4 There are four cakes in each box. (3x4=12)

To use arrays to help solve division problems



Year 3 – Division (When planning ensure you track back to year 2 and forwards to year4)

National Curriculum

Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables

Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Notes and guidance (non-statutory)

Pupils continue to practise their mental recall of multiplication tables when they are calculating mathematical statements in order to improve fluency. Through doubling, they connect the 2, 4 and 8 multiplication tables.

Pupils develop efficient mental methods, for example, using commutativity and associativity (for example, 4 × 12 × 5 = 4 × 5 × 12 = 20 × 12 = 240) and multiplication and division facts (for example, using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (for example, 30 × 2 = 60, 60 ÷ 3 = 20 and 20 = 60 ÷ 3).

Pupils develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division.

Pupils solve simple problems in contexts, deciding which of the four operations to use and why. These include measuring and scaling contexts, (for example, four times as high, eight times as long etc.) and correspondence problems in which m objects are connected to n objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).

Key vocab: share, share equally, one each, two each, three each, group in pairs, threes…., equal groups of, ÷, divide, divided by, divided into, left, left over, remainder, halve.

Key concepts

Division cannot be carried out in any order (whereas multiplication is commutative and can be).

We can use the inverse operation to find unknown values.

In the same way that multiplication is repeated addition, division is repeated subtraction.

Division can be partitive division and measurement division (also called quotient division). Thus the same division sentence can mean two different things. i.e. 24÷ 3 = 8. With partitive division this would be 24 items put into 3 groups (sharing). If it is measurement division it would be 24 put into 'groups of' 4 (grouping).

Learning objectives (see overleaf for exemplification)

To use number bonds for factor and products (using multiples of 3,4 and 8)
To identify missing factors
To derive related division facts from known multiplication facts
To use the distributive property strategy to divide 'friendly' numbers.
To divide a two digit number by a one digit number (in concrete with and without remainders)
To divide a two digit number by a one digit number using short division (no remainders)
To solve problems where items are shared equally (12 sweets between 4 children)
To solve problems where items are shared using knowledge of fractions (4 cakes shared between 8 children)
To know whether to round up or down depending on context.

Potential barriers/ misconceptions

Pupils are not secure in multiplication facts for 2,4,5,6,8,10 tables.

Pupils do not recognise the inverse of multiplication is division.

Pupils confuse the order of division and do not recognise that you need to begin with the largest number when writing a number sentence.

Pupils carry out division by sharing or grouping but cannot cope with a remainder and do not recognise that a remainder must always be less than the divisor.

Pupils associate X with multiplication and ÷ with division and can do calculations 8 X 2 and 16 ÷ 2 but are not able to find missing numbers in statements such as 6 X□= 12 and □- 5 = 3.

Example Questions

Lucy is looking at a number. She doubles the number and adds 3. Her answer is 15. What number is it?

Jane chose a number. She doubled it. Her answer was one hundred and two. What number did Jane chose?

A bottle of milk fills 6 cups. Mrs Brown wants to fill 50 cups with milk. How many bottles of milk does she need to buy? Show how you work it out.

Here are some signs: x = ÷ write a sign in each blank space to make this correct: 20 □4□5

There are 35 children. They get into teams of 5. How many teams are there altogether?

Write the answer: 45 ÷ 5 = □, Calculate 48 ÷ 3 and 56 ÷ 4

Ryan has some number cards. He holds up a card. He says: "If I multiply the number on this card by 5, the answer is 35". What is the number on the card? Ryan holds up a different card. He says: "If I multiply the number on this card by 6, the answer is 4". What is the number on the card?

Look at these three numbers: 5 12 60 Use all three numbers to make these correct: □x□= □ , □÷ □=□

Miss Blue needs 28 paper cups. She has to buy them in packs of 6. How many packs does she need to buy?

John is making cards. 1 sheet of paper makes 15 cards. John uses 5 sheets. How many cards does he make?

10 children can sit at 1 table. There are 43 children. How many tables are needed so each child can sit down?

What is the remainder when twenty-seven is divided by five?

Circle the two divisions which have an answer of 5 remainder 2: 17 ÷ 5 17 ÷ 3 22 ÷ 4 22 ÷ 5

James has 45 seeds. He plants 3 seeds in each pot. How many pots does he need?

Mental Maths

To count forwards and backwards in 3s, 4s and 8s
To count forwards and backwards in 3s, 4s and 8s from any given number
To have rapid recall of all division facts when given a multiplication fact.
To use repeated subtraction on the counting stick (i.e. 18÷3= count back 15, 12, 9, 6, 3, 0 = 6)
To divide any number by one or zero.
To divide any two digit even number by 2.
Use known facts to derive quickly:
doubles of numbers 1-100
doubles of 5,15, 25 up to 100
Doubles of 50, 100, 150, 200 up to 500
And all corresponding halves.
To divide any three digit multiple of 10 by 10 (340÷10, 890÷10 etc)
To know how to find quarters of a number by finding half of a half (quarter of 60 = 60÷2= 30÷2=15)

Year 3– Division

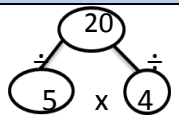
Progression (a combination of these models and images can be used for every objective)

To use number bonds for factor and products (using multiples of 3,4 and 8)

(Dividend ÷ divisor = quotient)

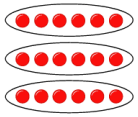
20÷ 4 = 5

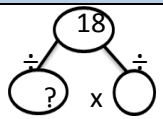
And 20 ÷ 5 = 4



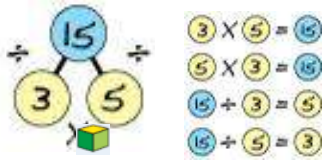
To identify missing factors

18 ÷ 6= □





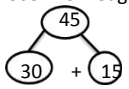
To derive related division facts from known multiplication facts



To use the distributive property strategy to divide 'friendly' numbers.

45 ÷ 3= □

Use knowledge of 45 to split the number into:



30÷ 3= 10

15÷ 3= 5

10+5 = 15

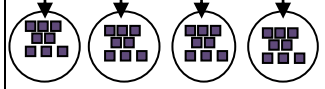
Therefor 45÷ 3 = 15

Friendly numbers are easier to calculate.

To divide a two digit number by a one digit number (in concrete with or without remainder)

Sharing (find the number of items in each group.)

Divide 34 cubes into 4 equal groups. How many in each group?



32÷ 4 = 8 r 2 (there are 8 in each group and a remainder of 2)

Grouping (Making equal groups)

Divide 22 stars into groups. There are 7 stars in each group. How many groups are there?

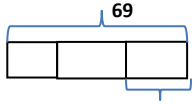
21÷ 7 = 3 r 1 (there are 3 complete groups of stars and one star remaining)

To divide a two digit number by a one digit number using short division (no remainders) (no regrouping)

There are 69 books to be shared between 3 classes.

How many books does each class get?

69 ÷ 3= □



$3\overline{)69}$ = 23

I could use my knowledge of tables to do:

60 ÷ 3= 20 (because if I know 3x2= 6 then 3x20 = 60 therefor 60÷ 3= 20)

9 ÷ 3= 3 (I know that 3x3 = 9)

20 + 3 = 23

Divide the tens by 3

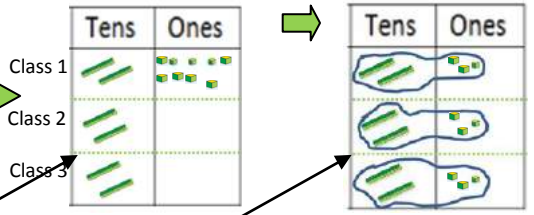
60÷ 3= 2 tens

$3\overline{)69}$ = 2

Then the ones by 3

9÷3 = 3 ones

$3\overline{)69}$ = 23

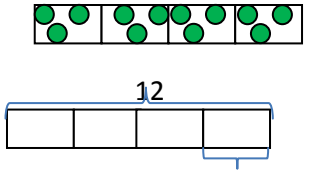


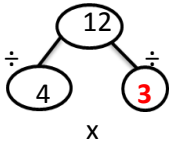
As we are dividing by three place value table is divided into three parts:

To solve problems where items are shared equally (using known facts)

How can we share 12 sweets between 4 children?

I could distribute 12 objects onto a bar model sharing equally:

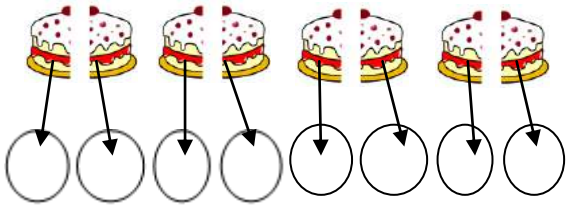




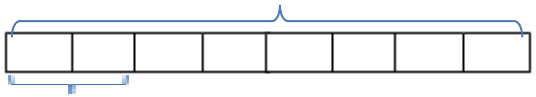
I know 4x3=12

Half and half again. I know half of 12 is 6 and half of 6 is 3. Each group is therefore worth 3.

To solve problems where items are shared using knowledge of fractions



4 cakes shared between 8 children **4**



To know whether to round up or down depending on context.

I have £50. Tickets cost £8. How many tickets can I buy?

ROUND DOWN: I can buy 6 tickets (£2 left over)

12 children can sit at 1 table. There are 52 children. How many tables are needed so each child can sit at a table?

ROUND UP: We will need five tables.

Year 4 – Division (When planning ensure you track back to year 3 and forwards to year 5)

National Curriculum

Recall multiplication and division facts for multiplication tables up to 12 × 12
Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
Recognise and use factor pairs and commutativity in mental calculations
Multiply two-digit and three-digit numbers by a one-digit number using formal written layout
Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Notes and guidance (non-statutory)

Pupils continue to practise recalling and using multiplication tables and related division facts to aid fluency.
Pupils practise mental methods and extend this to three-digit numbers to derive facts, (for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6).
Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers. Pupils write statements about the equality of expressions (for example, use the distributive law 39 × 7 = 30 × 7 + 9 × 7 and associative law (2 × 3) × 4 = 2 × (3 × 4)). They combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations for example, 2 x 6 x 5 = 10 x 6 = 60.
Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as the numbers of choices of a meal on a menu, or three cakes shared equally between 10 children.

Key vocab: share, share equally, one each, two each, three each, group in pairs, threes…., equal groups of, ÷, divide, divided by, divided into, left, left over, remainder, factor, quotient, divisible by, inverse.

Key concepts

Remainder needs to be rounded up or down according to the context.
Division can be partitive division and measurement division (also called quotient division). Thus the same division sentence can mean two different things. i.e. 24÷ 3 = 8. With partitive division this would be 24 items put into 3 groups (sharing). If it is measurement division it would be 24 put into 'groups of' 4 (grouping).

Potential barriers/ misconceptions

Pupils do not understand that division is grouping as well as sharing.
Pupils are muddled about the correspondence between multiplication and division facts, recording, for example: 3 X 5 = 15 so 3 ÷ 15 = 5. Pupils write a remainder that is larger than the devisor, for example: 36 ÷ 7 = 4 remainder 8. Pupils discard the remainder as do not understand its significance.
Pupils continue to subtract two when calculating 20 divided by 2 without using knowledge that 2 multiplied by 5 equals 10.

Learning objectives (see overleaf for exemplification)

To use number bonds for factor and products (To solve missing number sentences)
To make the link between sharing, arrays and short division.
To use known facts to derive facts involving 3 digit numbers (If I know 2x3 = 6 I can work out that 600÷3=200)
To use the distributive property strategy to divide 'friendly' numbers.
To divide a three digit number using short division (Regrouping in tens and ones)
To divide a three digit number using short division (Regrouping in tens, ones and hundreds)
To solve two step word problems

Example Questions

One length of swimming pool is 25 metres. Emily swims 5 lengths of the pool. How far does Emily swim altogether? William swims 225 metres in the pool. How many lengths does he swim?
On a sheet of stickers there are 5 circles, 2 stars and one rectangle (show the sheet). How many stickers are there altogether in 4 sheets? Rajesh needs 55 circles. How many sheets of stickers does he need? Tom has 10 sheets of stickers. How many more circles than rectangles does he have?
One rubber weighs the same as 60 paper clips. Once pencil sharpener weighs the same as 20 paperclips. How many pencil sharpeners weigh the same as one rubber? How many paperclips weigh the same as 2 rubbers and 4 pencil sharpeners together?
Halve twenty-seven. What is half of eight hundred and sixty? What number is exactly half way between fifty and eighty? What is half of seventy pounds? Divide forty-eight by eight. Calculate 56 ÷ 4
Charlotte puts 4 seeds in each of her pots. She uses 6 pots and has 1 seed left over. How many seeds did she start with?
Nineteen sweets are shared between some children. Each child receives six sweats and there is one sweet left over. How many children share the sweets?
50 children need 2 batteries each. There are 20 batteries in each box. How many boxes are needed?
50 children need one piece of chalk each. Chalk is sold in packs of 4. How many packs of chalk need to be bought?

Mental Maths

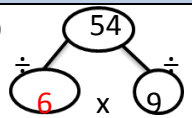
Rapid recall of multiplication facts to 12x12
To know all related division facts when given a multiplication fact (8x4 = 32 therefor 32÷4 = 8 32÷8 = 4)
Recognise and use factor pairs
To give statements about odd and even numbers (An odd digit cannot be divided exactly by two)
To know the divisibility of numbers (ring the numbers that divide exactly by four: 3, 8, 20, 27, 34, 36, 48, 50)
Recognise that a whole number is divisible by:
100 if the last two digits are 00; 10 if the last digit is 0; 2 if the last digit is 0,2,4,6,8; 4 if the last two digits are divisible by 4; 5 if the last digit is 5 or 0
Find all the pairs of factors of any number to 100 (i.e. pairs of factors of 24 are: 1 and 24, 2 and 12, 3 and 8, 4 and 6)
Relate division to fractions ($\frac{1}{2}$ of 10 is the same as 10÷2 and $\frac{1}{4}$ of 12 is the same as 12÷4)
To divide a whole number of pounds by 2, 4, 5 or 10 (£29 divided between 4 people = £7 each + £1÷4= 25p = £7.25 each)
Understand halving as the inverse of doubling. (if double 37 is 74 then half 74 is 37)
To use related facts to half (i.e. half of 28 = half of 20 is 10 and half of 8 is 4 = 10+4=14)
To find quarters and eighths by halving ($\frac{1}{8}$ of 56 is the same as half of 56 = 28 half again is 14, half again is 7 = 7)
To divide a four digit multiple of 1000 by 10 or 100 (8000÷ 100 = 80) (To find one tenth, one hundredth etc)

Year 4– Division — Progression (a combination of these models and images can be used for every objective)

To use number bonds for factor and products (To solve missing number sentences)

(Dividend ÷ divisor = quotient)

54÷ 9 = □

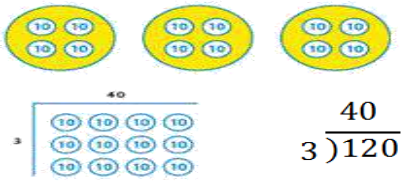


And 54 ÷ □ = 9

(Inverse: 9 x 6 = 54 and 6 x 9=54)

To (use counters to) make the link between sharing, arrays and short division.

120 ÷ 3 = 40



$$3\overline{)120}\quad 40$$

To use known facts to derive facts involving 3 digit numbers (If I know 2x3 = 6 I can work out that 600÷3=200)

If I know that 9x 4 = 36 I also know that:

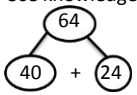
4 x 9 = 36
40 x 9 = 360
40 x 90 = 3600
Etc.

36 ÷ 4 = 9
36 ÷ 9 = 4
360 ÷ 4 = 9 0
360÷ 9 = 40
360 ÷ 40 = 9

To use the distributive property strategy to divide 'friendly' numbers.

64 ÷ 4= □

Use knowledge of 64 to split the number into:



Friendly numbers are easier to calculate.

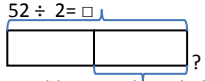
40÷ 4= 10
24÷ 4= 6
10+6 = 16

Therefor 64÷ 4 = 16

To divide a two digit number using short division (Regrouping in tens and ones) (Could also use place value counters)

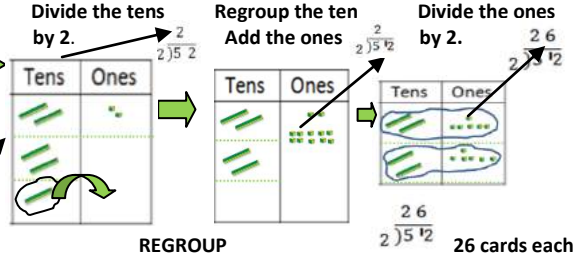
Farid has 52 football cards. He wants to share them equally with his friend. How many football cards does each boy get?

52 ÷ 2= □



?

I could use my knowledge of distributive properties:
52 = 40 + 12
40÷ 2 = 20
12 ÷ 2 = 6
20+6 = 26

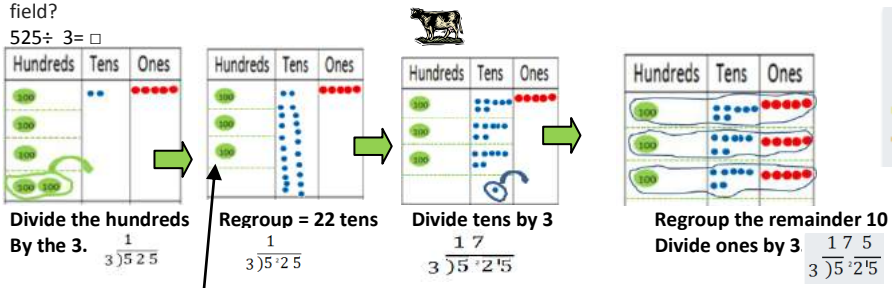


As we are dividing by two place value table is divided into two parts.

To divide a three digit number using short division (Regrouping in tens, ones and hundreds)

There are 525 cows on the farm. They need to be put into three different fields. How many cows will there be in each field?

525÷ 3= □



$$3\overline{)525}\quad 175$$

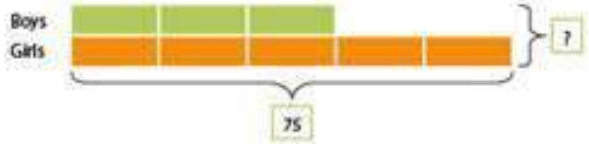
As we are dividing by three place value table is divided into three parts.

To solve two step word problems

A present for Auntie costs £30. Riza and his four cousins paid for the present.
How much did each child pay?

Her card cost them each an extra 20p.
How much was the card?
30÷ 5 = 6
Each child pays £6 for Aunties present
20 x 5 = 100 = The card cost £1.00

There are 3/5 as many boys as girls. There are 75 girls.
How many children altogether?



5 units = 75
1 unit= 75÷ 5 = 15
Boys= 15x3= 45
Girls = 15 x5= 75
75 + 45 = 120 children altogether

Year 5 – Division (When planning ensure you track back to year 4 and forwards to year 6)

National Curriculum

Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers . Know and use the vocabulary of prime numbers, prime factors and composite (nonprime) numbers

Establish whether a number up to 100 is prime and recall prime numbers up to 19

Multiply and divide numbers mentally drawing upon known facts

Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

Recognise and use square numbers and cube numbers, and the notation for squared(^{2}) and cubed (^{3})

Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes

Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign

Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Notes and guidance (non-statutory)

Pupils practise and extend their use of the formal written methods of short multiplication and short division. They apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations.

They use and understand the terms factor, multiple and prime, square and cube numbers.

Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (for example, $98 \div 4 = \frac{98}{4} = 24$ r 2 $= 24\frac{1}{2} = 24.5 \approx 25$).

Pupils use multiplication and division as inverses to support the introduction of ratio in year 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1000 in converting between units such as kilometres and metres.

Distributivity can be expressed as $72 \div 6 = (60+12) \div 6 = (60 \div 6) + (12 \div 6) = 10+2 = 12$

They understand the terms factor, multiple and prime, square and cube numbers and use them to construct equivalence statements (for example, $4 \times 35 = 2 \times 2 \times 35$; $3 \times 270 = 3 \times 3 \times 9 \times 10 = 9^2 \times 10$).

Pupils use and explain the equals sign to indicate equivalence, including in missing number problems (for example, $13 + 24 = 12 + 25$; $33 = 5 \times \square$).

Key vocab: share, share equally, one each, two each, three each, group in pairs, threes…., equal groups of, ÷, divide, divided by, divided into, left, left over, remainder, factor, quotient, divisible by, inverse.

Key concepts

Remainder needs to be rounded up or down according to the context.

Sharing is better for dividing smaller numbers and grouping is better for dividing larger numbers.

With positive whole numbers division makes a number smaller

Potential barriers/ misconceptions

Pupils recognise what calculation to do when word problems include the words 'times' or 'share', but are less confident when other language is used such as 'product', 'divided by', 'remainder' and mistakenly associate 'how many?' and 'how much?' with addition or subtraction.

Understand multiplication as repeated addition and division as repeated subtraction but not as scaling up and down to prepare the way for later work in measures and on ratio.

Pupils do not understand that ÷ 10 and then ÷ 10 again is the same as ÷ 100

When dealing with remainders, pupils have little understanding of how to represent as a fraction or a decimal.

Learning objectives (see overleaf for exemplification)

To use number bonds for factor and products and to identify missing factors (using fractions and decimals).

Divide whole numbers by 10,100 and 1000

Divide decimals by 10, 100 and 1000

To divide by powers of 10 (in scale drawings).

To divide by powers of 1000 (in converting between units such as km and m)

To solve division problems with decimals using place value counters.

Divide numbers up to 4 digits by a one digit number (with remainders)

To use the distributive property strategy to divide 'friendly' numbers.

To interpret remainders appropriately for the context (rounding up or down- see year 6 exemplification)

To interpret non-integer answers to division by expressing results in different ways

To solve multi step word problems

Example Questions

Jane chooses two numbers. She adds the two numbers together and divides the result by 2. Her answer is 44. One of Jane's numbers is 12. What is Jane's other number?

Charles makes a sequence of numbers. His rule is: "Find the last number and then add 10". Write the next numbers in his sequence: 36 28 24 _ _ In this sequence each number is double the previous number. Write in the missing numbers: _ _ 3 6 12 24 48 _

How many sevens are there in two hundred and ten? Five times a number is two hundred. What is the number? Divide three hundred and ninety by ten. Divide nine thousand three hundred by one hundred.

Write in the missing number: $3400 \div \square = 100$

Divide thirty-one point five by ten. Divide nought point nine by one hundred. What is nought point two six divided by ten? Ten times a number is eighty-six. What is the number?

Calculate $942 \div 6$. Calculate $364 \div 7$. Calculate $847 \div 7$

Mental Maths

To identify all factor pairs of a number.

To identify common factors of two numbers.

To recall prime numbers up to 19

To establish whether a number up to 100 is prime

Multiply and divide numbers mentally using known facts. (i.e. $240 \div 3 = 80$ because I know $24 \div 3 = 8$)

To use and understand the terms factor, multiple and prime, square and cube numbers

To know that dividing by four is the same as finding a quarter etc. (and $\frac{1}{3}$ of 24 is $24 \div 3$ or $\frac{24}{3}$)

To divide any number by 10,100, 1000

To round up or down according to context (see year 3 exemplification)

To double all whole numbers and decimals knowing that halving is the inverse.

Find sixths by halving thirds and twentieths by halving tenths.

CONCRETE → PICTORIAL → ABSTRACT (2 + 1 = 3)

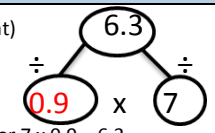
Year 5– Division

Progression (a combination of these models and images can be used for every objective)

To use number bonds for factor and products and to identify missing factors (using fractions and decimals). See year 4 for whole numbers.

(Dividend ÷ divisor = quotient)

6.3÷ 7 = □

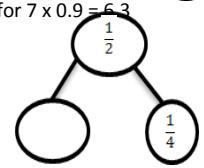


I know that 7 x 9 = 63 therefor 7 x 0.9 = 6.3

$\frac{1}{2} \div \frac{1}{4} = \frac{1}{4}$

Because I know that

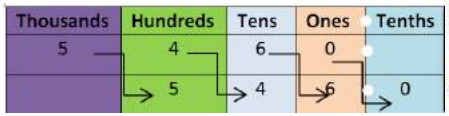
$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ $(2 \times \frac{1}{4})$

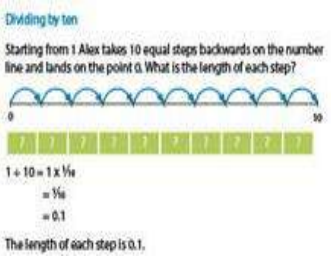


Divide whole numbers by 10,100 and 1000

To divide by powers of 10 (in scale drawings).

To divide by powers of 1000 (in converting between units such as km and m)





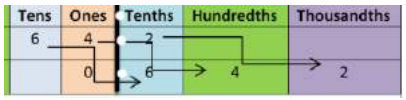
5460 ÷ 10 = 546

(The place value shifts to the right and
The value gets ten times smaller)

Divide decimals by 10, 100 and 1000

To divide by powers of 10 (in scale drawings).

To divide by powers of 1000 (in converting between units such as km and m)



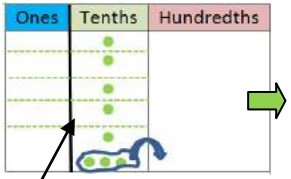
64.2 ÷ 100 = 0.642

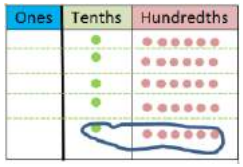
(The place value shifts two places to the right and
The value gets one hundred times smaller)

To solve division problems with decimals using place value counters.

A piece of ribbon is 0.8m long. Five children want part of the ribbon each for their hair. If the ribbon is shared equally how long is each piece?

0.8 ÷ 5 = 0.16





$$5\overline{)0.8^{3}0} \quad 0.16$$

Each piece of Ribbon is 0.16m or 16cm.

Divide the ones by 5 hundredths

Divide the tenths by 5

= 1 tenth r 3tenths (regroup the 3)

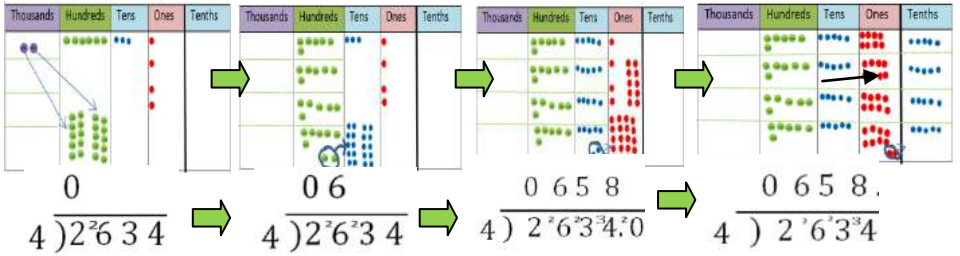
3 tenths = 30

Divide by 5 = 6 hundredths

As we are dividing by five place value table is divided into five parts.

Divide numbers up to 4 digits by a one digit number (with remainders)

There are 2634 people waiting for a boat. There are four boats. Each boat holds the same number of people. How many people will there be on each boat? How many people left over? 2634÷ 4= 658 r2



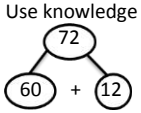
Remainder **2**

As we are dividing by four place value table is divided into four parts.

To use the distributive property strategy to divide 'friendly' numbers.

72 ÷ 6= □

Use knowledge of 72 to split the number into:



Friendly numbers are easier to calculate.

72÷ 6= (60+12) ÷ 6 =

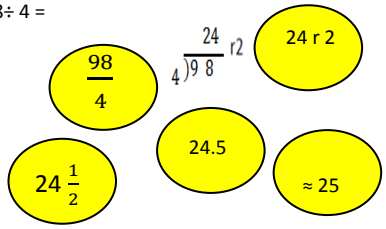
(60÷ 6) + (12÷ 6) =

10+2 = 12

Therefor 72÷ 6 = 12

To interpret non-integer answers to division by expressing results in different ways

98÷ 4 =



$$4\overline{)9\,8} \quad 24\ r2$$

To solve multi step word problems

Tejan and Sidney are playing top trumps. They have 96 cards altogether.
Tejan lost 24 cards to Sidney during the game.
At the end of the game, Sidney had twice as many cards as Tejan.
How many cards did Sidney have at the beginning?

3 units = 96
1 unit = 96 ÷ 3 = 32
2 units = 2 x 32 = 64
Sidney had 64 cards after the game

Before the game: 64 – 24 = 40 Sidney had 40 cards at the beginning.

Year 6 – Division

(When planning ensure you track back to year 5 for progression)

National Curriculum

Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context. Perform mental calculations, including with mixed operations and large numbers. Identify common factors, common multiples and prime numbers

Use their knowledge of the order of operations to carry out calculations involving the four operations

Solve problems involving addition, subtraction, multiplication and division. Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Notes and guidance (non-statutory)

Pupils practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of columnar addition and subtraction, short and long multiplication, and short and long division.

They undertake mental calculations with increasingly large numbers and more complex calculations. Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency.

Pupils round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures.

Pupils explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.

Common factors can be related to finding equivalent fractions.

Key vocab: share, share equally, one each, two each, three each, group in pairs, threes…., equal groups of, ÷, divide, divided by, divided into, left, left over, remainder, factor, quotient, divisible by, inverse

Key concepts

Division can be carried out mentally or using a written method and can be used to find out other associated facts.

Remainder needs to be rounded up or down according to the context.

Sharing is better for dividing smaller numbers and grouping is better for dividing larger numbers.

Remainder needs to be rounded up or down according to the context.

Division can be partitive division and measurement division (also called quotient division). Thus the same division sentence can mean two different things. i.e. 24÷ 3 = 8. With partitive division this would be 24 items put into 3 groups (sharing). If it is measurement division it would be 24 put into 'groups of' 4 (grouping).

Potential barriers/ misconceptions

Pupils lack understanding that division is grouping as well as sharing. Lack of times tables knowledge.

Pupils ignore decimal point when calculating then 'slot back in' – due to generalisation of adding decimals.

Pupils misunderstand the concept of making a no. 10/100/1000 times smaller, prefer to learn 'knock off a zero' and when a number ends in a different digit simply know that off. Ignore decimal point or 'move it'.

Pupils have a limited range of multiplication and division facts.

Pupils misuse half understood rules about multiplying and dividing by powers of ten and the associative law, for example: 145 X 30 = 145 000 Pupils have difficulty interpreting, when appropriate, a remainder as a fraction, for example: 16 ÷ 3 = 5 ⅓

Pupils interpret division only as sharing and not grouping (repeated subtraction) so are unable to interpret calculations such as: 12 ÷ ½. Pupils may not be confident in making reasonable estimates for multiplication or division calculations.

Example Questions

Mohamed thinks of a number. "Halve my number and then add 17. The answer is 23". What is Mohamed's number?

Write the number that is exactly half way between eight point six and eight point seven.

Divide four point eight by eight. Divide four point two by six. Divide four point two by seven.

Circle the best estimate of the answer to: 72.34 ÷ 8.91 6 7 8 9 10 11

Bacon rashers are put in packets of 12. The packets are then put into boxes. Each box contains 180 rashers of bacon. How many packets of bacon are there in each box?

Kojo buys a pack of 24 cans of lemonade for £6:00. What is the cost of each can?

Here are five calculations: A) 720 ÷ 64, B) 820 ÷ 75, C) 920 ÷ 80, D) 1020 ÷ 90, E) 1120 ÷ 100 write the letter of the calculation that has the greatest answer. Write the letter of the calculations that has an answer closest to 11.

When a number is divided by seven, the answer is three, remainder four. What is the number?

What is the smallest whole number that is divisible by five and by three?

Calculate 900 ÷ (45 X 4), Solve 50 ÷ □= 2.5 Divide thirty-one point five by ten. Calculate 504 ÷ 21 Calculate 848 ÷ 16

I pay £16.20 to travel to work each week. I work for 45 weeks each year. How much do I pay to travel to work each year? Show your working. I could buy one season ticket that would let me travel for all 45 weeks. It would cost £630. How much per week?

Learning objectives (see overleaf for exemplification)

To use number bonds for factor and products & To identify missing factors

Divide numbers up to 4 digits by 1 digit then 2 digit whole number using short division.

Interpret remainders as whole number remainders, fractions or rounding.

To use the distributive property strategy to divide 'friendly' numbers.

Divide numbers up to 4 digits by 1 digit whole number using long division.

Divide numbers up to 4 digits by 2 digit whole number using long division.

To interpret remainders appropriately for the context

To understand the order of operations

To solve word problems

Mental Maths

Identify common factors & multiples

To identify prime numbers

Rapid recall of all multiplication tables (and related number families)

To divide any number by 10, 100, 1000 (knowing that the place value changes)

To find one hundredth or one thousandth of an amount by dividing by 100 or 1000

To relate fractions to division (dividing by the denominator)

To know doubles of numbers including decimals and corresponding halves

To recognise that if 5x60 = 300 than $\frac{1}{5}$ of 300=60 and $\frac{1}{6}$ of 300 = 50

To halve a decimal fraction less than 1 with one or two decimal places. (half of 0.7)

Use knowledge that in exact multiples of (and prove):

100 the last two digits are 00 and 10 the last digit is zero and 5 The last digit is 0 or 5

25 The last two digits are 00, 25, 50 or 75

2 The last digit is 0,2,4,5,8,

3 The sum of the digits is divisible by 3

4 The last two digits are divisible by 4

6 The number is even and divisible by 3.

8 The last 3 digits are divisible by 8

9 The sum of the digits is divisible by 9.

CONCRETE → PICTORIAL → ABSTRACT (2 + 1 = 3)

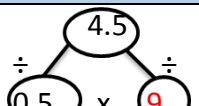
Year 6– Division

Progression (a combination of these models and images can be used for every objective)

To use number bonds for factor and products (using fractions, decimals, time)

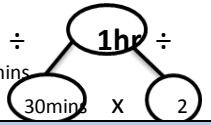
(Dividend ÷ divisor = quotient)

4.5÷ 0.5 = □



I know that x 0.5 doubles the dividend. (0.5 x2 = 9)

1hr ÷ 30mins = □



Because I know that 30 x 2 = 60mins

To identify missing factors

There are six types of missing number division problems:

1) a ÷ b = □
2) a ÷ □ = c
3) □ ÷ b = c
4) □ = b ÷ c
5) a = b ÷ □
6) a = □ ÷ c

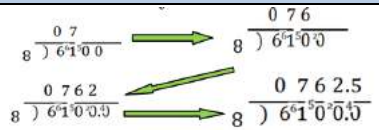
Divide numbers up to 4 digits by 1 digit whole number using short division.

6100 ÷ 8 =

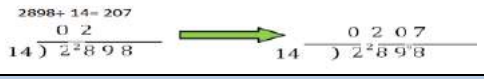
762.5

762 r 4

$762\frac{4}{8}$

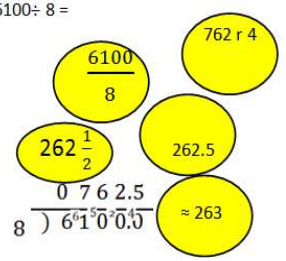


Divide numbers up to 4 digits by 2 digit whole number using short division.



Interpret remainders as whole number remainders, fractions or rounding.

6100÷ 8 =



To use the distributive property strategy

6345 ÷ 3 =

How can we find friendly numbers that are divisible by 3?

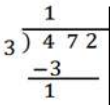
6345 = 6000 + 300 + 40 + 5 (not friendly)

6345 = 6000 + 300 + 30 + 15 (friendly)

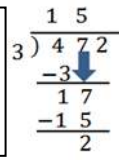
6345 = (6000÷ 3) + (300÷ 3) + (30÷ 3) + (15÷ 3) =

2000 + 100 + 10 + 5 = 2115

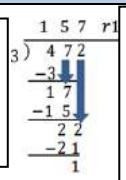
Divide numbers up to 4 digits by 1 digit whole number using long division.



How many 3s in 400?
100 (written as 1 in h's)
3 x 100 = 3 (00) subtract from 4(00) in 472. = 1(00)



1(00) left regrouped as 10 tens. 'Bring down' 7 tens = 17 (0) = 17 tens
17 tens ÷ 3 = 5 tens (at top)
5(0)x3= 15(0) then 17(0) – 15 (0) = 2(0)

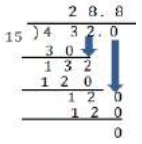


2 tens left regrouped as 20 ones. 'Bring down' 2 ones = 22.
22÷ 3= 7 (at top) 3x7=21 (written below 22).
22-21 = 1 (we can't do 1÷ 3 = remainder 1.

To interpret remainders appropriately for the context

Problem	Appropriate remainder
113 litres of water poured equally into 4 ten litre containers. How much water in each?	113÷4 = 28.25 = 28.25 litres
113 children in 4 equal groups. How many groups are formed? How many are not in a group?	113÷4 = 28 r1 28 groups and 1 child not in a group.
Taxis that hold 4 people are used to drive 113 people home. How many taxis are needed for everyone to get home?	113÷4 = 28 r1 29 taxis are needed
The class put 113 cookies into boxes of 4. How many were left over to eat?	113÷4 = 28 r1 1 cookie to eat
113 cookies in boxes of 4. How many boxes are needed? What fraction of the incomplete box is filled?	113÷4 = $28\frac{1}{4}$ 28 boxes are needed The last is $\frac{1}{4}$ filled.

Divide numbers up to 4 digits by 2 digit whole number using long division



432 ÷ 15 =
1) 28 r 12
2) $28\frac{12}{15} = 28\frac{4}{5}$
3) 28.8

To understand the order of operations

Step 1: Do the operation in brackets
Step 2: Do operation involving indices (i.e. 3^2)
Step 3: Multiply & divide in order from left to right
Step 4: Add & subtract in order from left to right.

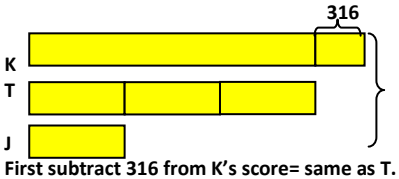
Division *Does not equal*

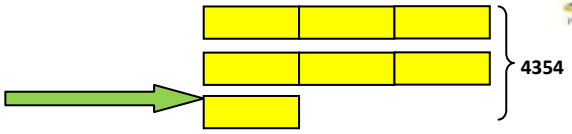
Remember: (a÷b) ÷ c ≠ a ÷ (b÷c)

112÷ 4 ÷ 2 =
Left to right: 112÷ 4 = 28
then 28÷ 2 = 14
NOT: 112 ÷ 4÷2 =112 ÷ 2 = 56

To solve word problems

KJ, Tion and Jade scored 4670 points playing their computer game. Tion scored 316 points less than KJ. Tion scored three times as many points as Jade. How many points did Tion score?

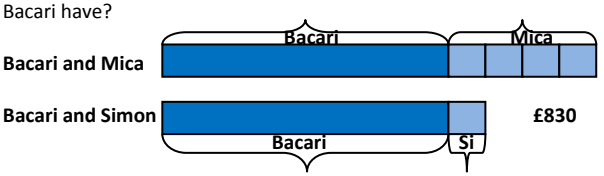




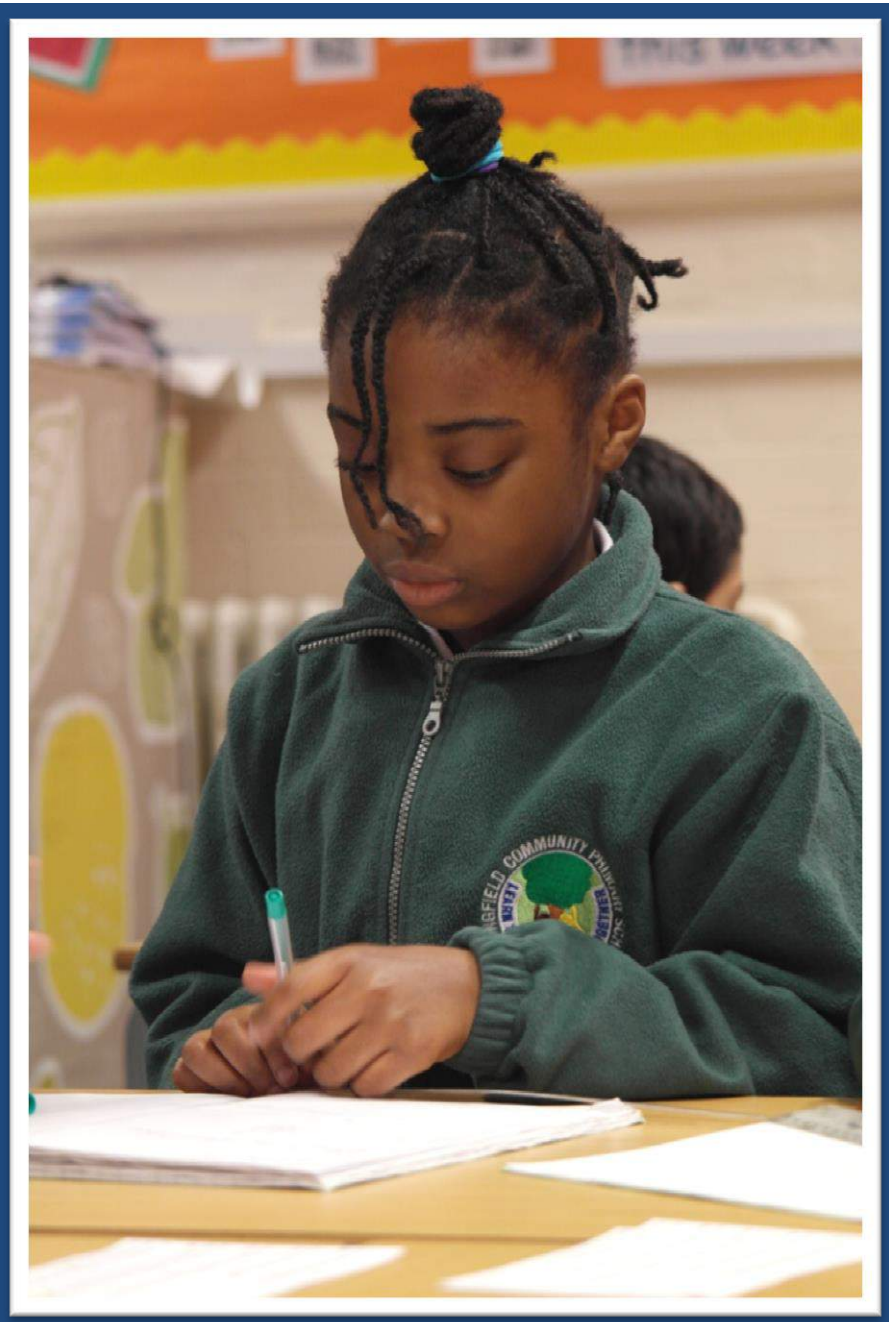
First subtract 316 from K's score= same as T.
4670 – 316 = 4354 points

7 units = 4354 points
1 unit = 4354 ÷ 7 = 622 points
3 units = 3 x 622 = 1866 points Tion scored 1866

Bakari and Mica had £1250. Bacari and Si had £830. Mica had four times as much as Si. How much did Bacari have?



£1250 - £830 = £420 (The difference between the amount Mica and Si had was £420)
3 units= £420 1 unit = £420 ÷ 3 = £140 **Si had £140.** £830 - £140 = £690 **Bacari had £690**

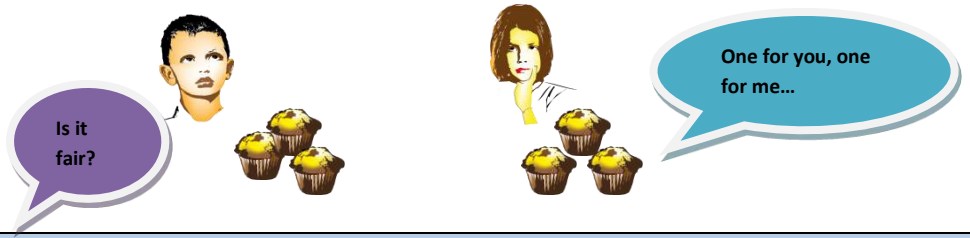
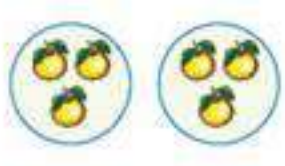
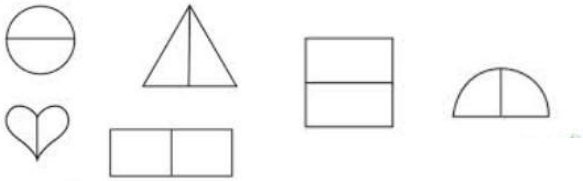
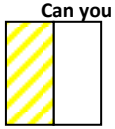
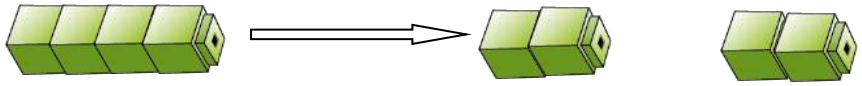


Chapter 6

Fractions, decimals & percentage

EYFS 2 - Fractions (When planning ensure you track forwards to year 1)	
Early Learning Goal 11 They solve problems, including doubling, halving and sharing.	**KS1 ready** Recognise, find and name a half as one of two equal parts of an object, shape or quantity. Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.
Key vocab: half, share equally, shape, object, amount, split, break, part, whole. **Key concepts** Fractions are bits of an object. Fractions are part of a number. Half is one of two equal parts. **Potential barriers/misconceptions** Pupils do not see a fraction as part of an object. Pupils do not understand how to share equally. Pupils do not have 1-1 correspondence. Pupils cannot identify fair and unfair sharing.	**Learning objectives** (See overleaf for exemplification) To share a quantity of objects between two. To identify half a group of objects To identify half a shape (not always circular) To put together halves to make whole shapes To break an object in half.
Example Questions How many shoelace holes are there? How many slices of bread to we need to make 4 whole sandwiches? Can you cut this sandwich in half? How many eggs will fill this box? Can you give me half the eggs? Can you cut the cake in half? How many pieces of cake are there now? Put half the sheep in the field. Half the ducks jumped in the pond. How many ducks in the pond?	**Mental Maths** **To revisit mental maths strategies in calculation strands** To count in ones from any single digit number (forwards and backwards) To calculate 'half the class' by splitting children equally into two lines. To count in twos up to 20. To identify a half-way point. To begin to count to ten in halves. (zero, half, one, one and a half, two, two and a half, three, three and a half…)

CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

EYFS 2 - Progression (a combination of these models and images can be used for every objective	
To share a quantity of objects between two.	**To identify half a group of objects**
	**Half of six is three**  
To identify half a shape	**To put together halves to make whole shapes**
 **Can you shade in half the shape?** 	 
To break (cut) an object in half.	
  	 Half of four is two

Year 1 - Fractions (When planning ensure you track back to Reception and forwards to year 2)

National Curriculum

Recognise, find and name a half as one of two equal parts of an object, shape or quantity.
Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.

Notes and guidance (non-statutory)

Pupils are taught half and quarter as 'fractions of' discrete and continuous quantities by solving problems using shapes, objects and quantities. For example, they could recognise and find half a length, quantity, set of objects or shape. Pupils connect halves and quarters to the equal sharing and grouping of sets of objects and to measures, as well as recognising and combining halves and quarters as parts of a whole.

Key vocab: half, share equally, shape, object, amount, part, equal parts, whole, one whole, one half, one quarter.

Key concepts

Fractions can be seen as:
Fractions as part of a whole (half a shape, half an object)
Fractions as part of a set (a quarter of 20- a fraction of a quantity)
Fractions are units divided into equal parts.
Instead of just using paper to show fractions a number of materials must be used: paper, pizza, fraction disks, and fruit.

Potential barriers/misconceptions

Pupils do not see a fraction as an equal part of a whole.
Pupils do not understand how to share equally.
Pupils cannot identify fair and unfair sharing.
Pupils must have strong conceptual understanding of seeing fractions as part of a whole before seeing fractions as part of a set.

Learning objectives (See overleaf for exemplification)

To recognise half an object (as one of two equal parts)
To recognise a quarter of an object (as one of two equal parts)
To recognise half a shape (as one of two equal parts)
To recognise a quarter of a shape (as one of two equal parts)
To identify half a quantity (to share equally between 2)
To identify a quarter of a quantity (to share equally between 4).
To know that a quarter is the same as half and half again.
To place fractions on a number line.
To identify halves (use Cuisenaire rods)

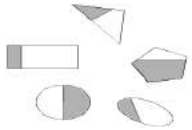
Example Questions

Here is a set of 12 pencils. How many is half the set?

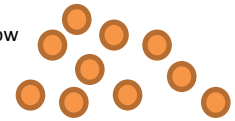
Shade one quarter of each shape:

There are twenty children in a classroom. Half of them are girls. How many are boys?

Tick the shape that has more than half shaded.



Look at these counters. Can you share them? How many would you and your friend get each?



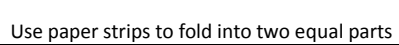
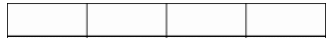
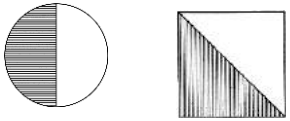
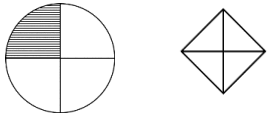
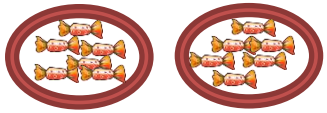
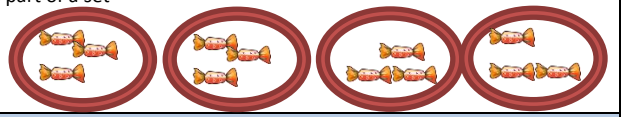
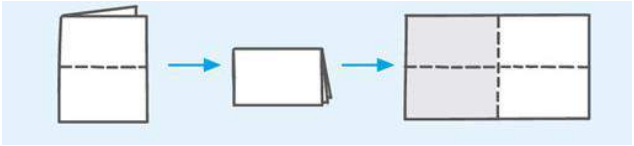
If the whole is 8 what is half them amount?

Mental Maths

To revisit mental maths strategies in calculation strands

To find half of any even number up to 20
To find a quarter of any even number up to 20 by halving and halving again.
To say what fraction of a shape is shaded (half, quarter).
To count in twos forwards and backwards from any even number to 20.
To count in halves. (Zero, half, one, one and a half, two, two and a half…)

CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

Year 1 - Progression (a combination of these models and images can be used for every objective)	
To recognise half an object (as one of two equal parts)	**To recognise a quarter of an object (as one of four equal parts)**
These are representations of fractions as part of a whole (to be manipulated in the concrete i.e. cutting) $\frac{1}{2}$   Use paper strips to fold into two equal parts	These are representations of fractions as part of a whole (to be manipulated in the concrete i.e. cutting) $\frac{1}{4}$   Use paper strips to fold into four Equal parts by halving and halving again.
To recognise half a shape (as one of two equal parts)	**To recognise a quarter of a shape (as one of four equal parts)**
These are representations of fractions as part of a whole $\frac{1}{2}$ 	These are representations of fractions as part of a whole $\frac{1}{4}$ 
To identify half a quantity (to share equally between 2)	**To identify a quarter of a quantity (to share equally between 4).**
These are representations of fractions as part of a set $\frac{1}{2}$ Of 12 	These are representations of fractions as part of a set $\frac{1}{4}$ Of 12 
To know that a quarter is the same as half and half again.	**To place fractions on a number line.**
	

To identify halves (use Cuisenaire rods)

Using the orange rod to represent 1, the yellow rods represent the halves. 10 units

$\frac{1}{2}$ 5 units + 5 units

Different rods can be used to represent 1. e.g. quarter of brown rod = red rod.

$\frac{1}{4}$ 

Year 2 - Fractions (When planning ensure you track back to year 1 and forwards to year 3)

National Curriculum

Recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity

Write simple fractions for example, $\frac{1}{2}$ of 6 = 3 and recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$

Key vocab: half, share equally, shape, object, amount, part, equal parts, whole, one whole, one half, one quarter, one third, three quarters, equivalent.

Key concepts

Fractions can be seen as:

Fractions as part of a whole (half a shape, half an object)

Fractions as part of a set (a quarter of 20- a fraction of a quantity)

Fractions are units divided into equal parts.

The symbol $\frac{1}{2}$ represents 1 out of 2 parts.

$\frac{2}{2}$ is a whole.

Potential barriers/misconceptions

Pupils are unable to relate fractions to the language of sharing equally.

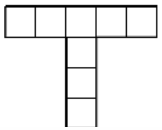
It is challenging for some children to identify equal parts and unequal parts when looking at shaded shapes.

Pupils are able to see fractions of an object but find it challenging to find fractions of an amount.

Focus should be on the understanding of a fraction. Children should be provided with many concrete opportunities to 'make' factions using unifix and other equipment.

Example Questions

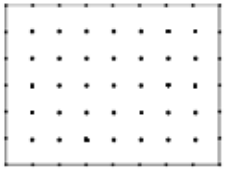
Shade one quarter of this shape:



What is half of this amount?

Mary eats half of these cherries. How many cherries does she eat?

Divide this shape into four equal parts:



Notes and guidance (non-statutory)

Pupils use fractions as 'fractions of' discrete and continuous quantities by solving problems using shapes, objects and quantities. They connect unit fractions to equal sharing and grouping, to numbers when they can be calculated, and to measures, finding fractions of lengths, quantities, sets of objects or shapes. They meet $\frac{3}{4}$ as the first example of a non-unit fraction.

Pupils should count in fractions up to 10, starting from any number and using the $\frac{1}{2}$ and $\frac{2}{4}$ equivalence on the number line. This reinforces the concept of fractions as numbers and that they can add up to more than one.

Learning objectives (See overleaf for exemplification)

To divide shapes into equal parts.

To know that $\frac{1}{2}$ and $\frac{1}{2}$ is equal to a whole.

To identify fractions of a length. (using halves, thirds and quarters)

To identify fractions of a shape. (using halves, thirds and quarters)

To identify fractions of a set of objects by sharing equally. (between two, three and four)

To identify fractions of a quantity. (using halves, thirds and quarters)

To identify all the different ways to make $\frac{1}{2}$

To recognise $\frac{3}{4}$ of a length, shape and object.

To recognise equivalent fractions.

To place fractions on a number line

To count in fractions

To use the bar model to show fractions

Mental Maths

To revisit mental maths strategies in calculation strands

To count in fractions up to ten starting from any number:

Count in halves: 0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$

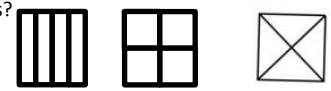
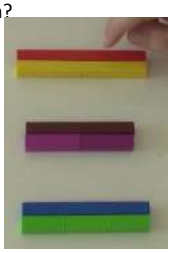
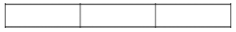
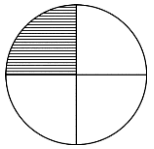
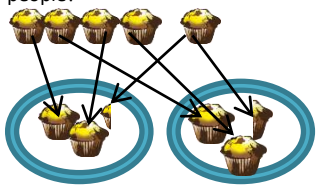
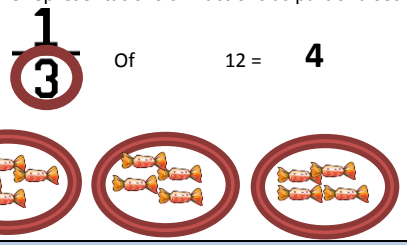
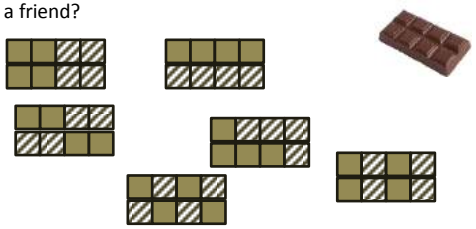
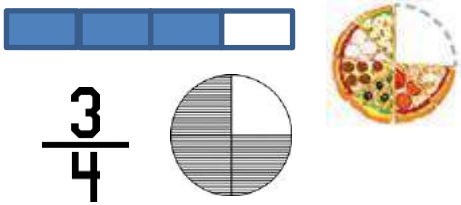
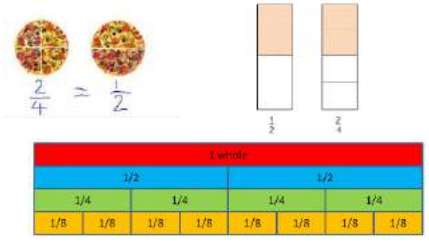
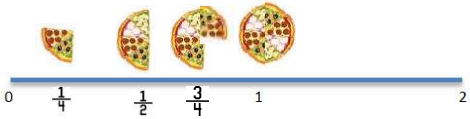
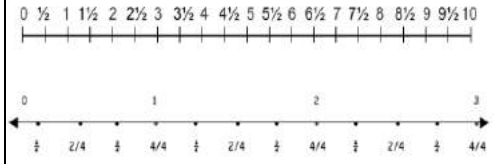
Count in quarters: 0, $\frac{1}{4}$, $\frac{2}{4}$ (same as $\frac{1}{2}$), $\frac{3}{4}$, $\frac{4}{4}$, (same as 1 whole), $1\frac{1}{4}$, $1\frac{2}{4}$ (same as $1\frac{1}{2}$) , $1\frac{3}{4}$, 2 …

Count in thirds: 0, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{3}$, (same as 1 whole), $1\frac{1}{3}$, $1\frac{2}{3}$, 2…

To start to make links with half is 50% and q quarter is 25%.

CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

Year 2 - Progression (a combination of these models and images can be used for every objective)

To divide shapes into equal parts.	**To know that $\frac{1}{2}$ and $\frac{1}{2}$ is equal to a whole.**	**To identify fractions of a length. (using halves, thirds and quarters)**	**To identify fractions of a shape. (using halves, thirds and quarters)**
Can you divide a square piece of paper into 4 equal parts?  Can you divide this strip of paper into three equal parts? 	The muffin is divided into 2 equal parts.  If I eat the 2 parts, then I will have eaten 2 out of 2 equal parts = $\frac{2}{2}$ (one whole) If I only eat one part, then I have eaten 1 out of 2 equal parts. This is written as $\frac{1}{2}$ If I eat $\frac{1}{2}$ now then the other $\frac{1}{2}$ later. I will have eaten $\frac{2}{2}$ I will have eaten the whole muffin.	Can you find $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length? (using Cuisenaire rods)  Use paper strips to fold into equal parts: 	The circle is divided into four equal parts (whole).  One part of the four equal parts is shaded. $\frac{1}{4}$ is shaded.
To identify fractions of a set of objects by sharing equally. (between two, three and four)	**To identify fractions of a quantity. (using halves, thirds and quarters)**	**To identify all the different ways to make $\frac{1}{2}$**	**To recognise $\frac{3}{4}$ of a length, shape and object.**
Can you share these four cakes equally between 3 people? 	These are representations of fractions as part of a set  1/3 Of 12 = 4	How could you cut up your chocolate to share it with a friend? 	 3/4 Three out of four equal parts are shaded.
To recognise equivalent fractions.	**To place fractions on a number line**	**To count in fractions**	**To use the bar model to show fractions**
			This bar model shows a whole with three equal parts.  2 parts are orange and 1 part is green. What fraction of the whole is green? Number of orange parts = 2 Number of green parts = 1 Fraction of the whole in orange is $\frac{2}{3}$ Fraction of the whole in green is $\frac{1}{3}$

Year 3 - Fractions (When planning ensure you track back to year 2 and forwards to year4)

National Curriculum

Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators

Recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators

Recognise and show, using diagrams, equivalent fractions with small denominators

Add and subtract fractions with the same denominator within one whole [for example, $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$]

Compare and order unit fractions, and fractions with the same denominators

Solve problems that involve all of the above.

Notes and guidance (non-statutory)

They begin to understand unit and non-unit fractions as numbers on the number line, and deduce relations between them, such as size and equivalence. They should go beyond the [0, 1] interval, including relating this to measure.

Pupils understand the relation between unit fractions as operators (fractions of), and division by integers.

They continue to recognise fractions in the context of parts of a whole, numbers, measurements, a shape, and unit fractions as a division of a quantity.

Pupils practise adding and subtracting fractions with the same denominator through a variety of increasingly complex problems to improve fluency.

Key vocab: half, share equally, shape, object, amount, part, equal parts, whole, one whole, one half, one quarter, one third, three quarters, two thirds, one tenth, equivalent, denominator, numerator,

Key concepts Fractions can be seen as:

Fractions as part of a whole (half a shape, half an object)

Fractions as part of a set (a quarter of 20- a fraction of a quantity)

Fractions are units divided into equal parts.

A unit fraction has one as the numerator ($\frac{1}{7}$) A non-unit fraction has >1 as the numerator ($\frac{2}{7}, \frac{3}{7}$…)

Potential barriers/misconceptions

Pupils struggle with the idea of a fraction being an equal part of something.

Pupils think that the bigger the numerator, the bigger the fraction.

Pupils think that the size of a fraction depends solely on the denominator and you can ignore the numerator.

Pupils think $\frac{3}{4}$ is always more than $\frac{1}{2}$ and do not make reference to the whole.

Pupils struggle moving on from finding the value of a shape to the more abstract concept of finding a fraction of a number.

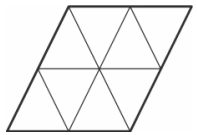
When comparing fractions pupils must ensure they use a whole that is the same size.

The whole can be more than one. i.e. the whole is 20, half is 10.

Example Questions

Colour $\frac{1}{2}$ of this shape.

Shade $\frac{1}{4}$ of this shape.



Five is a quarter of a number.
What is the number?

Put a ring around the fraction which is equal to one-half.

$\frac{1}{20}$ $\frac{1}{100}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{5}{10}$

Learning objectives (See overleaf for exemplification)

To identify unit fractions of objects, shapes and length. (a unit fraction has 1 as the numerator)
To identify non-unit fractions of objects, shapes and length. (a non-unit fraction has >1 as the numerator)
To calculate fractions of a quantity
To recognise equivalent fractions
To recognise that tenths arise from dividing an object into ten equal parts
To compare fractions (fractions with the same denominator)
To order fractions (fractions with the same denominator)
To compare fractions with different denominators
To recognise equivalent fractions (see exemplification year 4)
To add like fractions (fractions with the same denominator)
To subtract like fractions
To solve word problems involving fractions

Mental Maths (ensuring the revisit of mental maths strategies in calculation strands)

To count in fractions up to ten starting from any number:

Count in halves: 0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$

Count in quarters: 0, $\frac{1}{4}, \frac{2}{4}$ (same as $\frac{1}{2}$), $\frac{3}{4}, \frac{4}{4}$, (same as 1 whole), $1\frac{1}{4}$, $1\frac{2}{4}$ (same as$1\frac{1}{2}$) , $1\frac{3}{4}$, 2 …

Count in thirds: 0, $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}$, (same as 1 whole), $1\frac{1}{3}$, $1\frac{2}{3}$, 2…

To count up and down in tenths

To divide one digit numbers by ten & to divide multiples of ten by ten.

To know that: two quarters are the same as one half
One half is equivalent to five tenths
Ten tenths make one whole
One whole is three quarters plus one quarter, three tenths plus seven tenths etc
One quarter is half of one half

To identify numbers that are half way between two digits (halfway between 3 and 4?)

To say any number between two whole numbers (between 6 and 7 = 6 and a quarter)

To identify on a number line that half is greater than a quarter, less than three quarters and to know that three quarters is between one half and one whole.

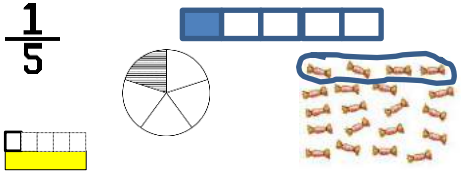
To choose any number on a number line to 100 and estimate where half that number is.

CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

Year 3 - Progression (a combination of these models and images can be used for every objective)

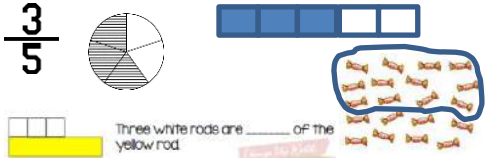
To identify unit fractions of objects, shapes and length. (a unit fraction has 1 as the numerator)

$\frac{1}{5}$



To identify non-unit fractions of objects, shapes and length. (a non-unit fraction has >1 as the numerator)

$\frac{3}{5}$



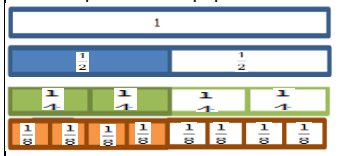
To calculate fractions of a quantity

$\frac{4}{9}$ of the block is green

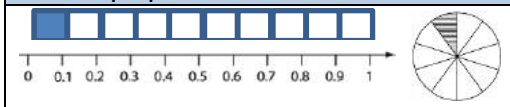
$\frac{5}{9}$ of the block is red

To recognise equivalent fractions

Use strips of folded paper to show equivalent fractions.



To recognise that tenths arise from dividing an object into ten equal parts



Each whole is divided into ten equal parts.

Each part is $\frac{1}{10}$ (one tenth) this is written as 0.1

$\frac{10}{10}$ = 1 whole

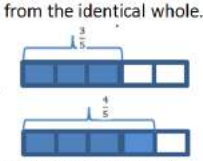
To compare fractions (fractions with the same denominator)

Fractions can be compared if taken from the identical whole.

There were two cakes at the party.

$\frac{3}{5}$ of the chocolate cake was eaten.

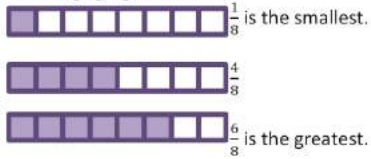
$\frac{4}{5}$ of the sponge cake was eaten.



Which one was most popular? How much is left of each?

To order fractions (fractions with the same denominator)

Arrange $\frac{1}{8}$ $\frac{6}{8}$ $\frac{4}{8}$ in order from the smallest.



$\frac{1}{8}$ is the smallest.

$\frac{4}{8}$

$\frac{6}{8}$ is the greatest.

To compare fractions (fractions with different denominators)

The denominators are different but must come from the same size whole in order to be compared.

Izabela eats $\frac{1}{2}$ the cake. Oliver eats $\frac{1}{5}$ of the cake. Who eats more?

$\frac{1}{2}$ (divide into 2 equal parts)

$\frac{1}{5}$ (divide into 5 equal parts).

Izabela eats more

To add like fractions (fractions with the same denominator)

$\frac{5}{8}$ and $\frac{3}{8}$ are like fractions because the denominator is the same.

Paul ate $\frac{5}{8}$ and Colin ate $\frac{3}{8}$. How much pizza did they eat altogether?

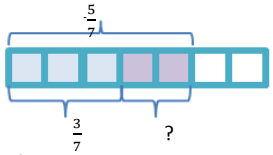
$\frac{5}{8}$ + $\frac{3}{8}$ = $\frac{8}{8}$ (whole)

They ate the whole pizza.

To subtract like fractions

Subtract $\frac{3}{7}$ from $\frac{5}{7}$

$\frac{5}{7}$ of the picture has been coloured by Ben and Lily. Ben coloured $\frac{3}{7}$ how much did Lily colour?



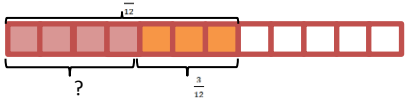
$\frac{5}{7}$ - $\frac{3}{7}$ = $\frac{2}{7}$ Lily coloured $\frac{2}{7}$ of the picture.

To solve word problems involving fractions

Lucy cut her pizza into twelfths. She gave $\frac{7}{12}$ to Mary.

Mary shared some of her pizza with Ollie and then had $\frac{3}{12}$ left.

How much pizza did Ollie get?



$\frac{7}{12}$ - $\frac{3}{12}$ = $\frac{4}{12}$

Ollie got $\frac{4}{12}$ of the pizza.

Year 4 - Fractions (including decimals) (When planning ensure you track back to year 3 and forwards to year 5)

National Curriculum

Recognise and show, using diagrams, families of common equivalent fractions.
Count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.
Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number.
Add and subtract fractions with the same denominator.
Recognise and write decimal equivalents of any number of tenths or hundredths.
Recognise and write decimal equivalents to $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$
Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths.
Round decimals with one decimal place to the nearest whole number.
Compare numbers with the same number of decimal places up to two decimal places.
Solve simple measure and money problems involving fractions and decimals to two decimal places.

Notes and guidance (non-statutory)

Pupils should connect hundredths to tenths and place value and decimal measure.
They extend the use of the number line to connect fractions, numbers and measures.
Pupils understand the relation between non-unit fractions and multiplication and division of quantities, with particular emphasis on tenths and hundredths.
Pupils make connections between fractions of a length, of a shape and as a representation of one whole or set of quantities. Pupils use factors and multiples to recognise equivalent fractions and simplify where appropriate (for example, $\frac{6}{9} = \frac{2}{3}$ or $\frac{1}{4} = \frac{2}{8}$).
Pupils continue to practise adding and subtracting fractions with the same denominator, to become fluent through a variety of increasingly complex problems beyond one whole.
Pupils are taught throughout that decimals and fractions are different ways of expressing numbers and proportions.
Pupils' understanding of the number system and decimal place value is extended at this stage to tenths and then hundredths. This includes relating the decimal notation to division of whole number by 10 and later 100.
They practise counting using simple fractions and decimals, both forwards and backwards.
Pupils learn decimal notation and the language associated with it, including in the context of measurements. They make comparisons and order decimal amounts and quantities that are expressed to the same number of decimal places. They should be able to represent numbers with one or two decimal places in several ways, such as on number lines.

Key vocab: Part, equal parts, fraction, whole, half, quarter, eighth, third, sixth, fifth, tenth, twentieth, proportion, in every, for every, decimal, decimal fraction, decimal point, decimal place.

Key concepts

Fractions make up equal parts of a whole. Unequal parts are not fractions of a whole. The numerator refers to the parts required. The denominator refers to the total number of parts in a whole.
The multiplying factor technique is used to find equivalent fractions.
Two fractions are equal when they are expressed as equivalent fractions.

Potential barriers/ misconceptions

Pupils think fractions and decimals are negative numbers.
Pupils think fractions are added together by adding the top numbers together then adding the bottom numbers together.
Pupils are unsure of the meaning of the value of the denominator and the numerator, particularly where the numerator is greater than 1.
Pupils do not understand the value of each digit or what the zero represents in a decimal number.
Pupils think that numbers with more decimal points are bigger than those with less e.g. 1.6 is less than 1.034.
Pupils do not understand the column to the right of the decimal place is ten times smaller (and just 'add/subtract zeros.) Pupils misread decimals e.g. read 2.47 is read as 'two point forty-seven'.

Example Questions

What fraction of these tiles is circled?

Circle the two fractions that are greater than $\frac{1}{2}$. $\frac{1}{8}$ $\frac{6}{10}$ $\frac{5}{8}$ $\frac{3}{10}$

Here is a part of a number line. Write in the two missing numbers:

Write these prices in order from smallest to largest: 99p £10.50 £0.75 £9 £2.05
Write these amounts in order: £70.07 £70.70 £7.70 £7.07
Which of these means $^{7}/_{10}$? A) 70 B) 7 C) 0.7 D) 0.07
0.4 is the same as: A) four B) four tenths C) four hundredths D) one-quarter
Circle the fraction that is the same as nought point five: $^{1}/_{2}$ $^{1}/_{4}$ $^{1}/_{3}$ $^{3}/_{4}$
Put a ring around the fraction which is equal to nought point four: $^{1}/_{4}$ $^{1}/_{40}$ $^{1}/_{400}$ $^{4}/_{10}$

Learning objectives (See overleaf for exemplification)

To identify equivalent fractions
Show equivalent fractions pictorially (and calculate equivalent fractions)
To compare fractions
To use factors and multiples to recognise equivalent fractions
To simplify fractions
Add and subtract like fractions (fractions with the same denominator).
To calculate the fraction of numbers and quantities
Recognise and write decimal equivalents of any number of tenths of hundredths
Recognise and write decimal equivalents to $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$
Compare numbers with the same number of decimal places (up to 2 decimal places)
Round decimals with one decimal place to the nearest whole number.
To connect fractions, decimals and measures (using a number line)

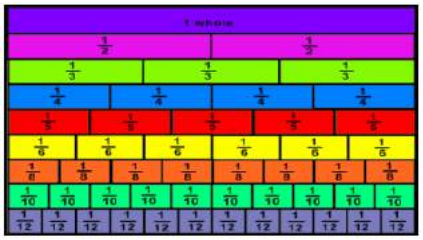
Mental Maths

Count from zero in steps of one tenth
Start at 5.1 (for example) and count on or back in steps of 0.1
Count up and down in hundredths;
Recognise that hundredths arise when dividing an object by one hundred and tenths from dividing one by ten.
To count in fractions forwards and backwards & to count in decimals forwards and backwards
Divide one digit numbers by 10 and 100 (identifying the value of the digits in the answer as ones, tenths & hundredths)
Divide a two digit numbers by 10 and 100 (identifying the value of the digits in the answer as ones, tenths & hundredths)
Round decimals with one decimal place to the nearest whole number. (and to round to the nearest £)
To multiply whole numbers by ten (and explain that the digits move one place to the left)
To divide whole numbers by ten (and explain that the digits move one place to the right)
To multiply integers less than 1000 by 100. (800x100=)
To know that finding half is equivalent to dividing by 2. Half 16 is 16 ÷ 2 = 8
To know that when sharing a cake/pizza etc between 4 you divide by four and each person receives a quarter $\frac{1}{4}$
To use multiplication facts to find: one tenth of 100, 30, 500 etc
one fifth of 15, 10, 35 etc
one tenths, one quarter, one fifth of £1 or 1m.
To know that 0.5 = $\frac{1}{2}$ that 0.25 = $\frac{1}{4}$ that 0.75 = $\frac{3}{4}$ and that 0.1 = $\frac{1}{10}$ (particulary in context of money and measures)

CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

Year 4 - Progression (a combination of these models and images can be used for every objective)

To identify equivalent fractions

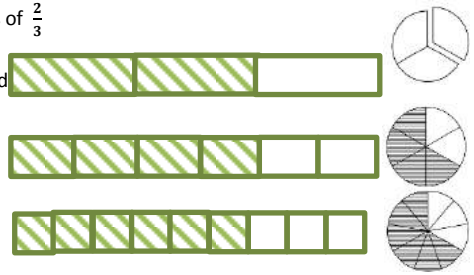


Continue the equivalent fraction sequence.
Can you see a pattern?

Show equivalent fractions pictorially

Equivalent fractions of $\frac{2}{3}$

$\frac{2}{3}$ of the bar is shaded

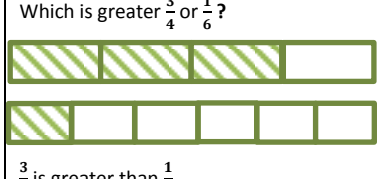


$\frac{2}{3} = \frac{4}{6}$

$\frac{2}{3} = \frac{6}{9}$

To compare fractions (Use fraction strips or fraction discs to help.)

Which is greater $\frac{3}{4}$ or $\frac{1}{6}$?



$\frac{3}{4}$ is greater than $\frac{1}{6}$

Draw two bars the same length. Divide each into the number represented by the denominator. (bar one = 4, bar two = 6)

To use factors and multiples to recognise equivalent fractions

Multiplying the numerator and the denominator by the same number will give an equivalent fraction.

The first three equivalent fractions of $\frac{2}{3}$ **(x2, x3, x4…)**

$\frac{2}{3} = \frac{4}{6}$ (x2 / x2)

$\frac{2}{3} = \frac{6}{9}$ (x3 / x3)

$\frac{2}{3} = \frac{6}{9}$ (x4 / x4)

Concept to be taught alongside visual representation using fraction strips.

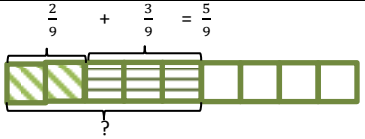
To simplify fractions

You can simplify fractions by dividing the numerator and denominator by the same number. This is called cancelling.

$$\frac{30 \div 6}{42 \div 6} = \frac{5}{7}$$

Add and subtract like fractions (fractions with the same denominator).

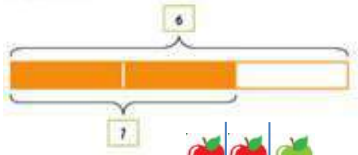
$\frac{2}{9} + \frac{3}{9} = \frac{5}{9}$



(What fraction is unshaded? = $\frac{4}{9}$)

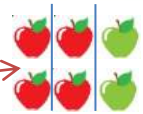
To calculate the fraction of numbers and quantities

$\frac{2}{3}$ of 6

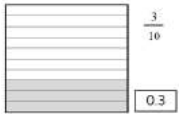


$\frac{2}{3}$ of the apples are red

(group split into **3** parts)

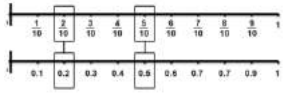


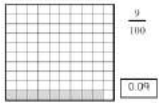
Recognise and write decimal equivalents of any number of tenths of hundredths



Tenths (pictorial)

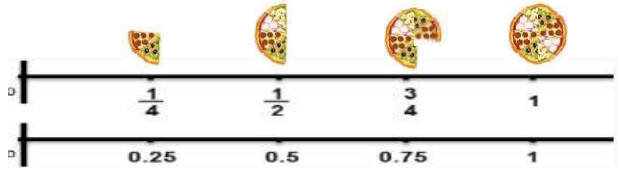
Tenths on a number line





Hundredths (pictorial)

Recognise and write decimal equivalents to $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$



Compare numbers with the same number of decimal places (up to 2 decimal places)

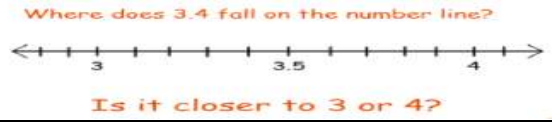
Using **>** (greater than) **<** (less than) and **=** (equal to)

Compare: 3.4 [ ] 3.45

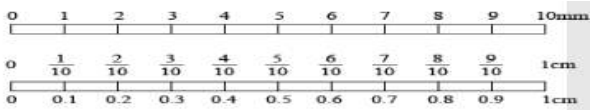
Think about it as: £3.40 [ ] £3.45

Therefore: 3.40 [<] 3.45

Round decimals with one decimal place to the nearest whole number



To connect fractions, decimals and measures (using a number line)



To use the bar model to help solve word problems

A computer game is £24 in the sale. It is $\frac{1}{4}$ cheaper than its original price.

£24 ÷3 = £8 (giving the value of 3 sections of the bar). The final section of the bar must also be £8 (same proportion). £8 x 4 = original price = £32

Year 5 - Fractions (including decimals and percentages) (When planning ensure you track back to year 4 and forwards to year 6)

National Curriculum

Compare and order fractions whose denominators are all multiples of the same number.

Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths. Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $\frac{2}{5}+\frac{4}{5}=\frac{6}{5}=1\frac{1}{5}$]

Add and subtract fractions with the same denominator and denominators that are multiples of the same number. Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams. Read and write decimal numbers as fractions [for example, $0.71=\frac{71}{100}$]

Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents.

Round decimals with two decimal places to the nearest whole number and to one decimal place.

Read, write, order and compare numbers with up to three decimal places.

Solve problems involving number up to three decimal places. Recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal. Solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{2}{5}, \frac{4}{5}$, and those fractions with a denominator of a multiple of 10 or 25.

Notes and guidance (non-statutory)

Pupils should be taught throughout that percentages, decimals and fractions are different ways of expressing proportions. They extend their knowledge of fractions to thousandths and connect to decimals and measures.

Pupils connect equivalent fractions > 1 that simplify to integers with division and other fractions > 1 to division with remainders, using the number line and other models, and hence move from these to improper and mixed fractions.

Pupils connect multiplication by a fraction to using fractions as operators (fractions of), and to division, building on work from previous years. This relates to scaling by simple fractions, including fractions > 1.

Pupils practise adding and subtracting fractions to become fluent through a variety of increasingly complex problems. They extend their understanding of adding and subtracting fractions to calculations that exceed 1 as a mixed number. Pupils continue to practise counting forwards and backwards in simple fractions.

Pupils continue to develop their understanding of fractions as numbers, measures and operators by finding fractions of numbers and quantities.

Pupils extend counting from year 4, using decimals and fractions including bridging zero, for example on a number line. Pupils say, read and write decimal fractions and related tenths, hundredths and thousandths accurately and are confident in checking the reasonableness of their answers to problems.

They mentally add and subtract tenths, and one-digit whole numbers and tenths.

They practise adding and subtracting decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1 (for example, 0.83 + 0.17 = 1).

Pupils should go beyond the measurement and money models of decimals, for example, by solving puzzles involving decimals. Pupils should make connections between percentages, fractions and decimals (for example, 100% represents a whole quantity and 1% is $\frac{1}{100}$, 50% is $\frac{50}{100}$, 25% is $\frac{25}{100}$) and relate this to finding 'fractions of'.

Key vocab proper/improper fraction, mixed number, numerator, denominator, equivalent, per cent, %

Key concepts To find fractions of amounts we divide the whole into equal amounts.

Mixed numbers are made up of whole numbers and proper fractions. A proper fraction is a number between 0 and 1. A proper fraction is part of a whole. (an improper fraction is equal to or >1)

Pupils should be taught throughout that percentages, decimals and fractions are different ways of expressing proportions. Per cent relates to 'number of parts per hundred. Percentages are written as a fraction with the denominator 100. Decimals are special types of fractions with denominators in t, h and th

Two fractions are related when the denominator of one is a multiple of the denominator of the other.

Potential barriers/misconceptions

Pupils think that when you multiply fractions and decimals the total gets bigger and when you divide they get smaller.

Pupils regard fractions and decimals as two abstract ideas. Unable to make links between the two. Pupils do not understand the value of each digit or what the zero represents in a decimal number.

Pupils consider hundredths to be greater than tenths.

Pupils think that numbers with more decimal points are bigger than those with less e.g. 1.6 is less than 1.034. Pupils do not understand the column to the right of the decimal place is ten times smaller.

Pupils do not understand what happens when you multiply or divide by ten.

Pupils view a percentage as a number rather than part of an amount. Pupils confuse tenths with hundredths. Pupils think percentages cannot be greater than 100.

Pupils are not able to multiply and divide by 10 and 100 securely to find 10% and 1% of a total.

Learning objectives (See overleaf for exemplification)

To identify equivalent fractions (including tenths and hundredths)

To compare and order fractions (whose denominators are multiples of the same number)

To calculate fractions of numbers and quantities.

Read and write decimal numbers as fractions.

To convert mixed numbers to improper fractions (and back)

To add and subtract fractions with the same denominator (see year 4)

To add and subtract fractions with denominators that are multiples of the same number.

To add and subtract decimals

To multiply fraction and mixed numbers by a whole number. (use diagrams to support)

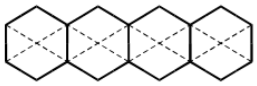
To convert fractions to percentages

Example Questions

Mark with arrows the points –1.5 and 0.45 on the number line.

Fill in the missing numbers in the boxes: $\frac{2}{12}=\frac{\square}{6}$ $\frac{1}{2}=\frac{12}{\square}$ $\frac{1}{\square}=\frac{6}{24}$

This diagram shows four regular hexagons. Shade in one third of the diagram.



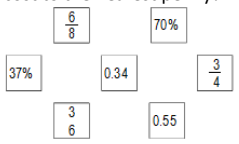
Write the total as a decimal. $4 + {}^{6}/_{10} + {}^{2}/_{100} =$

Fill in the blank to make this correct: 6.45 = 6 + 0.4 +_

Here are four digit cards: '9' '4' '1' '2' use each digit card once to make the decimal number nearest to 20:

Four apples cost ninety-five pence. How much does each apple cost to the nearest penny?

Tick each of the cards that shows more than a half.

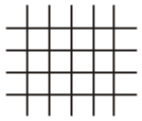


What is seven-tenths as a percentage?

What is thirty out of one hundred as a percentage?

What percentage of the bar is shaded?

Here is a grid made of squares. Shade 10% of this grid.



Mental Maths

Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents.

Round decimals with two decimal places to the nearest whole number and to one decimal place. (metres & £s)

Read, write, order and compare numbers with up to three decimal places.

To know percentage and decimal equivalents of $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{2}{5}, \frac{4}{5}$

To mentally add and subtract tenths, and one-digit whole numbers and tenths.

To multiply and divide whole numbers by 10 and 100 (and to explain what happens to the place value)

To know that one quarter is half of one half. One eighth is half of one quarter. One sixth is half of one third. One tenth is half of one fifth, one twentieth is half of one tenth.

To understand that finding one third is equivalent to dividing by three.

To know that when dividing 3 whole cakes by 4 each person gets $\frac{3}{4}$ or $3 \div 4$.

To identify a decimal fraction between 2 numbers (between 4.1 and 4.2)

To know that 10% = 0.1 = $\frac{1}{10}$, 25% = 0.25 = $\frac{1}{4}$, 50 % = 0.5 = $\frac{1}{2}$, 75% = 0.75 = $\frac{3}{4}$

CONCRETE + PICTORIAL + ABSTRACT 2 + 1 = 3

Year 5 - Progression (a combination of these models and images can be used for every objective)

To identify equivalent fractions (including tenths and hundredths)

Multiplying the numerator and the denominator by the same number will give an equivalent fraction.

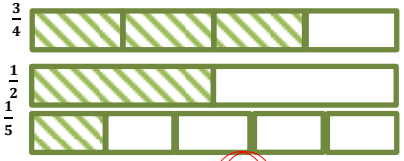
The first three equivalent fractions of $\frac{2}{7}$ **(x2, x3, x4…)**

$\frac{2}{7} = \frac{4}{14}$ (x2 / x2) $\quad \frac{2}{7} = \frac{6}{21}$ (x3 / x3) $\quad \frac{2}{7} = \frac{6}{28}$ (x4 / x4)

Concept to be taught alongside visual representation using fraction strips.

To compare and order fractions (whose denominators are multiples of the same number)

Put in order $\frac{3}{4}, \frac{1}{5}, \frac{1}{2}$

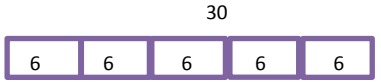


$\frac{3}{4} = \frac{15}{20}$ (x5 / x5) $\quad \frac{1}{2} = \frac{10}{20}$ (x10 / x10) $\quad \frac{1}{5} = \frac{4}{20}$ (x4 / x4)

Find the common lowest common denominator: 2, 4, 5= multiples of 20

To find fractions of numbers and quantities.

$\frac{1}{5}$ of 30 (the bar must be divided into 5 parts)

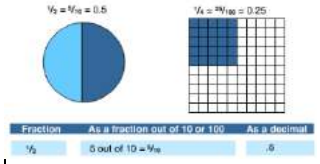


The same image can be used to find $\frac{2}{5}$ or $\frac{3}{5}$ of 30 etc.

This can be done in the concrete by children 'sharing equally' into five groups to discover that 30÷5 = 6. The find $\frac{2}{5}$ they then multiply by 2.

Read and write decimal numbers as fractions.

$0.71 = \frac{71}{100}$



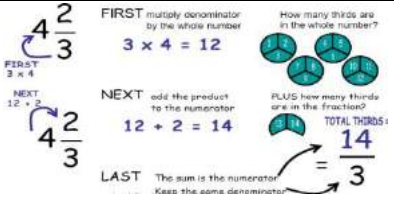
To convert improper fractions to mixed numbers

$\frac{6}{4} = 6 \div 4$

$4\overline{)6.^{2}0}$ $\quad 1.5$

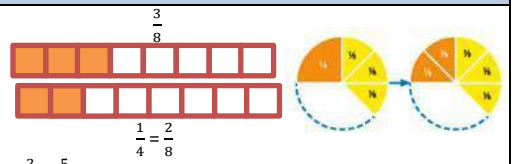
1.5 = 1 $\frac{1}{2}$ (or 1 $\frac{2}{4}$)

To convert mixed numbers to improper fractions



To add and subtract fractions with denominators that are multiples of the same number.

$\frac{3}{8}$ add $\frac{1}{4}$

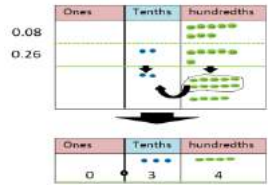


$\frac{3}{8} + \frac{1}{4} = \frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

8 and 4 are both multiples of 8.

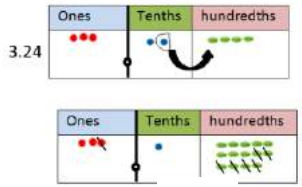
To add and subtract decimals

Add 0.08 and 0.26



0.08
+ 0.26
0.34

Subtract 1.06 from 3.24



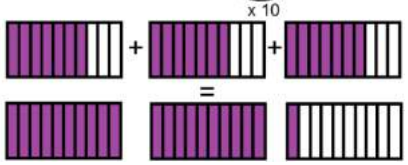
3.24
- 1.06
2.18

To multiply fractions and mixed numbers by a whole number. (use diagrams to support)

Product of a fraction and a whole number

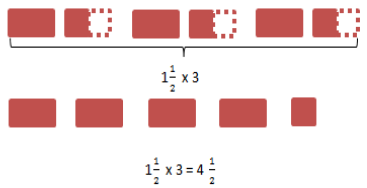
1. $3 \times \frac{7}{10} = \frac{21}{10}$

2. $\frac{3}{1} \times \frac{7}{10} = \frac{21}{10}$ (x 7, x 10)



Product of a mixed number and a whole number

$1\frac{1}{2} \times 3 =$

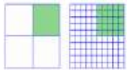


To convert fractions to percentages

1% is $\frac{1}{100}$, 50% is $\frac{50}{100}$, 25% is $\frac{25}{100}$

Jamal spent $\frac{1}{4}$ of his birthday money. What % is this?

Method 1



$\frac{1}{4} = \frac{25}{100} = 25\ \%$ (x 25)

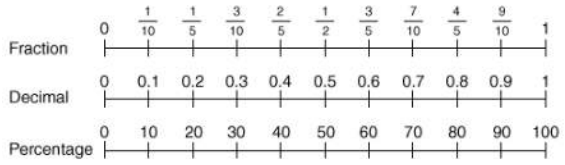
Method 2: $1 = \frac{4}{4}$ **(100%)** $\quad \frac{1}{4} = ?\ \%$

1 whole = 4 parts = 100% → 1 part = $\frac{100}{4}$ **% = 25 %**

Method 3:

$\frac{1}{4} = \frac{1}{4} \times 100\% = 25\%$

To recognise fraction, decimal and percentage equivalents and convert.



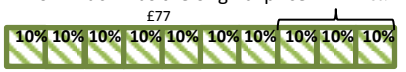
To use the bar model to help solve word problems

Jenny bought three strawberry laces from the sweet shop.
She ate $\frac{4}{9}$ of one lace. What fraction of the laces was left?

$3 - \frac{4}{9} = 2\frac{9}{9} - \frac{4}{9}$

$= 2\frac{5}{9}$

A computer game is reduced in the sale by 30%. Its reduced price is £77. How much was the original price.



£77 ÷ 7 = £11

The original cost (the whole bar) is £11 x 10 = £110

CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

Year 6 - Fractions (including decimals and percentages) (When planning ensure you track back to year 5 for progression)

National Curriculum

Use common factors to simplify fractions; use common multiples to express fractions in the same denomination.
Compare and order fractions, including fractions > 1
Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions. Multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$]. Divide proper fractions by whole numbers [for example $\frac{1}{3} \div 2 = \frac{1}{6}$]
Associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $\frac{3}{8}$]
Identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places. Multiply one-digit numbers with up to two decimal places by whole numbers. Use written division methods in cases where the answer has up to two decimal places. Solve problems which require answers to be rounded to specified degrees of accuracy. Recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.

Key vocab: proper/improper fraction, mixed number, numerator, denominator, equivalent, per cent, %

Key concepts

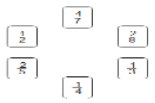
Pupils can use their understanding of the relationship between unit fractions and division to work backwards by multiplying a quantity that represents a unit fraction to find the whole quantity (for example, if 1/4 of a length is 36cm, then the whole length is 36 × 4 = 144cm). A whole number when divided by another can result in: a whole number with or without remainder, purely a proper fraction or a mixed number.

Potential barriers/misconceptions

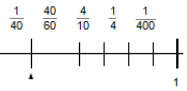
Pupils do not understand that fractions can be 'equivalent' (i.e. same size but split into different number of equal parts). Therefore pupils struggle with concept of reducing to a simpler equivalent.
Pupils may remember the rule 'whatever you do to the bottom, do to the top' (and vice versa) but due to lack of understanding why this works, cannot apply it in context.
Pupils do not understand that digits after the decimal point represent parts of a whole.
Pupils do not know where to place the decimal point when calculating.
Pupils misinterpret the values of digits and changing values when multiplying or dividing a decimal number by an integer. Pupils are unable to convert between units using decimals to two places (e.g. change 2.75 litres to 2750ml).

Example Questions

Draw one line to join two fractions which have the same value.



Put a ring around the fraction which is equivalent to 40%.



Look at these fractions. $\frac{1}{2}$ $\frac{1}{3}$ $\frac{5}{6}$

Mark each fraction on the number line.

Put a ring around the decimal which is equal to one-fifth: 0.1, 0.2 0.3, 0.4, 0.5
Write these numbers in order from smallest to largest: 3.03 3.3 3.23 3 3.2
Put a ring around the smallest number: 0.27 0.207 0.027 2.07 2.7
What number is exactly halfway between one point one and one point two?
Write a number that is bigger than nought point three but smaller than nought point four.
Round each decimal to the nearest whole number: 6.01 9.51 7.75
Round two point six nine four to one decimal place.
Write 80% as a fraction in its simplest form.
What is seven-tenths as a percentage?
What is twenty out of forty as a percentage?
A class is collecting money for charity. They want a total of £1000. By the end of April, they have collected £400.
What percentage of their total have they collected by the end of April?
Write these decimals as percentages: 0.25 0.6
Write these percentages as decimals: 42% 5%
John had £5. He gave 25% of it to charity. How much did he give?

Notes and guidance (non-statutory)

Pupils should practise, use and understand the addition and subtraction of fractions with different denominators by identifying equivalent fractions with the same denominator. They should start with fractions where the denominator of one fraction is a multiple of the other (for example, $\frac{1}{2} + \frac{1}{8} = \frac{5}{8}$) and progress to varied and increasingly complex problems.
Pupils should use a variety of images to support their understanding of multiplication with fractions. This follows earlier work about fractions as operators (fractions of), as numbers, and as equal parts of objects, for example as parts of a rectangle.
Pupils use their understanding of the relationship between unit fractions and division to work backwards by multiplying a quantity that represents a unit fraction to find the whole quantity (for example, if 1/4 of a length is 36cm, then the whole length is 36 × 4 = 144cm).
They practise calculations with simple fractions and decimal fraction equivalents to aid fluency, including listing equivalent fractions to identify fractions with common denominators.
Pupils can explore and make conjectures about converting a simple fraction to a decimal fraction (for example, 3 ÷ 8 = 0.375). For simple fractions with recurring decimal equivalents, pupils learn about rounding the decimal to three decimal places, or other appropriate approximations depending on the context. Pupils multiply and divide numbers with up to two decimal places by one-digit and two-digit whole numbers. Pupils multiply decimals by whole numbers, starting with the simplest cases, such as 0.4 × 2 = 0.8, and in practical contexts, such as measures and money. Pupils are introduced to the division of decimal numbers by one-digit whole number, initially, in practical contexts involving measures and money. They recognise division calculations as the inverse of multiplication.
Pupils also develop their skills of rounding and estimating as a means of predicting and checking the order of magnitude of their answers to decimal calculations. This includes rounding answers to a specified degree of accuracy and checking the reasonableness of their answers

Learning objectives (See overleaf for exemplification)

To compare and order fractions (including fractions >1)
To use common factors to simplify fractions
To use common multiples to express fractions in the same denomination
To add and subtract fractions with denominators that are multiples of the same number (see year 5)
To add and subtract fractions with different denominators and mixed numbers (using equivalent fractions)
To multiply simple pairs of proper fractions (writing the answer in its simplest form) e.g. $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$
To divide proper fractions by whole numbers. $\frac{1}{3} \div 2 = \frac{1}{6}$
To calculate decimal fraction equivalents (by dividing using a simple fraction)
To multiply one digit numbers with up to two decimal places by whole numbers.
To multiply up to 2 digit decimals by whole numbers.
To interpret remainders as whole number remainders, fractions or by rounding, as appropriate for the context.

Mental Maths

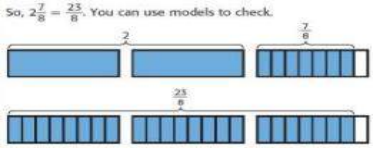
Identify the value of each digit in numbers given to three decimal places.
Multiply and divide numbers by 10, 100 and 1000 (giving answers to three decimal places)
Recall and use equivalences between simple fractions, decimals and percentages, with obvious connections e.g. 0.4= $\frac{4}{10}$ = 40%
Suggest a fraction that is greater than one quarter and less than one third.
Identify a number that is halfway between for example: 5 ¼ and 5 ½
To understand that finding one tenth is equivalent to dividing by 10.
To know how many halves in 1 ½, 3 ½, 9 ½, quarters in 1 ¼ , 2 ¾ , 5 ½ , etc
Suggest a decimal fraction between 4.17 and 4.18 (etc)
To know that 33% and 67 % are roughly one third and two thirds.
To match decimals, fractions and percentages.

CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

Year 6 - Progression (a combination of these models and images can be used for every objective)

To compare and order fractions (including fractions >1)

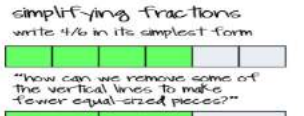
Compare $1\frac{1}{2}$ and $2\frac{7}{8}$ $= \frac{3}{2}$ and $\frac{23}{8}$

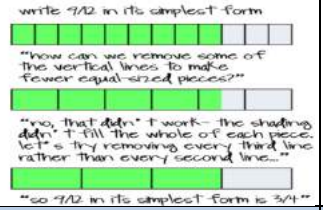


To use common factors to simplify fractions

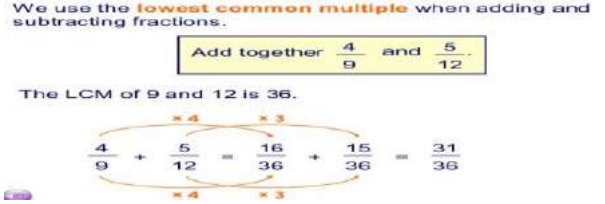
$\frac{4}{6} \overset{\div 2}{=} \frac{2}{3}$ (common factor=2)

$\frac{9}{12} \overset{\div 3}{=} \frac{3}{4}$ (common factor =3)





To use common multiples to express fractions in the same denomination



To add and subtract fractions with different denominators and mixed numbers (using equivalent fractions)

Find the sum of $2\frac{2}{3}$ and $3\frac{5}{9}$

X3

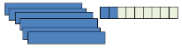
$\frac{2}{3} = \frac{6}{9}$

X3

$2\frac{2}{3} + 3\frac{5}{9} = 2\frac{6}{9} + 3\frac{5}{9}$

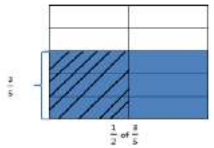
$= 5\frac{11}{9}$

$= 6\frac{2}{9}$



To multiply simple pairs of proper fractions (writing the answer in its simplest form) e.g. $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

The product of proper fractions:



Due to commutative laws the answer to $\frac{3}{5} \times \frac{1}{2}$ would also equal $\frac{3}{10}$

$\frac{3}{4} \times \frac{8}{9} = \frac{3 \times 8}{4 \times 9} = \frac{24}{36} = \frac{2}{3}$

divide both the numerator and the denominator by the common factor (3):

$\frac{3}{4} \times \frac{8}{9} = \frac{3^{1}}{4} \times \frac{8}{9^{3}}$

divide both the numerator and the denominator by the common factor (4):

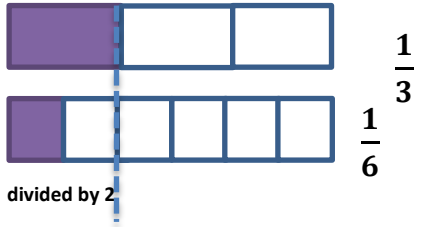
$\frac{3^{1}}{4^{1}} \times \frac{8^{2}}{9^{3}} = \frac{1 \times 2}{1 \times 3} = \frac{2}{3}$

To divide proper fractions by whole numbers. $\frac{1}{3} \div 2 = \frac{1}{6}$

$\frac{1}{3} \div 2 = \frac{1}{6}$

$\frac{1}{3} \div \frac{2}{1}$

$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$



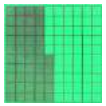
To calculate decimal fraction equivalents (by dividing using a simple fraction)

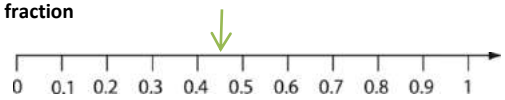
To convert simple fraction to a decimal fraction

$\frac{9}{20}$ as a decimal $= \frac{9 \times 5}{20 \times 5}$

$= \frac{45}{100}$

$= 0.45$



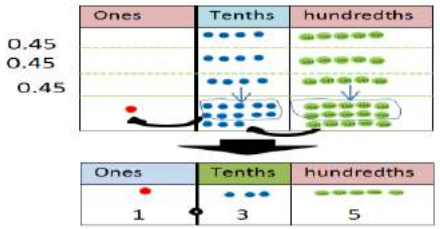


Or using division:

$$20\overline{)9.^{9}0\,^{1}0} = 0.45$$

To multiply one digit & 2 digit decimal numbers with up to two decimal places by whole numbers.

0.45 x 3



Line up decimal points.

$$\begin{array}{r} 7.21 \\ \times\ 2.31 \\ \hline 721 \\ 2163 \\ +\ 1442 \\ \hline 16.6551 \end{array}$$

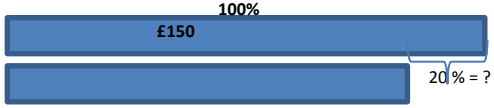
To interpret remainders as whole number remainders, fractions or by rounding, as appropriate for the context.

To use the bar model to help solve word problems (percentage)

The price of a skateboard is normally £150. During the sale it was reduced by 20%. How much was the discount? How much did it cost in the sale?

Method 1:

Usual price



100 % = £150

1 % = $\frac{150}{100}$ = £1.50

20 % = 20 x £1.50 = £ 30 The discount was £30. Therefor £150 - £30 was paid = £120.

Discount = 20% of £150

$= \frac{20}{100}$ x £ 150 = £30 The discount was £30

Chapter 7

Ratio and Proportion

CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

Year 6 – Ratio & proportion

National Curriculum

Pupils should be taught to: solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts
solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison
solve problems involving similar shapes where the scale factor is known or can be found
solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

Notes and guidance (non-statutory)

Pupils recognise proportionality in contexts when the relations between quantities are in the same ratio (for example, similar shapes and recipes).
Pupils link percentages or 360° to calculating angles of pie charts.
Pupils should consolidate their understanding of ratio when comparing quantities, sizes and scale drawings by solving a variety of problems. They might use the notation a:b to record their work.
Pupils solve problems involving unequal quantities, for example, 'for every egg you need three spoonfulls of flour', 'of the class are boys'. These problems are the foundation for later formal approaches to ratio and proportion.

Key vocab: for every, to every, in every, as many as, ratio, proportion

Key concepts

Ratio is a way of comparing the relative sizes of two quantities or sets of items.
A given ratio does not indicate the actual sizes of the quantities involved.
A comparison model can be drawn to represent the two quantities given the ratio.
Ratios are parts of a whole.
Ratios compare two or more parts.
The whole number of parts can be found by adding the ratio.
Ratios can be equivalent.
Ratios can be reduced to lowest terms.
Proportion compares a part to the whole where ratio compares a part to a part.
Proportion uses the language '1 in every 5' where ratio uses '1 to 4' or '1 for every 4'.

Potential barriers

Pupils will often confuse the terms 'ratio' and 'proportion' and need a clear understanding of when each is appropriate.
Fraction and percentage work may be introduced too early, before the pupils have a clear understanding of the meaning of the terms.
Pupils may need to have it highlighted to them that a given ratio may not always indicate the actual size of the quantities involved (i.e. number of items in each set).
Pupils may not know that when comparing quantities the information stated first is the first part of the ratio e.g. when making purple I used red and blue paints at a ratio of 3:2, the red is the 3 and the blue the 2.

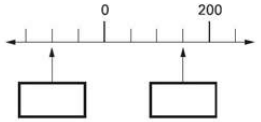
Learning objectives (see overleaf for exemplification)

To use ratio to compare two things
To find equivalent ratios
To compare three quantities using ratios
To follow simple recipes involving basic proportions
To read a simple scale on a map e.g. 1cm = 100cm , 250:1 means 1cm = 2.5m.
To solve problems involving missing values. (using integer multiplication and division facts).
To solve problems involving percentages
To use percentages for comparison
To use the scale factor to solve problems involving shapes
To use knowledge of fractions and multiples to solve problems involving unequal sharing

Example Questions

Two letters have a total weight of 120 grams. One letter weighs twice as much as the other. Write the weight of the heavier letter.

Jenny is going to make some cordial. The finished drink should be 1⁄3 cordial and 2⁄3 water.
Jenny puts 100ml of cordial in a glass. How much water should she put with it?
Two rulers cost eighty pence. How much do three rulers cost?
In a country dance there are 3 boys and 2 girls in every line. 42 boys take part in the dance. How many girls take part?
Here is part of a number line. Write the missing numbers in the boxes.



To make some dough, Gemma mixes 5 cups of flour with every 3 cups of water.

A) Find the ration of the amount of flour used to the amount of water used.

B) If Gemma wants to make 5 times the amount of dough as above, how many cups of water and how many cups of flour does she need?

C) If she used 21 cups of water, how many cups of flour are needed to make the same amount of dough.

Mental Maths

To discuss and complete statements linked to ratio and proportion:
-In every week I spend 5 days at school. In every 2 weeks I spend X days at school and in every 3 weeks I spend Y days at school.
- For every 2 bags of crisps you buy you get one sticker. How many stickers do you get for 6 bags?
- John has 1 stamp for every 2 that Mark has. What other statement s can you make?
Solve simple problems involving 'in every' or 'for every':
- Chicken must be cooked for 50 mins for every kg. How long does it take to cook a 3kg chicken?
- At the gym there are 2 boys for every 3 girls. There are 15 girls at the club. How many boys are there? If there are twelve boys at the club how many girls are there now?
- Zara uses 3 tomatoes for every 1/2 litre of sauce. How much sauce does she make from 15 tomatoes? How many tomatoes does she need for 1 litre of sauce?
- A mother seal is fed 5 fish for every 2 fish given to her baby. Alice fed the seal 15 fish. How many fish did her baby get? Alice fed the baby seal 8 fish. How many fish did its mother get?
- For every 50p coin Mum gives to Dad, he gives her five 10p coins. Dad gave mum twenty-five 10p coins. How many 50p coins did mum give him?

Use multiplicative reasoning to solve simple ratio and proportion questions:
- Kate shares out 12 sweets. She gives Jim 1 sweet for every 3 sweets she takes. How many sweets does Jim get?
-Dee mixes 1 tin of red paint with 2 tins of white. She needs 9 tins altogether. How many tins of red paint does she need?

CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

Year 6 - Progression (a combination of these models and images can be used for every objective)

To use ratio to compare two things

There is 1 chocolate muffin and 2 cherry cakes.

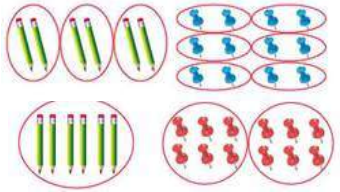
The ratio of chocolate muffins to cherry cakes is 1:2.

Standard eggs

Free range eggs

The ratio of standard eggs to free range eggs is 2:1. One carton has 10 eggs. The ratio does not give the number of eggs!

To find equivalent ratios



The ratio is 6 pencils to 12 pins.

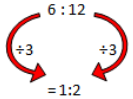
6:12

This can be simplified to 3 groups of pencils to 6 groups of pins

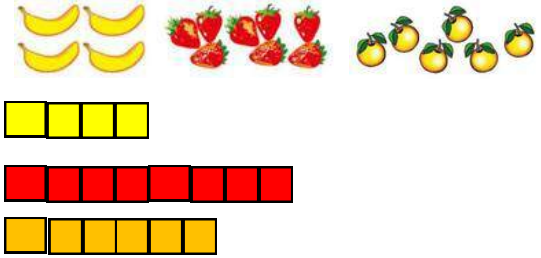
3:6

In its simplified form this would be 1 group of pencils to 2 groups of pins:

1:2 (these are all equivalent ratios)



To compare three quantities using ratios



The ratio of bananas to strawberries to oranges is:

4 : 8 : 6

2 is a common factor: ÷2

2 : 4 : 3 in its simplest form.

To solve word problems involving ratio.

A safari park has 96 big cats altogether. 60 of them are Lions. What is the ratio of the number of Lions to the number of Tigers? 60

Lions

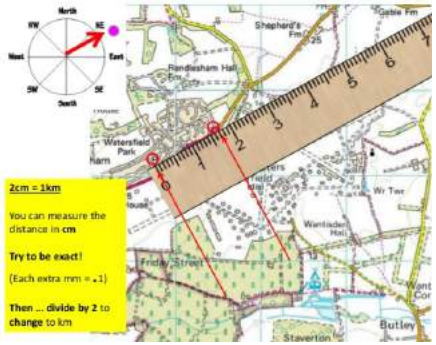
Tigers 96

Number of tigers = 96—60 = 36

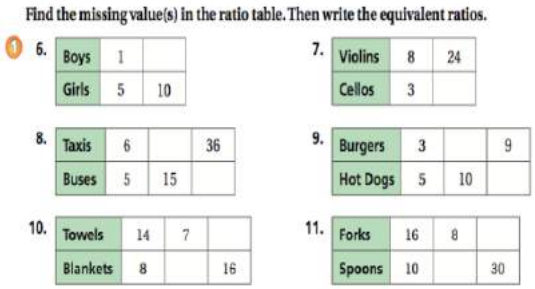
60:36 = common factor 12 = 5:3

The ratio of the number of Lions to the number of tigers is 5:3.

To follow simple recipes involving basic proportions
To read a simple scale on a map e.g. 1cm = 100cm , 250:1 means 1cm = 2.5m.



To solve problems involving missing values. (using integer multiplication and division facts).



Find the missing value(s) in the ratio table. Then write the equivalent ratios.

6.			
Boys	1		
Girls	5	10	

7.			
Violins	8	24	
Cellos	3		

8.			
Taxis	6		36
Buses	5	15	

9.			
Burgers	3		9
Hot Dogs	5	10	

10.			
Towels	14	7	
Blankets	8		16

11.			
Forks	16	8	
Spoons	10		30

To solve problems involving percentages
To use percentages for comparison

When Jonny was 10 years old his mass was 36kg. Two years later, his mass had increased by 30%. Find Jonny's mass when he was 12 years old.

?% ?%

10 years old ? kg

12 years old

100% = 36kg

1% = 36 ÷ 100 = ? Kg

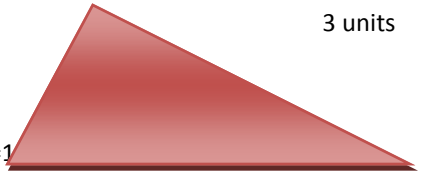
? % = ? X ? = ? Kg

Jimmy's mass when he was 12 years old was ? Kg.

To use the scale factor to solve problems involving shapes

The sides of a triangle are in the ratio 2:3:4. The sum of all the sides of the triangle is 162cm. Find the length of the longest side of the triangle. The sum of the 3 sides = 9 units

2 units 3 units



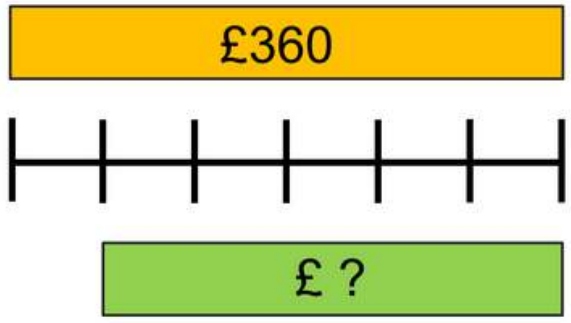
9 units = 162 cm

1 unit = 162 ÷ 9 =1

4 units = 4 x 18 = 72 cm

The length of the longest side = 72cm

To use knowledge of fractions and multiples to solve problems involving unequal sharing



Chapter 8

Algebra

Year 6 – Algebra (When planning ensure you track back to year 5 for progression)

National Curriculum

Use simple formulae
Generate and describe linear number sequences
Express missing number problems algebraically
Find pairs of numbers that satisfy an equation with two unknowns

Notes and guidance (non-statutory)

Pupils should be introduced to the use of symbols and letters to represent variables and unknowns in mathematical situations that they already understand, such as:
missing numbers, lengths, coordinates and angles
formulae in mathematics and science
equivalent expressions (for example, a + b = b + a)
generalisations of number patterns

Key vocab:variable, expression, equations/functions, expanding brackets, coefficient, BIDMAS, rearranging, simplifying, substitution.

Key concepts

Although only mentioned formally in year 6, children are exposed to algebra from a young age through patterns and sequencing.
The use of symbols & letters to represent variables must be introduced in known contexts: lengths, angles etc
Inverse can be used for missing number calculations.
A letter in algebra stands for 'whatever number is chosen', that is, a variable.
When a number and a letter are side by side we multiply.
It is important to reinforce through own modelling and language that the equals sign means 'is the same as'.

Potential barriers/ misconceptions

Pupils do not know the inverse to each operation.
Pupils cannot understand that a number can be represented as a letter or symbol.
Pupils think that a letter and a number together make a higher digit number e.g. 3n is 35 not 3 x 5.
Pupils calculate the value of the same letter differently in one calculation.
Pupils are unable to read coordinates.
Avoid the fruit salad approach to explaining algebraic statements: 3a+5b as 3 apples and 5 bananas– or anything that reinforces the ideas that the letters stand for objects of specific numbers.

Learning objectives (see overleaf for exemplification)

To use simple formula to generate, express and describe:
-Linear number sequences
-Mathematical formula
-Missing number, lengths, coordinates and angles problems
-equivalent expressions (a+b = b + a)

To find pairs of numbers that satisfy and equation with two unknowns
To find all possibilities of combinations of two variables.

Example questions

Find the value of t 33 – 8t = 15
Find the value of u 7 + 4u = 70 – 3u

P and q each stand for whole numbers.
P + q = 1000 p is 150 greater than q. Calculate the numbers p and q.
M stands for a whole number greater than 10 and less than 20. N stands for a whole number greater than 2 and less than 10. What is the smallest number that m x n could be? What is the largest number that m – n could be?
K stands for a whole number. K + 7 is greater than 100. K – 7 is less than 90. Find all numbers that k could be.
When m equals 20, what is the value of ten plus three m?
Here are five number cards 'A' 'A' 'A' 'B' 'B' A and B stand for two different whole numbers. The sum of all the numbers on all five cards is 30. What could be the values of A and B?
6(2x + 4) =
7(-5 + 2) =
$3 + 12 \div (4 - 1) \times 2^2 =$
3 (2x – 5y) + 10 = 5 – 2(4x – 2y)

Mental maths

To express a relationship in symbols to start to use simple formula:
- Use symbols to write a formula for the number of months m in years y.

- Write a formula for the cost of c chews at 4p each.

- write a formula for the nth term of this sequence: 3, 6, 9, 12, 15 …

- The perimeter of a rectangle is 2 x (l+w) Where l is the length and w is the width. What is the perimeter if l=8cm and b=5cm.

- The number of bean sticks needed for a row which is m meters long is 2m + 1. How many bean sticks do you need for a row which is 60 meters long?

Year 6 - Progression (a combination of these models and images can be used for every objective)

To use simple formula to generate, express and describe linear number sequences.

Sometimes, rather than finding the next number in a linear sequence, you want to find the 41st number, or 110th number, say. Writing out 41 or 110 numbers takes a long time, so you can use a general rule.

To find the value of any term in a sequence, use the nth term rule

The rule for this sequence of numbers is **'add 3 each time'.**

1 4 7 10 13 16 ...

The sequence continues in the same way.

Mary says,

'No matter how far you go there will never be a multiple of 3 in the sequence'.

Is she correct?
Circle Yes or No.

Explain how you know.

To use simple formula to generate, express and describe mathematical formula

Only like terms can be added or subtracted.

Collect all the like terms together, eg, re-write the expression

3g + 2k + 5g + 4k - g

with all the g s and all the k s together:

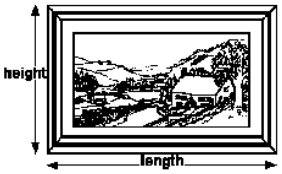
3g + 5g - g + 2k + 4k

When you add or subtract terms, keep each term with their + or - sign.

To use simple formula to generate, express and describe missing number, lengths, coordinates and angles problems

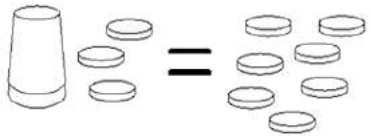
Here are some picture frame sizes.

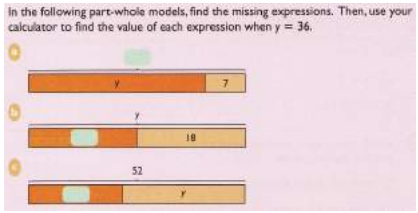
height in cm	10	12	14	16
length in cm	16	20	24	28



For each frame, the length is **twice** the height, **subtract 4.**

What is the **length** of a frame which has a **height** of **36cm**?





To use simple formula to generate, express and describe equivalent expressions (a+b = b + a)

Find the value of ***u*** in this equation.

7 + 4*u* = 70 – 3*u*

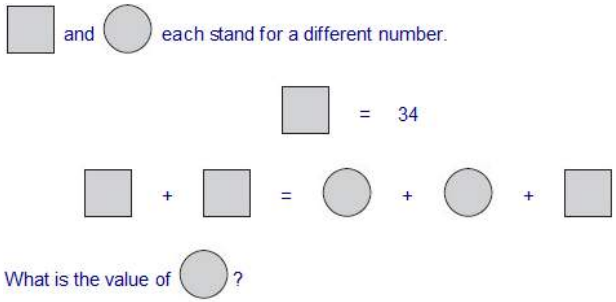
3 x a + 4 = 6

is the same as

3 x a = 6 - 4

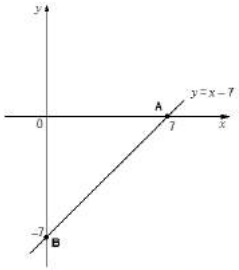
- inverse relations (e.g. 4 × 5 = 20 and 20 ÷ 5 = 4)
- associative law (2 × 3) × 4 = 2 × (3 × 4)
- distributive law 39 × 7 = 30 × 7 + 9 × 7

To find pairs of numbers that satisfy and equation with two unknowns



To find all possibilities of combinations of two variables.

The diagram shows the graph of $y = x - 7$



Write the coordinates of one point on the line **between A and B**.

(,)

Chapter 9

Statistics

CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

It is advised that Y1 teachers also use this strand to get their children 'year 2 statistics ready'

Year 2 - Statistics (When planning ensure you track forwards to year 3)

National Curriculum

Interpret and construct simple pictograms, tally charts, block diagrams and simple tables.
Ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity.
Ask and answer questions about totalling and comparing categorical data.

Notes and guidance (non-statutory)

Pupils record, interpret, collate, organise and compare information (for example, using many-to-one correspondence in pictograms with simple ratios 2, 5, 10)

Key vocab: sort, set, represent, graph, table, list, count, label, most/least common/popular.

Key concepts

Pupils should experience all four stages in handling data: collecting it, organizing it, representing it and interpreting it.
The skills of handling data are best taught through purposeful enquiry across curriculum (science etc)
Motivation is higher when the data is collected by the pupils themselves.
Block graphs can be introduced initially using squares of coloured papers stuck onto large sugar paper.
Pupils should be encouraged to ask and answer questions about data at all stages.

Potential barriers/misconceptions

Pupils find it difficult to select appropriate data to collect.
Pupils do not understand one object can represent many things.
Pupils often confuse the x and y axis and need time to understand what they represent.
Pupils struggle with comparing more than one piece of data.
Handing data only takes place in maths and pupils are unable to link this to other areas of the curriculum

Learning objectives (see overleaf for exemplification)

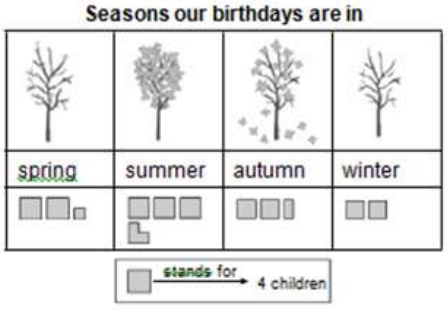
To replace accordingly with pictograms/tally charts/block diagrams/simple tables
To interpret _____
To count the number of objects in each category and sort the categories by quantity,
To compare categorical data
To construct a _____

To make pictograms and graphs where one symbol represents more than one unit.
To read and interpret a simple key
To ask and answer questions about categorical data.
To read the scale on a graph.
To sort objects using more than one criteria (Carroll diagrams)
To sort objects using more than one criteria (Venn diagrams)

Example Questions:

This table shows the ages of some children. Who is the youngest? How many children are older than Harriet?

Name	Age	
Fred	7 years	4 months
Harriet	7 years	0 months
Isla	6 years	10 months
Julian	7 years	6 months
Kate	6 years	11 months
Asim	6 years	11 months



There is an even number of birthdays in 2 seasons. Which seasons are they? How many children have a birthday in the summer?

Mental Maths

To count 'up' a counting stick in intervals of 1, 2, 5, 10

To organise lists: Make a list of all the multiples of 10 between 10 and 100.
Make a list of five different numbers that are more than 70.
Make a list of if all the odd numbers from 15 to 35.

To quickly count up scores when voting takes place.

Respond to questions: How can we find out?
What information shall we collect and how?
How shall we organise it?

To quickly read key information from a graph and respond to questions such as 'do most children walk to school?'.

Test a hypothesis such as: Children in our class are in bed by half past seven.

Year 2 - Progression (a combination of these models and images can be used for every objective)

To interpret/construct pictograms

A way of representing discrete data in which each member is depicted and may represent just one or a number of individuals.

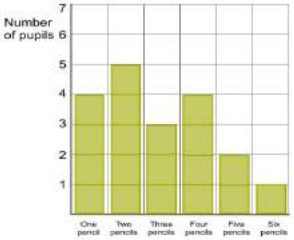
To interpret interpret/construct tally charts

A simple way of counting, making a mark for each item, every fifth makes a group of five.

Vehicles on the road	Number of vehicles
Car	𝍸
Bike	III
Lorry	II
Bus	IIII
Motorbike	II

To interpret interpret/construct block diagrams

An introduction to representing discrete data. Each member is represented by an individual square in columns.

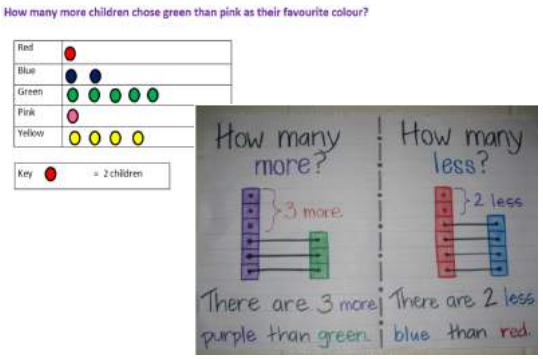


To interpret interpret/construct simple tables

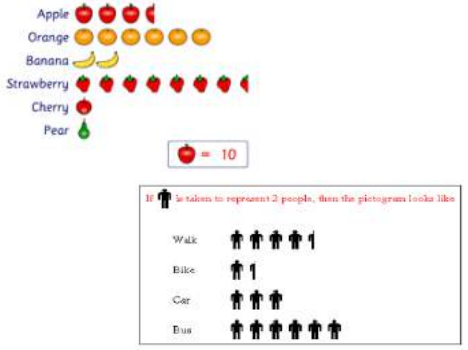
Name of team	Number of wins
Argentina	3
Brazil	5
England	1
France	1
Germany	3
Italy	4
Spain	1
Uruguay	2

To count the number of objects in each category and sort the categories by quantity.

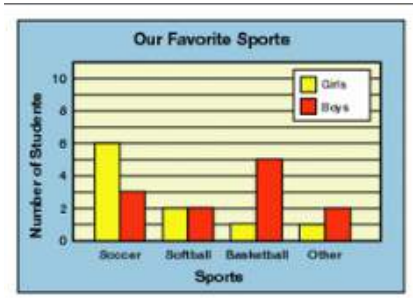
To compare categorical data.



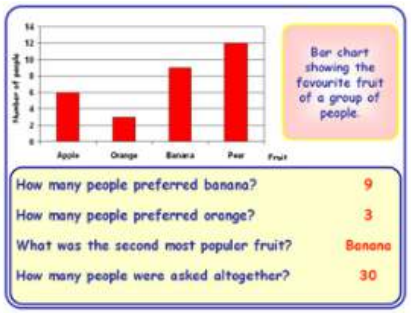
To make pictograms and graphs where one symbol represents more than one unit.



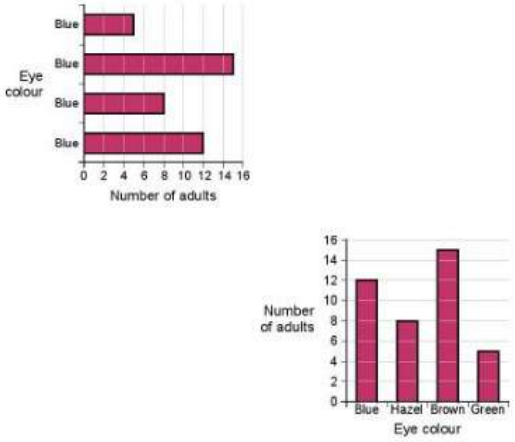
To read and interpret a simple key



To ask and answer questions about categorical data.



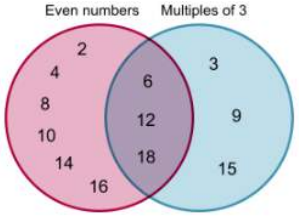
To read the scale on a graph.



To sort objects using more than one criteria (Carroll diagrams)

	multiple of 4	not a multiple of 4
more than 20	24	41
not more than 20	8	1

To sort objects using more than one criteria (Venn diagrams)



CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

Year 3 - **Statistics** (When planning ensure you track back to year 2 and forwards to year 4)

National Curriculum

Interpret and present data using bar charts, pictograms and tables

Solve one-step and two-step questions [for example, 'How many more?' and 'How many fewer?'] using information presented in scaled bar charts and pictograms and tables.

Notes and guidance (non-statutory)

Pupils understand and use simple scales (for example, 2, 5, 10 units per cm) in pictograms and bar charts with increasing accuracy.

They continue to interpret data presented in many contexts.

Key vocab: sort, set, represent, graph, chart, pictogram, diagram, table, list, count, tally, axis, label, title.

Key concepts

Pupils should experience all four stages in handling data: collecting it, organizing it, representing it and interpreting it.

The skills of handling data are best taught through purposeful enquiry across curriculum (science etc)

Motivation is higher when the data is collected by the pupils themselves.

Pupils should be encouraged to ask and answer questions about data at all stages.

When pupils are collecting (ungrouped) discrete data, use variables that have no more than a dozen values.

(i.e. favourite food– first agree with the class a menu of six possibilities to choose from rather than free choice.)

Learning objectives (see overleaf for exemplification)

To interpret and present data using bar charts
To interpret and present data using pictograms
To interpret and present data using tables
To recognise importance of titles and labels when sorting data
To solve one step questions using statistical information.
To solve two step questions using statistical information
To understand and use simple scales
To classify shapes, numbers and objects into a Venn diagram.
To classify shapes, numbers and objects into a Carroll diagram.

Potential barriers/misconceptions

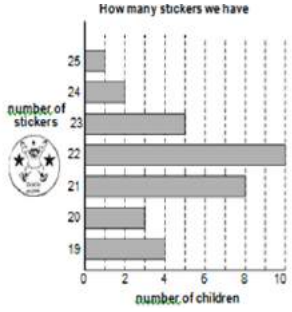
Pupils find it difficult to think of a line of enquiry.
Pupils find it difficult to select appropriate data to collect.
Pupils do not understand one object can represent many things.
Pupils often confuse the x and y axis and need time to understand what they represent.
Pupils struggle with comparing more than one piece of data.
Pupils lack meaningful experience of handling data.

Example Questions

The tally chart shows the number of children in each class. The tally for Class 3 is covered up. Complete the tally for Class 3.

Class	Tally	Total
Class 1	𝍸 𝍸	10
Class 2	𝍸 𝍸 𝍸 𝍸 ||	22
Class 3		13
Class 4	𝍸 𝍸 𝍸 ||	17

How many children have 23 stickers? How many children have fewer than 21 stickers?



Mental Maths

To count 'up' a counting stick in intervals of 1, 2, 5, 10

To count up a counting stick in intervals of any number.

To quickly count up scores when voting takes place.

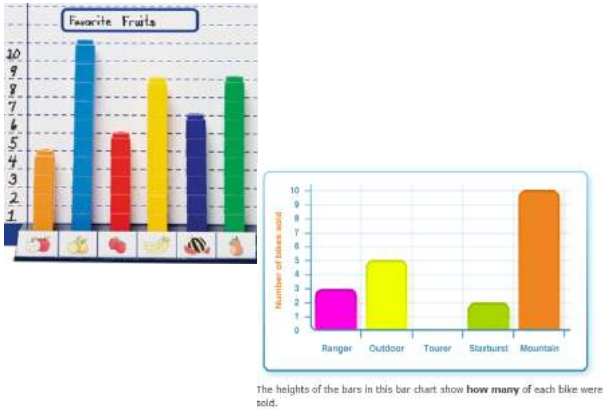
Respond to questions: How can we find out?
What information shall we collect and how?
How shall we organise it?

To hypothesise: How would the graph be different if …
(in relation to travel to school) it were a wet day
December
If there were no buses?
If we asked year six.

CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

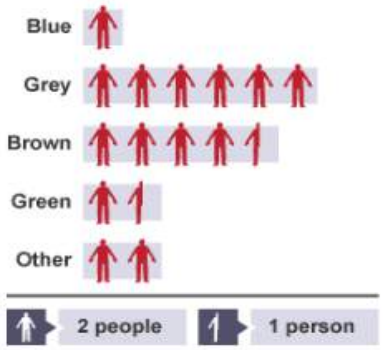
Year 3 - Progression (a combination of these models and images can be used for every objective)

To interpret and present data using bar charts



The heights of the bars in this bar chart show **how many** of each bike were sold.

To interpret and present data using pictograms



To interpret and present data using tables

A table is used to write down a number of pieces of data about different things.

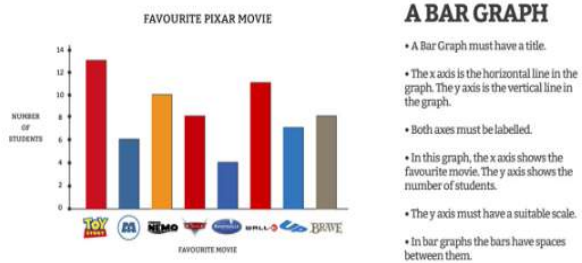
Table example

Name	Colour	Number of gears	Price
Ranger	Silver	5	£140
Outdoor	Blue	10	£195
Tourer	Red	15	£189
Starburst	Silver	15	£215
Mountain	White	5	£129

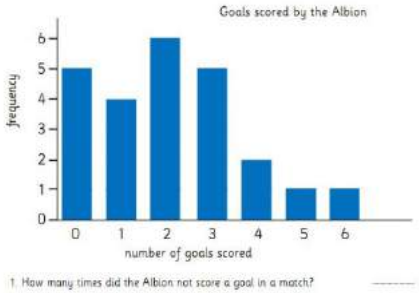
The **title** of the table tells us what the table is about.

The **headings** tell us what data is in each column.

To recognise importance of titles and labels when sorting data



To solve one step questions using statistical information.



1. How many times did the Albion not score a goal in a match? _____

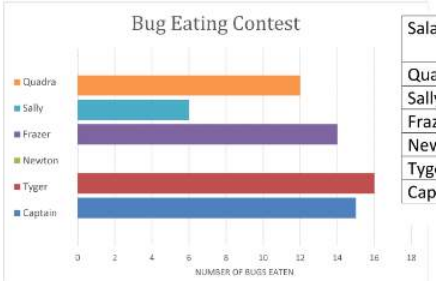
To solve two step questions using statistical information

Here are the heights of some of the world's tallest trees.

Tree	Height (m)
Australian Mountain Ash	100
Coast Douglas Fir	99
Coast Redwood	116
Giant Sequoia	95
Sitka Spruce	97
Tasmanian Blue Gum	91

If you climbed half-way up the Coast Redwood tree, how many more metres would you have to climb before you reached the top? _____ m

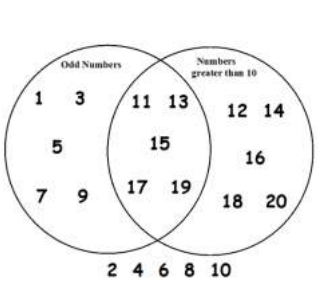
To understand and use simple scales

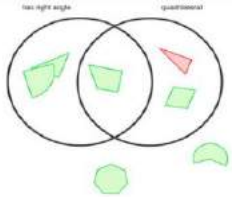


Salamander	Bugs eaten
Quadra	
Sally	6
Frazer	14
Newton	7
Tyger	16
Captain	

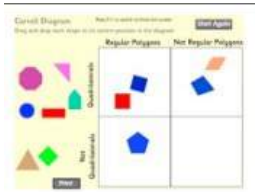
A scale increasing by 2 is appropriate as the highest total is 16.

To classify shapes, numbers and objects into a Venn diagram.





To classify shapes, numbers and objects into a Carroll diagram.



	Odd numbers			Even numbers		
Less than 20	1	3	5	2	4	6
	7	9	11	8	10	12
	13	15	19	14	16	18
Not Less than 20	21	23	25	22	24	26
	27	29	31	28	30	32
	33	35	37	34	36	38

Year 4 - Statistics (When planning ensure you track back to year 3 and forwards to year 5)

National Curriculum

Interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs.

Solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs.

Notes and guidance (non-statutory)

Pupils understand and use a greater range of scales in their representations.

Pupils begin to relate the graphical representation of data to recording change over time.

Key vocab: sort, set, represent, graph, chart, pictogram, diagram, table, list, count, tally, axis, label, title.

Key concepts

Pupils should experience all four stages in handling data: collecting it, organizing it, representing it and interpreting it. The skills of handling data are best taught through purposeful enquiry across curriculum (science etc)

Motivation is higher when the data is collected by the pupils themselves.

When pupils are collecting (ungrouped) discrete data, use variables that have no more than a dozen values.

(i.e. favourite food– first agree with the class a menu of six possibilities to choose from rather than free choice.)

Graphs and tables of data can be collected from newspapers and advertising– are these helpful or misleading?

Potential barriers/ misconceptions

Pupils find it difficult to select appropriate data to collect.

Pupils often confuse the x and y axis and need vocabulary reinforced.

Pupils struggle with comparing more than one piece of data.

Pupils lack meaningful experience of handling data.

Pupils may not label graph accurately (title, axes)

Intervals on the scale are often assumed to be in intervals of 1. Pupils don't see that the scale changes.

When drawing graphs accuracy is important both on the axes/scale etc and the data presented.

Learning objectives (see overleaf for exemplification)

To interpret and present data in a bar chart

To interpret and present data in a time graph

To solve comparison problems using information presented (in a range of tables/graphs).

To solve sum problems using the information presented (in a range of tables/graphs).

To solve finding the difference problems using the information presented (in a range of tables/graphs).

To understand and use a range of scales.

To understand the recording of change over time.

To record change over time in a range of graphs.

To record data into Venn and Carroll diagrams.

Example Questions

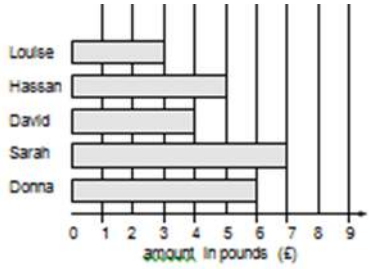
Chris did a survey of the number of people who went into shops in one hour. How many people went into the Supermarket in the hour?

How many more people went into the Post Office than the Shoe shop?

Here is part of a bar chart of the information. Draw in the missing bar.

Number of people who went into a shop
|||| stands for 5 people

Shoe shop	|||| ||||	
Newsagent			
Post Office			||||		|||| || |||| ||
Bread shop	|||| ||||		
Supermarket			|||| |||| ||



Five children collect money to plant trees. Here is a bar chart of the amounts they have raised so far. Their target is £40 altogether. How much more money do they need to reach the target?

Mental Maths

To count 'up' a counting stick in intervals of 2, 3, 5.

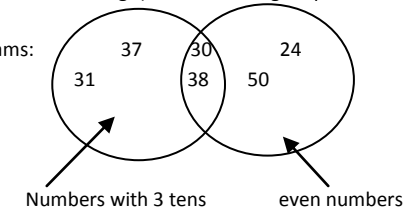
To count up a counting stick in intervals of any number.

To count up a counting stick in decimal intervals 0.5, 1.0, 1.5...

To quickly count up scores when voting takes place.

To interpret data from a pictogram using multiplicative reasoning. (i.e. if each image represents 5 people and there are 4 images then 5x4 = 20 = 20 people)

To sort numbers using rapid recall into venn diagrams:



To sort numbers using rapid recall into carrol diagrams :

	odd		not odd	
Numbers with 3 tens	37	31	38	28
Numbers that do not have 3 tens.	23	25	26	20

CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

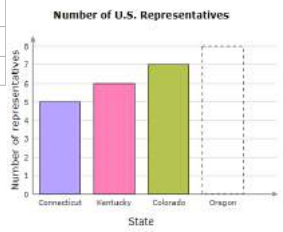
Year 4 - Progression (a combination of these models and images can be used for every objective)

To interpret and present data in a bar chart

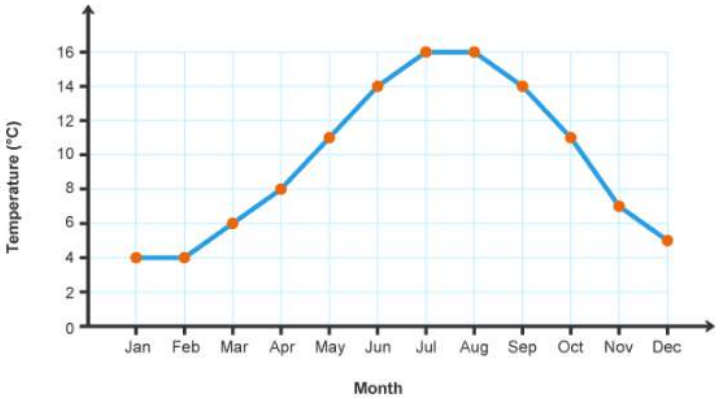
A government class learnt about the number of representatives for each state, based on the 2000 census.

Use the data in the table to complete the bar graph below.

Number of U.S. Representatives	
State	Number of representatives
Connecticut	5
Kentucky	6
Colorado	7
Oregon	5



To interpret and present data in a time graph



To solve comparison problems using information presented (in a range of tables/graphs).

National Team	Nº of Summer Olympics taken part in.	Gold	Silver	Bronze	Total
United States (USA)	25	929	729	638	2296
Soviet Union (URS)	9	395	319	296	1010
Germany (GER)	22	247	284	320	851
Great Britain (GBR)	26	207	255	253	715
France (FRA)	26	191	212	233	636
Italy (ITA)	25	190	157	174	521
China (CHN)	8	163	117	105	385
Hungary (HUN)	24	159	141	159	459
East Germany (GDR)	5	153	129	127	409

Which country has won the most amount of gold medals?

Can you order them in order of no. of Olympics taken part in?

To solve sum problems using the information pre-sented (in a range of tables/graphs).

National Team	Nº of Summer Olympics taken part in.	Gold	Silver	Bronze	Total
United States (USA)	25	929	729	638	
Soviet Union (URS)	9	395	319	296	1010
Germany (GER)	22	247	284	320	851
Great Britain (GBR)	26	207	255	253	
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Hungary (HUN)	24	159	141	159	459
East Germany (GDR)	5	153	129	127	409

Calculate the **total** number of medals for both the USA the UK.

To solve finding the difference problems using the information presented (in a range of tables/graphs).

National Team	Nº of Summer Olympics taken part in.	Gold	Silver	Bronze	Total
United States (USA)	25	929	729	638	2296
Soviet Union (URS)	9	395	319	296	1010
Germany (GER)	22	247	284	320	851
Great Britain (GBR)	26	207	255	253	715
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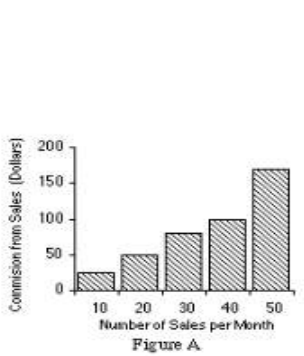
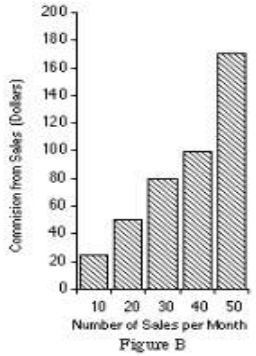
China (CHN)	8	163	117	105	385
Hungary (HUN)	24	159	141	159	459
East Germany (GDR)	5	153	129	127	409

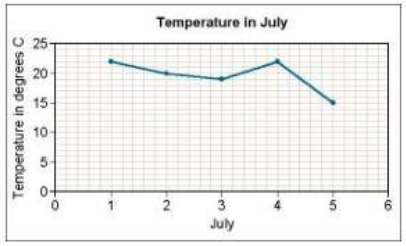
How many more summer Olympics have the USA taken part in than China?

To understand and use a range of scales.

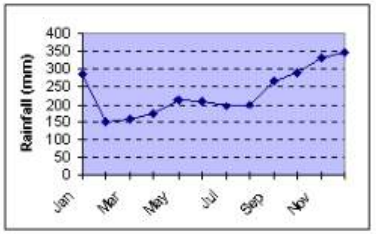


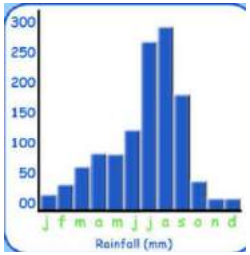


To understand the recording of change over time.

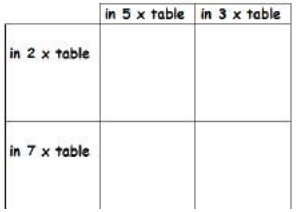


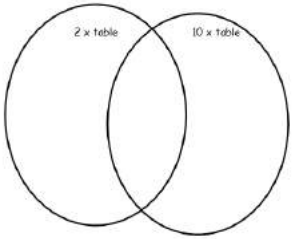
To record change over time in a range of graphs.





To record data into Venn and Carroll diagrams.





CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

Year 5 - **Statistics** (When planning ensure you track back to year 4 and forwards to year 6)

National Curriculum

Solve comparison, sum and difference problems using information presented in a line graph
Complete, read and interpret information in tables, including timetables.

Notes and guidance (non-statutory)

Pupils connect their work on coordinates and scales to their interpretation of time graphs.
They begin to decide which representations of data are most appropriate and why.

Key vocab: sort, set, represent, graph, chart, pictogram, diagram, table, list, count, tally, axis, label, title. classify, maximum/minimum value, range, outcome.

Key concepts

Pupils should experience all four stages in handling data: collecting it, organizing it, representing it and interpreting it. The skills of handling data are best taught through purposeful enquiry across curriculum (science etc)
Motivation is higher when the data is collected by the pupils themselves.
When pupils are collecting (ungrouped) discrete data, use variables that have no more than a dozen values. (i.e. favourite food– first agree with the class a menu of six possibilities to choose from rather than free choice.)
Pupils should be encouraged to collect their own examples of tables, timetables and graphs to analyse.

Potential barriers / misconceptions

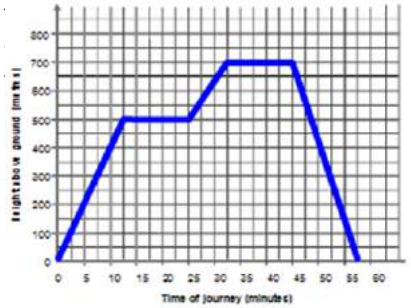
Pupils will not need to learn how to draw pie charts for themselves as these can be generated on the computer. They will need to learn how to interpret them. Barriers will arise if pupils attempt to accurately draw pie charts. Equally pie charts are inaccessible if pupils enter too many variables.

If pupils are collecting data arising from a continuous variable (such as height in cm) get them first to record the measurements to the nearest something (e.g. cm) and then group the results and handle it like a grouped discrete data, but draw the columns in the bar char without gaps.
Pupils don't understand that it is not appropriate to join the tops of the bars when the values in between have no meaning.

Learning objectives (see overleaf for exemplification)

To solve comparison problems using information in a line graph.
To solve sum problems using information in a line graph
To solve difference problems using information in a line graph.
To complete, read and interpret information in tables (including time tables)
To make links with coordinates
To choose the appropriate representations of data.

Example Questions



t what height above the ground was the balloon

oor

Kelly chooses a section of a newspaper. It has 50 words in it. She draws a bar chart of the number of letters in each word.
What fraction of the 50 words have more than 6 letters?
What is the mode for the number of letters used in a word?

Mental Maths

To count up and down a scale in intervals of any number.

Test the hypothesis about the frequency of an event by collecting data quickly: Reading paper, voting, internet…

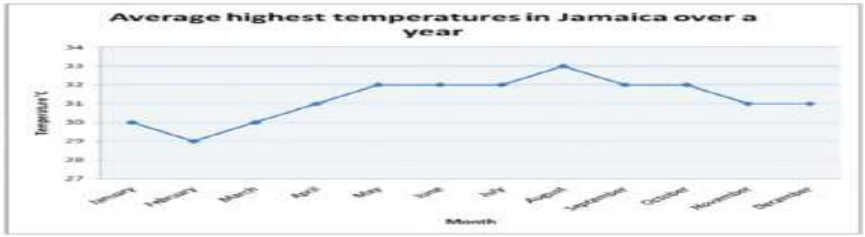
To be able to analyse data from a bar chart and respond rapidly to questions such as:
(in the context of goals scored by a team in the last season):
-How many matches were played in total?
-What was the maximum number of goals they scored in a match?
-In how many matches did they score >3 goals?
- What was the most common number of goals?

To develop an understanding of the mode (most common item)
To develop an understanding of the range (difference between greatest and least).

To discuss questions such as:
-How can we find out if this is true?
- What information shall we collect?
- How shall we organise it?

Year 5 - Progression (a combination of these models and images can be used for every objective)

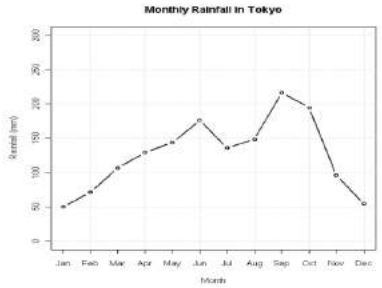
To solve comparison problems using information in a line graph.



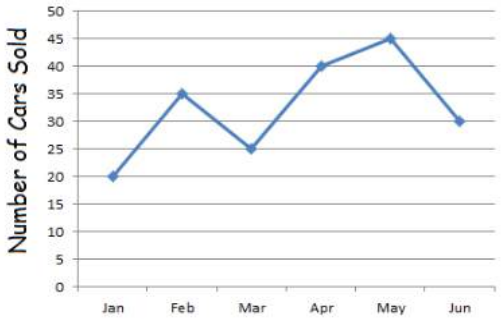
Which months had the highest and lowest temperatures?

To solve sum problems using information in a line graph

What was the total rainfall for January and February?



To solve difference problems using information in a line graph.



How many more cars were sold in May than in January?

To complete, read and interpret information in tables (including time tables)

PROBLEM SOLVING - THE CAT FAMILY (METRIC)

Here are facts about some of the members of the cat family.

Name	Average Weight (kg)	Speed (kph)
Cheetah		121
Lynx	26	47
Lion	190	77
Cougar	73	69
Leopard	67	58
Tiger		

1) Use the information below to complete the information in the table:

- The cheetah is 7kg lighter than a leopard.
- The tiger can run 13kph faster than a lynx.
- The tiger is 37kg heavier than a lion.

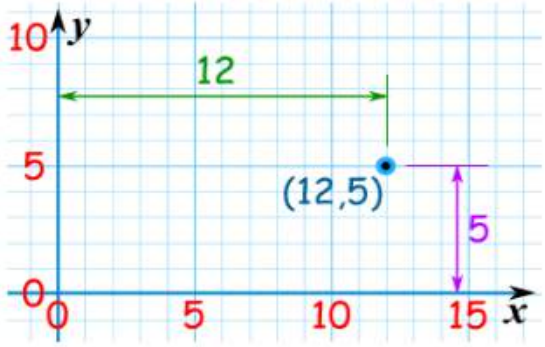
2) Put the animals in order of weight, from lightest to heaviest.

_____ _____ _____ _____ _____ _____

Bus timetables:

	1st	2nd	3rd	4th	5th	6th
Depot	07:30	07:45	08:00	08:15	08:30	08:45
Green St	07:40	07:55	08:10	08:25	08:40	08:55
High St	07:45	08:00	08:15	08:30	08:45	?
Central Park	07:48	08:03	08:18	08:33	08:48	09:03
Railway Station	07:53	08:08	08:23	08:38	?	09:08
Shopping Centre	08:00	08:15	08:30	08:45	09:00	09:15
Brown St	08:08	08:23	08:36	08:51	09:06	09:21
Church St	08:08	08:23	08:38	08:53	09:08	09:23
St Georges School	08:15	08:30	08:45	09:00		
Library	08:20	08:35	08:50	09:05	09:20	09:35
Hospital	08:25	08:40	08:55	09:10	09:25	09:40
Friary Walk	08:33	08:48	09:03	09:18	09:33	09:48
St Marys School	08:42	08:57	09:12			
Forest Rd	08:48	09:03	09:18	09:33	09:48	10:03
Swimming Pool	09:00	?	09:30	09:45	10:00	10:15

To make links with coordinates



To choose the appropriate representations of data.

Bar graphs—suitable for make comparisons among data
Double bar graphs—suitable for comparing two sets of data
Line graphs—suitable for showing change over time
Double line graphs—suitable for comparing two sets of data over time
Histograms—suitable for showing data in equal intervals
Circle graphs—suitable for showing data that are parts of a whole
Pictographs—suitable for showing data that are multiples of a number

Year 6- **Statistics** (When planning ensure you track back to year 5 for progression)

National Curriculum

Interpret and construct pie charts and line graphs and use these to solve problems

Calculate and interpret the mean as an average

Notes and guidance (non-statutory)

Pupils connect their work on angles, fractions and percentages to the interpretation of pie charts.

Pupils both encounter and draw graphs relating two variables, arising from their own enquiry and in other subjects.

They should connect conversion from kilometres to miles in measurement to its graphical representation.

Key vocab: sort, set, represent, graph, chart, pictogram, diagram, table, list, count, tally, axis, label, title. classify, maximum/minimum value, range, outcome, statistics, average, distribution, mode, median, mean.

Key concepts

Pupils should experience all four stages in handling data: collecting it, organizing it, representing it and interpreting it. The skills of handling data are best taught through purposeful enquiry across curriculum (science etc.)

Motivation is higher when the data is collected by the pupils themselves.

Pupils should use line graphs for statistical data only where the variable along the horizontal axis is time. (e.g. midday temp over month, pupils who walk every day etc.).

The average is a representative figure for a set of numbers, enabling us to make comparisons between different

Potential barriers

Pupils will not need to learn how to draw pie charts for themselves as these can be generated on the computer. They will need to learn how to interpret them. Barriers will arise if pupils attempt to accurately draw pie charts. Equally pie charts are inaccessible if pupils enter too many variables.

If pupils are collecting data arising from a continuous variable (such as height in cm) get them first to record the measurements to the nearest something (e.g. cm) and then group the results and handle it like a grouped discrete data, but draw the columns in the bar char without gaps.

Pupils are not always clear that for grouped discrete data the bars may be labelled with the range that they represent but not the divisions between the bars.

Pupils must understand that when drawing conclusions from statistics, such as averages, this can be uncertain or misleading.

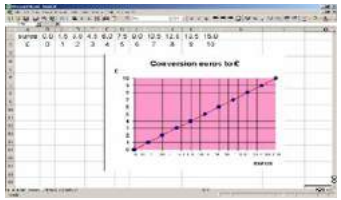
Learning objectives (see overleaf for exemplification)

To interpret line graphs.
To construct line graphs
To solve problems using line graphs.
To interpret pie charts
To construct pie charts (using a computer programme).
To solve problems using pie charts
To connect angles and pie charts
To connect fractions and percentages with pie charts
To calculate and interpret the mean as the average.
To draw graphs relating to two variables.
To connect conversion from km to miles in measurement to its graphical representation.
To choose the appropriate representations of data.

Example Questions

Table and pie chart of favourite sport of Year 6 girls, produced in Microsoft Excel.

Table and conversion graph for euros to pounds, produced in Microsoft Excel:



Mental Maths

To count up and down a scale in intervals of any number.

Test the hypothesis about the frequency of an event by collecting data quickly: Reading paper, voting, internet…

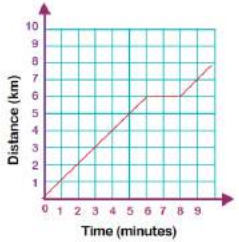
To know the percentage equivalent to common fractions and vice versa (1/4, 1/2, 1/5, 3/4 etc)

To look at a pie chart and answer questions such as:
(in the context of ages of the population of an area)
- What fraction (percentage) of the population is 16 or under? 60 or over?
-Why do you think there are more people aged 16 or under living here than aged 60 or over?

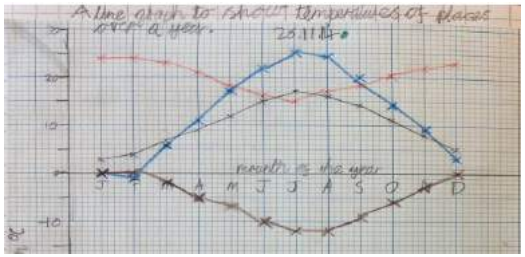
To use mental addition and division skills to find the mean.

Year 6 - Progression (a combination of these models and images can be used for every objective)

To interpret line graphs.



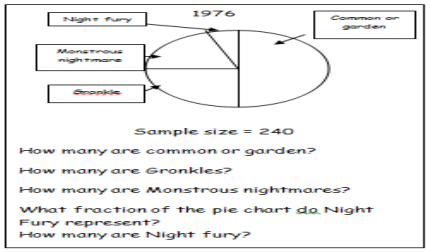
To construct line graphs



To solve problems using line graphs.

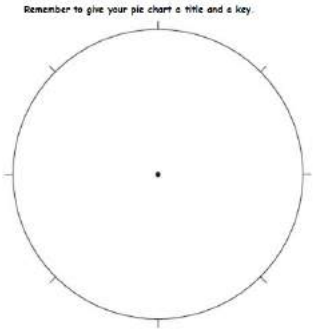
What was the most popular?
What was the median of the data represented?
What was the range of the data?
What was the mean number?

To interpret pie charts

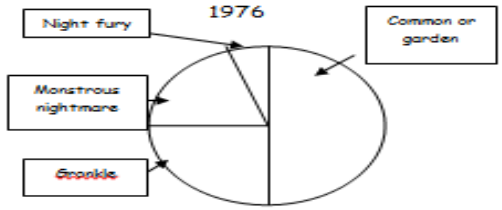


To construct pie charts

If drawing by hand , ensure circle is 'marked' and data accessible to divide up easily.



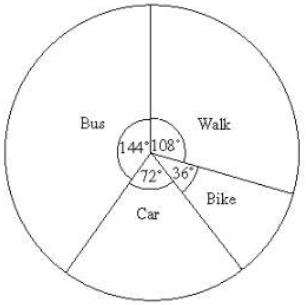
To solve problems using pie charts



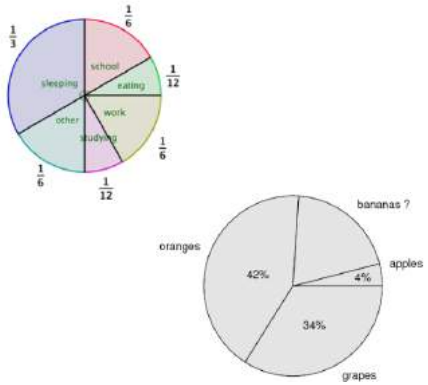
What was the most popular?
What was the median of the data represented?
What was the range of the data?
What was the mean number?

To connect angles and pie charts

To calculate missing angles subtract from 360'

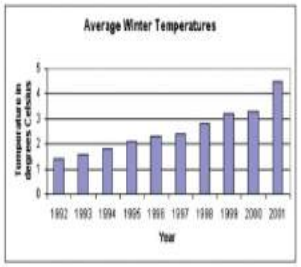


To connect fractions and percentages with pie charts



To calculate and interpret the mean as the average.

The mean value can be useful for comparing things. For example, you can find the goal average for a football team by finding the mean value of the goals scored per match. When you compare the goal averages of two teams you are comparing mean values.

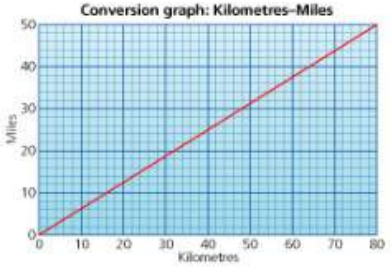


Mean values are also helpful for working out weather patterns, as shown in this graph (left).

If the mean winter temperature increases every year it may indicate that global warming is a serious problem.

To draw graphs relating to two variables.

To connect conversion from km to miles in measurement to its graphical representation.



Can you create your own conversion graph for pounds to US dollars?

To choose the appropriate representations of data.

Bar graphs—suitable for make comparisons among data
Double bar graphs—suitable for comparing two sets of data
Line graphs—suitable for showing change over time
Double line graphs—suitable for comparing two sets of data over time
Histograms—suitable for showing data in equal intervals
Circle graphs—suitable for showing data that are parts of a whole
Pictographs—suitable for showing data that are multiples of a number

Chapter 10

Measurement

EYFS 1 - **Measurement** (When planning ensure you track forwards to EYFS 2)

Early Learning Goal 12
Pupils use everyday language to talk about size, weight (mass), capacity, position, distance, time and money to compare quantities and objects and to solve problems.
They recognise, create and describe patterns.
They explore characteristics of everyday objects and shapes and use mathematical language to describe them.

Key vocab: measure, size, compare, long, short, weigh, heavy, light, full, empty, time, days of the week, before, after, big, little, thin, thick.

Key concepts
We always introduce new aspects of measurement through direct comparison and activities involving ordering.
If two objects are the same length, they stay the same length when moved (conservation of length).
The length of something can be shown in different ways.
We measure the amount of liquid by describing the volume. (full, half full, empty) (more/less)
The tallest container does not necessarily have the largest volume.
There is a difference between night time and day time.
We do different things at night time and day time.
We exchange money when buying items.

Potential barriers/misconceptions
Children may find it difficult reading or counting discrete quantities.
Pupils are not able to measure and count accurately.
Conservation of measure is not yet embedded therefore when an amount of water gets poured from one container to another, for example, pupils will think that the amount has changed.
Children may not have a conceptual understanding of key vocabulary,

Learning objectives (see overleaf for exemplification)
Can say what is different and what is the same.
Begins to categorise (sort) objects according to properties such as size.
Begins to use the language of size.
Shows an interest in shape and space by making arrangements with objects.
Experiments with capacity (which holds more/less).
Anticipates specific time-based events, such as meal times and home time.
Understands some talk about immediate future e.g. 'later' or 'soon'.
Understands some talk about the immediate past e.g. 'before'.
Uses money in role play .
Exchanges money for objects.

Example Questions
I am giving you six strips. Find two strips in your set which are the same length. Show them to me. Now find a strip in your set which is longer than this one.
Find a piece of ribbon which is longer than this one.
Guess first, then check:
- How far up the wall you can reach?
- How far you can throw the bean bag?

Find a bucket which holds more than this one.
Guess first, then check if all the water in the bowl will go into the bucket, or whether there is too much.
How many glasses will fill the jug?
Which do you put on first, your shoes or your socks?
What will we be doing later this afternoon?
At what time of year do the leaves fall off the trees?
What is your bedtime?

Mental Maths
To count in 1ps to 5 pence
To count in 1ps to 10 pence
Count in £1 coins. £1, £2 up to £5
Count in £1 coins up to £10.

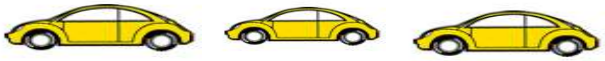
To compare the heights of children at the front of the class= taller, shorter
To discuss capacity by looking at cups of water/milk. Describing as half full, full or empty.

What did you do today?
What will you do tomorrow?
How old are you?

Listen and discuss: The very hungry caterpillar
Mr Wolf's week
Can't you sleep little bear?

Hickory Dickory dock and other nursery rhymes
We go to lunch at 12:00– Twelve O'clock.

EYFS 1 Progression (a combination of these models and images can be used for every objective)

Can say what is different and what is the same.	**Begins to categorise objects according to properties such as size** **Begins to use the language of size..**	**Shows an interest in shape and space by making arrangements with objects.**
Which cars are the same size? How do you know?  Providing cars in the concrete to support: 		
Experiments with capacity	**Anticipates specific time-based events, such as meal times and home time.**	**Understands some talk about immediate future e.g. 'later' or 'soon'.**
Which container holds more? Less? How do you know?   Can you fill it? It is full Can you empty it? It is empty	Using key events as marker points in day. Before/after lunch. 	Sequencing of familiar stories also supports language. 
Understands some talk about the immediate past e.g. 'before'.	**Uses money in role play**	**Exchanges money for objects .**
		

EYFS 2 - **Measurement** (When planning ensure you track forwards to year 1)

Early Learning Goal 12

Pupils use everyday language to talk about size, weight (mass), capacity, position, distance, time and money to compare quantities and objects and to solve problems.
They recognise, create and describe patterns.
They explore characteristics of everyday objects and shapes and use mathematical language to describe them.

KS1 ready

Compare, describe and solve practical problems for time [for example, quicker, slower, earlier, later].
Measure and begin to record time (hours, minutes, seconds).
Sequence events in chronological order using language [for example, before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening].
Recognise and use language relating to dates, including days of the week, weeks, months and years.
Tell the time to the hour and half past the hour and draw the hands on a clock face to show these times.

Key vocab: measure, size, compare, long, short, weigh, heavy, light, full, empty, time, days of the week, before, after, big, little, thin, thick.

Key concepts

We always introduce new aspects of measurement through direct comparison and activities involving ordering.
We use scales to measure things. Mass can be measured in non standard units. Mass can be described in units.
If two lumps of dough with the same mass are rearranged (moulded into lumps)- they will still have the same mass (conservation of mass). .
There is a difference in length between objects. We can use different things to measure the length of an item.
The number of units tells us how long an item is. The length of something can be shown in different ways.
We measure the amount of liquid by describing the volume.
The tallest container does not necessarily have the largest volume.
There is a difference between night time and day time.
Time measures the duration it takes to do something.

Potential barriers/ misconceptions

Children may find it difficult reading or counting discrete quantities.
Ensure that children are 'accurate' when using scales and measuring implements.
Pupils are not able to measure and count accurately.
Pupils do not understand the mathematical language e.g. confuse volume and capacity.
Pupils struggle with the practical reading of measurements e.g. do not hold container level.
Pupils do not understand the measurements between marked divisions on a scale.
Pupils confuse the minute and hour hands. Pupils cannot grasp that digital clocks and timers have a number scale based on 60 not 100. Pupils do not understand the vocabulary related to time.
Pupils struggle to understand a new scale out of context.

Learning objectives (see overleaf for exemplification)

Uses every language to compare quantities and objects.
Uses everyday language to solve problems.
Orders two items by mass (using everyday language)
Orders two or three items by length or height.
Uses everyday language to compare quantities and objects and talk about distance.
Orders two items by capacity (using everyday language).
Orders and sequences familiar events.
Uses everyday language related to time (days of week & begins to identify o'clock).
Measure short periods of time in different ways.
Uses everyday language to talk about money.
Demonstrates understanding that £1 has greater value than pennies.
Know and name different coins – 1p, 2p, 5p, 10p, 20p, 50p, £1 and £2.
Can use 1p, 5p and 10p coins to make amounts up to 20p.

Example Questions

How many cubes will balance the parcel on the scales?
I'll put a box on one side of the balance (scale). Find a box which is heavier than this one. Now find a box which is lighter than this one.
Guess first, then check: how far up the tree you can reach? How far you can throw the ball?
Find me three objects that are longer/shorter than this pencil.
How many jumbo bricks do you need to make a tower that is as tall as you?
Find a bucket which holds more than this one.
Guess first, then check if all the water in the bowl will go into the bucket, or whether there is too much.
How many glasses will fill the jug?
These cards tell a story of how some children built a snowman. Put the cards in order.
Find and show me the card that shows Sarah eating her school lunch.
Find me a card which shows what Sarah does before/after school lunch.
Look at these pictures. Point to a picture which shows something that you think happened in the morning.
Point to a picture which shows something that you think happened in the afternoon.
Point to a picture which shows something that you think happened in the evening?

Mental Maths

To count in pennies – 1p, 2p, 3p etc.
To count in 1p's to 5 pence then to 10 pence
To count in £1 coins. £1, £2 up to £5
To count in £1 coins up to £10.
To count in 2p's to 10 then to 20.
To 'count on'. Jim had £5 he spent £3 how much did he have left. £3 in our heads, count on '4,5' He has £2 left.
To count how much altogether: is 1p and 1p and 1p and 1p? 5p and 5p?
To count up how much altogether: Sunil spent 1p and 2p on sweets. How much is this altogether?
Chews cost 2p each. How much would 2 chews cost? 2p, 4p
To add £1s. (in the pound shop I buy… 2 tins of paint, 2 brushes and 1 magazine– how much does it cost?)
To calculate how to pay (an apple costs 6p which 2 coins could I use?).
To estimate: How far you can throw beanbag? How far you can jump? Which box is heavier? How full the bottle will be if I pour in the water from the jug?
To recall the days of the week in order: Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday, Monday.
To know what day it is tomorrow. What day it was yesterday.
To know key times in the day: 12:00 lunch time, 3:30 home time, 8:00 bed time etc

EYFS 2 - Progression (a combination of these models and images can be used for every objective)

Uses every day language to compare quantities and objects.

Uses everyday language to solve problems.

Let's guess (estimate)
Let's measure
Let's compare

Orders two items by mass (using everyday language)

The yellow bag is heavier than the blue bag.

The blue bag is lighter than the yellow bag.

Orders two or three items by length or height.

Children must first compare two objects: shorter / longer.
Then three objects: Long, longer, longest

Uses everyday language to compare quantities and objects and talk about distance.

Orders items by capacity (using everyday language)

Full, half full, empty

More , less .

Orders and sequences familiar events.

Uses everyday language related to time (days of week & begins to identify o'clock).

Measure short periods of time in different ways.

How many marbles can you get in your pot in this time?
How quickly can you collect 20 marbles?

Demonstrates understanding that £1 has greater value than pennies .

Which coin would you rather have? Why?

Know and name different coins – 1p, 2p, 5p, 10p, 20p, 50p, £1 and £2.

Can use 1p, 5p and 10p coins to make amounts up to 20p.

Year 1 - **Measurement** (When planning ensure you track back to Reception and forwards to year 2)

National Curriculum

Compare, describe and solve practical problems for:
lengths and heights [for example, long/short, longer/shorter, tall/short, double/half]
mass/weight [for example, heavy/light, heavier than, lighter than]
capacity and volume [for example, full/empty, more than, less than, half, half full, quarter]
time [for example, quicker, slower, earlier, later]
measure and begin to record the following:

Lengths and heights, mass/weight, capacity and volume, time (hours, minutes, seconds)
Recognise and know the value of different denominations of coins and notes
Sequence events in chronological order using language [for example, before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening]
Recognise and use language relating to dates, including days of the week, weeks, months and years
Tell the time to the hour and half past the hour and draw the hands on a clock face to show these times

Notes and guidance (non-statutory)

The pairs of terms: mass and weight, volume and capacity, are used interchangeably at this stage.

Pupils move from using and comparing different types of quantities and measures using non-standard units, including discrete (for example, counting) and continuous (for example, liquid) measurement, to using manageable common standard units.

In order to become familiar with standard measures, pupils begin to use measuring tools such as a ruler, weighing scales and containers.

Pupils use the language of time, including telling the time throughout the day, first using o'clock and then half past.

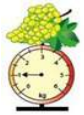
Key vocab: measure, size, compare, estimate, nearly, roughly, days of the week, seasons, O'clock, half past.

Key concepts

We always introduce new aspects of measurement through direct comparison and activities involving ordering.
We compare masses using a balancing scale. Mass can be measured in non standard units. Mass can be described in units. The size of the object does not always determine it's mass. i.e it could be large and light.
To measure length we use a start line. Time is a form of measurement that indicates a duration of time.
The capacity of a container is the amount of space it can hold. The volume of a container is the amount of space it contains.

Potential barriers

Pupils may find it difficult relating standard to non-standard units.
Pupils lack the language required for the comparison of measurements (long, longer, longest)
Pupils believe the bigger the object the greater its mass. Pupils confuse the minute and hour hands.
Pupils associate coin size directly with value.
When liquid is transferred from one container to another sized container pupils think the capacity has changed.



Learning objectives (see overleaf for exemplification)

To compare and order length
To measure using a start line
To measure in non standard units
To compare and order mass
To weigh mass in non standard units
To compare and order capacity and volume
To sequence events in chronological order.
To tell the time to the hour
To tell the time to the half hour
To recognise and know the value of different coins and notes
To exchange money
To solve problems involving money (making amounts in different ways).

Example Questions

How much do the grapes weigh?
Use the balance to find out which of these three boxes is heaviest, lightest, and which is in between.
5 children use cubes to balance one of their shoes. This table shows the number of cubes they need.

	Cubes
Roma	16
Tina	13
Gareth	18
Ali	20
Susan	15

Whose shoe is heaviest? Whose shoe is two cubes lighter than Gareth's shoe?
Draw a line 12 centimetres long. Use a ruler.
Tick the side of the shape which is 7centimetres long. Use a ruler.
Find two strips which, put together, are the same length as your longest strip.
Put these in order of duration (length): week, month, year, day, hour.
One clock shows a time between 3 o'clock and 5 o'clock in the same afternoon. Tick it.
Draw a tick on the clock which shows half past three. Show as clock faces: 3:00, 4:00, 3:30, 6:15, 2:00, 4:30
Tom's school starts at 9 o'clock. Tom went to the doctors in the morning and got to school half an hour late. Draw the time Tom got to school on the clock below. (Provide blank clock face).
January is the first month in the year. What is the fourth month in the year?
I am going to ask each of you in turn to find a coin for me in this container. Put your coin back in the container after your turn. Child A: a 5p, coin Child B: a 20p coin, Child C: a 10p coin, Child D: a 2p coin
Amy has these coins in her purse. How much is in Amy's purse?
Amy spends 10p. How much does she have left?

Mental Maths

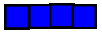
Recognise and use language relating to dates including :
days of the week, weeks, months and years, in context and by recall.
To justify statements: I can pay for anything from 1p to 5p if I have two 2p and one 1p coins. (3p=1p&2p etc)
In context of classroom shop use mental strategies to:
Find totals and give change: How much altogether is 5p+2p+1p? Chews cost 5p each. How much do 3 chews cost?
Rosie spent 5p and 3p. How much change from 10p does she get. (count on from 8…)
To solve problems around what to buy and how to pay:
Apples are 6p each. What do two apples cost? Which two coins could pay exactly?
Describe different ways of paying 7p exactly. 13p?
To use mental strategies to solve measurement problems in classroom contexts:
The classroom is 15m long. The library is 12m long. How much longer is the classroom?
On the scales 8 bricks balance an apple. 4 bricks balance a pear. How many bricks balance both the apple and the pear?
A full jug holds 6 cups of water. How many cups of water do two jugs hold?
How long is it from 2 o'clock to 6 o'clock?
It is seven o'clock. What time was it 2 hours ago?
To suggest a unit you would use measure: the height of a table, the weight of a parcel, across the classroom.
To know that: 1 week= 7 days 1 day = 24 hours
To know in order the days of the week. (To identify what day it is today, yesterday, tomorrow, two days ago).
To know what time it will be in one hour. (i.e. it is 3 O'Clock now. What time will it be in one hour?)

Year 1- Progression (a combination of these models and images can be used for every objective)

To compare and order length

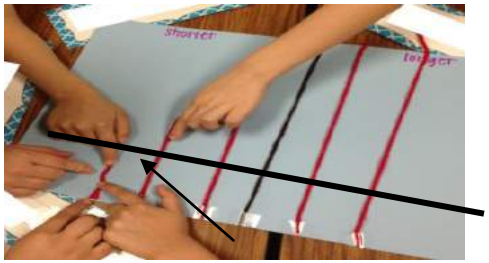
Can you make a taller tower?
Can you make a shorter tower?

Can you make a longer train?
Can you make a shorter train?



Short can be used to describe height and length.
Long is for length and tall for height.

To measure using a start line



Children line up all objects with the start line.

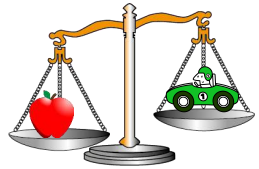
To measure in non standard units

Unifix blocks
1 unifix= 1 unit
The dinosaur is 7 units long.
OR
Measure in paperclips… 1 clip = 1 unit

Investigation can be carried out with any non standard unit as long as object used to measure is the same. (either blocks or clips)

To compare and order mass

Use balancing scales to measure.



The apple is heavier than the car.
The car is lighter than the apple.
What object is as heavy as the apple?

To weigh mass in non standard units

1 bear = 1 unit

The mass of the car is 7 units.

To compare and order capacity and volume

The 'amount of water' in the container is the 'volume of water'.

The volume of water remains the same when it is poured into different containers.

What can you say about the volume of water in these jars?

How could you check?

To sequence events in chronological order.

To tell the time to the hour

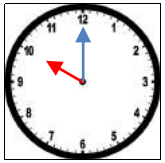
Hour hand
Minute hand

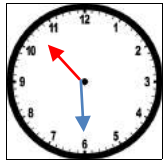
When the minute hand is at 12 we say it is o'clock.

Children should show times on model clocks with moving hands.

To tell the time to the half hour

When the minute time is at 6:00, we say it is half past.





Notice the hour hand has moved too.

Can you make these times on your clock?
Tom feeds his cat at 5:30.
Jeff gets on the bus at 9:30.

To recognise and know the value of different coins and notes

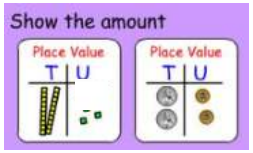
Can you put these coins and notes in order from smallest to largest? (size then value)

What could you buy with each coin?
What could you buy with each note?

To exchange money

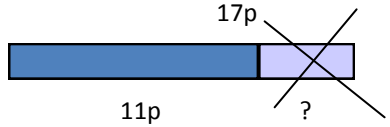
How many ways can you make £1?

Using 10p coins and 1p coins can you find the equivalent in diennes?



How many 1p coins would you exchange for 10p?
How many 10p coins would you exchange for £1.00
How many 1p coins would you exchange for £1.00?

To solve problems involving money



Year 2 - **Measurement** (When planning ensure you track back to year 1 and forwards to year 3)

National Curriculum

Choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels.
Compare and order lengths, mass, volume/capacity and record the results using >, < and =
Recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value

- find different combinations of coins that equal the same amounts of money
- solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change.
- compare and sequence intervals of time
- tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times. Know the number of minutes in an hour and the number of hours in a day

Notes and guidance (non-statutory)

Pupils use standard units of measurement with increasing accuracy, using their knowledge of the number system. They use the appropriate language and record using standard abbreviations.

Comparing measures includes simple multiples such as 'half as high'; 'twice as wide'.

Pupils become fluent in telling the time on analogue clocks and recording it.

They become fluent in counting and recognising coins. They read and say amounts of money confidently and use the symbols £ and p accurately, recording pounds and pence separately.

Key vocab: measure, size, compare, estimate, nearly, roughly, seasons, months, days of the week.

Key concepts

We always introduce new aspects of measurement through direct comparison and activities involving ordering.
The four operations can be used to solve problems involving measures.
Length is a concept of measure to determine how short or long an object is.
The measures m and cm are units of measure for length. The Kg and g are units of measure for mass.
An object can be heavier or lighter than another based on the masses of the two objects.
The decimal point separates the pounds from the pence.

Potential barriers

Pupils do not use the correct vocabulary for time intervals.
Pupils do not read the time intervals as jumps of five.
Pupils to not grasp that the clock is a circular number line.
Pupils do not understand how to find time intervals by using a number line.
Pupils cannot make the link between 100p and £1.
Pupils do not know how to calculate intervals on a scale.

Learning objectives (see overleaf for exemplification)

To measure and compare lengths and heights in metres (> < =) .
To measure and compare lengths and heights in centimetres.
To solve length problems using the four operations .
To measure and compare masses in kilograms (> < =) .
To measure and compare masses in grams (> < =) .
To solve mass problems using the four operations.
To measure and compare temperature (> < =).
To measure and compare volume (> < =) .
To recognise and use coins and notes and compare amounts.
To select different combinations of coins to make a particular value.
To calculate giving change up to and including £1.00.
To exchange pence for pounds.
To compare and sequence intervals of time.
To tell and write the time to quarter past/to and five minutes.

Example Questions

How much does the watermelon weight?

How heavy is Peter?

Measure two lines. How much longer is line A than line B?
Sita says, 'On my 3rd birthday, I was 95cm tall. Now I am 28cm taller'. How tall is Sita now?
Finish the sentence (kilograms, litres, metres, hours): *I can measure the length of the classroom in….*
Three sticks fit along one side of this book. Estimate how many sticks fit around all four sides of the book.
Look at this clock. (show clock face at 2:30). What time will the clock show two hours later? Show as clock faces: 6:00, 4:30, 8:30, 11:30.
Selma's watch shows this time (show as digital watch): 3:30. Henry's watch shows the same time. Draw the hands on his watch (provide blank analogue clock face).
How many months are there in one year? A week has 7 days. How many weeks are there in 35 days?
Emily leaves at nine fifteen. It takes half an hour to get to the shop. What time does Emily get to the shop?
Tom leaves school at (show as analogue): 2:00 and gets home at (show as analogue): 2:20. How long does he take to get home?

How much does the bottle hold? (2 centimetres, 2 kilograms, 2 litres, 2 metres, 2 grams)

Desi and Ella share this money equally. How much do they each get?

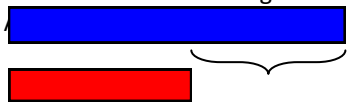
Mental Maths

To know the number of minutes in an hour & the number of hours in a day.
To know the order of months and seasons of the year. (reciting months and knowing 'last month was, next month is'
To partition money. (To know that £1.45 is the same as £1 and 45p)
To know the place value of money. (How many pence is £1.50? Write 125p in £ and p)
To find totals and give change: I have £14. I am given another £9. How much do I have now?
-A pear costs 15p more than an apple. An apple costs 12p. How much is the pear?
To know what to buy and how to pay: Investigate ways of paying 50p using only silver coins.
-Ruth has two coins of the same value. How much might she have altogether?
-You have three 10p and three 5p coins. You use two coins. What might the lolly cost? What if you used three coins?
To solve measurement problems: My cat is 30 cm tall. My dog is 25 cm taller. How tall is my dog?
-You have 50litres of water. How many 10l buckets can you fill?
To calculate time duration: Sue got on the bus at 9 o'clock. The journey took half an hour. What time did she arrive?
-James walked from 9:45 until 10:15. For how many minutes did he walk?
To know that: 1m= 100cm, 1kg=1000g and 1litre = 1000ml
To make direct comparisons: identifying objects that are more/less than 1m, 1cm, 10 cm, ><1kg, holding ><1 litre.
To suggest what could be measured using: metres, centimetres, kilograms, grams, litres...

Year 2 - Progression (a combination of these models and images can be used for every objective)

To compare lengths and heights in metres (> < =)

A blue ribbon is 9m long



?

Which ribbon is longer?
How much longer? 9m-6m = 3m
9m > 6m

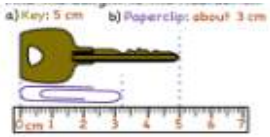
What if the objects are too big to put side by side?

To compare lengths and heights in centimetres.

Measure with a ruler starting at 0

Use string to measure curved lines.

Compare two lengths:



What is the difference?

To solve length problems using the four operations .

Izabela has 4 strips of paper. Each one is 3cm long. She pastes them alongside each other. How long is the strip now?

3cm
4 x 3 = 12cm
Also problems involving:
Addition/subtraction/division

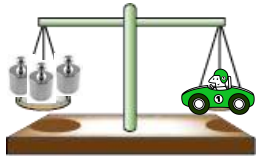
To compare masses in kilograms (> < =)

This big bunch of grapes is heavy as a mass of 1kg.

The mass of the s
More than 1kg.

Can you find objects that are
> 1kg =1kg and <1kg

To compare masses in grams (> < =)

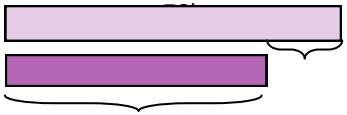


The car has a mass of 30g.
30g<1kg. What else is lighter than 1kg?

Can you measure out 350g for this recipe?

To solve mass problems using the four operations

Tommy's mass is 78kg. Jessica is 12kg lighter than Tommy.
What is Jessica's mass?



12kg

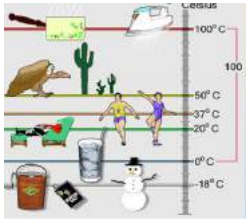
?kg

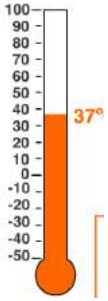
78—12 = 66 Jessica's mass is 66kg

To measure and compare temperature (> < =)

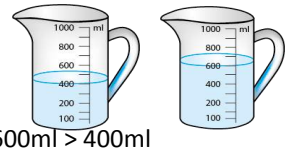
Human body = 37'C

Other common temps:





To measure and compare volume (> < =)



600ml > 400ml

Ordering capacity of containers:

To recognise and use coins and notes and compare amounts.

62p > 56p

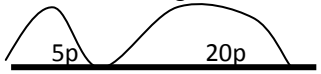
To select different combinations of coins to make the same amount of money.

Use number bonds to 10p.
How many ways?

£1 (using number bonds to make one pound, £10 etc)

To calculate giving change

Using the shopkeeper method to count on.
An orange costs 75p. I give the shopkeeper £1.
He will then 'count on' in order to calculate change.



5p + 20p = 25p change

To exchange pence for pounds.

Fair swap/ exchange:

100 pennies = £1:00

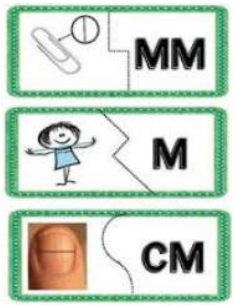
Count to 100 in collected 1ps.

To compare and sequence intervals of time.

To tell and write the time to five minutes.

Count around the clock face in 5s.

To identify common measures



Year 3 - **Measurement** (When planning ensure you track back to year 2 and forwards to year 4)

National Curriculum

Pupils should be taught to:

Measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml).

Measure the perimeter of simple 2-D shapes.

Add and subtract amounts of money to give change, using both £ and p in practical contexts.

Tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks.

Estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes and hours; use vocabulary such as o'clock, am/pm, morning, afternoon, noon and midnight.

Know the number of seconds in a minute and the number of days in each month, year and leap year.

Compare durations of events [for example, to calculate the time taken by particular events or tasks].

Notes and guidance (non-statutory)

Pupils continue to measure using the appropriate tools and units, progressing to using a wider range of measures, including comparing and using mixed units (for example, 1 kg and 200g) and simple equivalents of mixed units (for example, 5m = 500cm).

The comparison of measures includes simple scaling by integers (for example, a given quantity or measure is twice as long or 5 times as high) and this connects to multiplication.

Pupils continue to become fluent in recognising the value of coins, by adding and subtracting amounts, including mixed units, and giving change using manageable amounts. They record £ and p separately. The decimal recording of money is introduced formally in year 4.

Pupils use both analogue and digital 12-hour clocks and record their times. In this way they become fluent in and prepared for using digital 24-hour clocks in year 4.

Key vocab: o'clock, am/pm, morning, afternoon, noon and midnight, measuring scale, estimate, division.

Key concepts

We always introduce new aspects of measurement through direct comparison and activities involving ordering – this will consolidate conceptual understanding.

We can convert units of measure using multiplicative reasoning.

Measures from length to money is an excellent way to introduce and consolidate decimal notation.

Potential barriers

Pupils are unclear that inter-scale means to 'scale up' or 'scale down' i.e. inter scale by 5– get 5 times bigger.

Some pupils will struggle to compare using mixed units. i.e. compare 2kg and 750g.

Pupils struggle with counting using the shopkeeper method e.g. Change I would get from £10 if I spent £8.50. they may write £2.50 instead.

Recording of money is done separately as £3 and 25p. Pupils will not yet be confident understanding the decimal place and need to consolidate the value of both pounds and pence before recording as £3.50

Pupils do not understand the relationship between the digital and analogue display.

Pupils to not grasp that the clock is a circular number line.

In order to calculate intervals of time pupils will need to be secure in their understanding of the number line.

Learning objectives (see overleaf for exemplification)

To measure and compare lengths in m, cm and mm.

To use multiplication and division to scale by integers.

To measure and compare mass in Kg and g.

To measure and compare volume in l/ml.

To solve measurement problems using both addition and subtraction.

To convert between different units of measure.

To measure and calculate perimeter of 2D shapes.

To calculate change given in both £ and p.

Tell and write the time from an analogue clock (standard clock and with Roman numerals).

To match digital and analogue clocks.

To read and record time to the nearest minute.

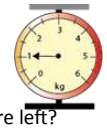
To compare time in seconds, minutes and hours.

To convert hours and minutes.

To calculate and compare duration of events.

Example Questions

The scale shows the weight of a kitten. How much does the kitten weigh?



Emily is making a cake. She has 450g of flour on the scale. She added sugar and then the scale shows 600g. How much sugar did she add?

The height of the door is most likely to be: 1 metre, 2 metres, 5 metres, 10 metres, 100 metres.

Kate has a piece of ribbon one metre long. She cuts 30 cm off. How many centimetres of ribbon are left?

This was the time on Samira's watch when she set off for a walk (Show on analogue clock with no numbers): 10:15. What time did the watch show 20 minutes before this? What time did it show an hour and a half after?

Tom does English and Maths homework each week. It takes him a total of two and a half hours. He spends 80 minutes doing English homework. How many minutes does he spend doing Maths homework?

A week has 7 days. How many weeks are there in 35 days?

My watch shows three-fifty PM. What time will it show in thirty minutes?

Katie's glass holds a quarter of a litre when it is full. She nearly fills it to the top with juice. Tick the approximate amount of juice she puts in the glass (4 millilitres, 20 mililitres, 120 millilitres, 220 millilitres, 420 millilitres).

This jug has water in it. Ravi pours 150 millilitres out of the jug. How much water will be left in the jug?

Lewis makes a call from a telephone box. He has £2 in coins. He uses these five coins to make the call.

How much money has he got left from the £2?

Mental Maths

To know the number of seconds in a minute.

To know the number of days in each months.

To know the number of days in a year and leap year. (365 days, 52 weeks or 12 months)

To use decimal notation for money. (How many pence is £9.05? What is 465p in £ and pence?)

To find totals and give change: It costs 75p for a child to swim. How much does it cost for two children?

-A set of paint costs £3. Parveen saves 20p a week. How many weeks must she save to buy the paints?

- Dad bought three packets of biscuits at 70p each. What was his change from £3?

To know what to buy and how to pay: Which 5 coins make 74p? What other amounts can you make with 5 diff coins?

To solve measurement problems in context: Two rolls of tape are 35cm and 41cm. Total? Difference?

- An egg weighs 50 grams. How much would six eggs weigh?

- A big potato weighs 1/4 kg. What would be the weight of 10 potatoes?

- A bottle holds 35ml of medicine. A teaspoon holds 5ml. How many teaspoons of medicine in the bottle.

- Mark got into the pool at 3:30pm. He swam for 40 mins. What time did he get out?

To know that: 1kilometre = 1000 metres, 1 metre = 100 cm, 1 kilogram = 1000 grams and 1 litre = 1000 millilitres.

To know that 3.5m = 3 and a half metres and that 3.05m is 3 metres and 5 cm.

To suggest items that could be measured using: kilometres, metres, centimetres, kilograms, grams, litres, millilitres.

To know own date of birth and say who is older/younger.

To count around the clock in 5s.

To know the days of the week, months and seasons in order..

Year 3 - Progression (a combination of these models and images can be used for every objective)

To measure and compare lengths in m, cm and mm (using measuring tapes)

The Python is 1m 24cm

1m 23 cm

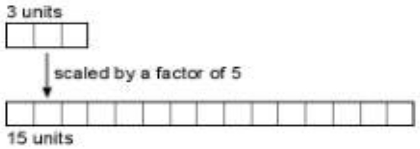
1m = 100cm

23 cm

1m 23cm = 100cm + 23cm
= 123cm
The snake is 123cm long.

To use multiplication and division to scale by integers.

Ed's house is 3 km from school. Tyreis' house is is 5 times further away.
How many km does Tyreis live from school??



To measure and compare mass in Kg and g (using scales)

The mass of the potatoes =1kg 268g.

1kg 268g

1kg =1000g

268g

268g

The mass of the potatoes is 1268g

To measure and compare volume in l/ml (using scaled containers)

The amount of water = volume
The capacity of a container is the full amount of water it can hold.

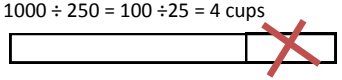
The container holds 1l 400ml

1l = 1000ml
1l 400ml = 1000ml + 400ml
= 1400ml

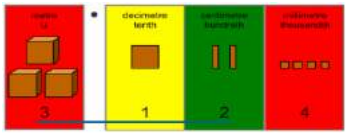
To solve measurement problems using both addition and subtraction.

Rosie had 100ml of juice in a jug.
She completely filled some cups with all the juice in the jug. The capacity of each cup was 250ml. How many cups did she use?

1000ml— 250ml—250ml—250 ml—250ml
(repeated subtraction = 4 cups)
Or 1000 ÷ 250 = 100 ÷25 = 4 cups



To convert between different units of measure.



Use diennes to show place value of measures.

1000g = 1 kilogram
1000ml = 1 litre
100cm = 1 metre

To measure and calculate perimeter of 2D shapes.

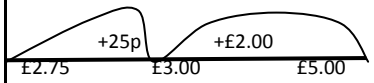
Use rubber bands and geoboards to investigate perimeter of 2D shapes.

One peg to the next = 1 unit

Perimeter of square = 4 units/ 4 cm²
Perimeter of triangle = 6 units/ 6cm²
Perimeter is a measure of length

To calculate change given in both £ and p.* (must consolidate coin recognition)

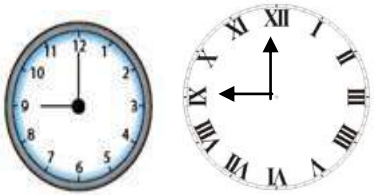
Use a number line to 'count on' and find the difference when calculating change:



25p + £2.00 = £2.25

Tell and write the time from an analogue clock. (standard clock and with Roman numerals)

The minute hand moves one complete round in 60 minutes. The hour hand moves from 8 to 9 in one hour.



To match digital and analogue clocks.

Analogue Digital

9:42 or nine forty-two.

And use 24 hour:

To read and record time to the nearest minute

Once secure in nearest 5 min:

Focus on 1 min intervals on clock:

To compare time in seconds, minutes and hours.

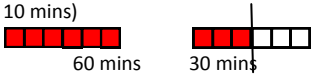
To order times from earliest to latest:

To compare seconds: 60 s > 42 s
To compare minutes: 45 mins < 57 mins
To compare hours: 5 hours > 3 hours

Joan took 3 hours to get home. Julie took 120 minutes to get home. Who took longer? 120mins = 2 hours.
Joan took longer.

To convert hours and minutes

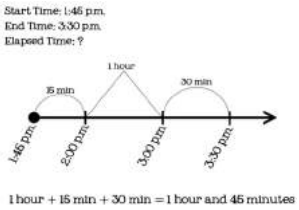
Jamal spent 3 hours on the computer. How many minutes is this?
3h = 60 mins + 60 mins + 60 mins
(3x6 = 18 therefor 3 x 60 = 180)
= 180 minutes
The football lasts for 90 minutes. How long is this in hours?
(= 10 mins)



1 hour 30 mins or one hour and a half.

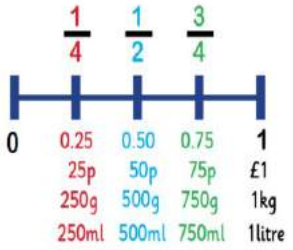
To calculate and compare duration of events.

The game started at 1:45pm and finished at 3:30pm. How long did it last for?



Yesterday's game lasted for 1 hour 30 mins (90 mins) so was shorter.

To identify fractions of measures



CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

Year 4 - **Measurement** (When planning ensure you track back to year 3 and forwards to year 5)

National Curriculum

Pupils should be taught to:

Convert between different units of measure [for example, kilometre to metre; hour to minute].

Measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres.

Find the area of rectilinear shapes by counting squares.

Estimate, compare and calculate different measures, including money in pounds and pence.

Read, write and convert time between analogue and digital 12- and 24-hour clocks.

Solve problems involving converting hours to minutes, minutes to seconds, years to months, weeks to days.

Notes and guidance (non-statutory)

Pupils build on their understanding of place value and decimal notation to record metric measures, including money.

They use multiplication to convert from larger to smaller units.

Perimeter can be expressed algebraically as 2(a + b) where a and b are the dimensions in the same unit.

They relate area to arrays and multiplication.

Key vocab: unit, measuring scale, division, estimate, fortnight, leap year, perimeter.

Key concepts

We always introduce new aspects of measurement through direct comparison and activities involving ordering.

Visualise and measure in compound units (kg and g).

We can convert units of measure using multiplicative reasoning.

Perimeter can be expressed algebraically as 2(a + b) where a and b are the dimensions in the same unit.

Measures from length to money is an excellent way to introduce and consolidate decimal notation.

The area of an object can be related to arrays and multiplication.

Potential barriers

Pupils do not apply place value knowledge to a 'measure' context.

Pupils find it difficult reading scale and understanding that the intervals may change depending on the scale.

Pupils do not have sufficient understanding of decimals and fractions and so find it difficult to relate this to measures.

Pupils are not able to use formulas to calculate area and perimeter and therefore confuse the two concepts.

Pupils count a square as being within a shape, even if is not fully within.

Learning objectives (see overleaf for exemplification)

To convert units of measure.

To measure and calculate the perimeter of rectilinear shapes.

To find the area of rectilinear shapes (by counting squares).

To estimate, compare and calculate measures.

To calculate money in pounds and pence using four operations.

To convert time between analogue and digital clocks (12 hour and 24 hour).

To solve problems involving converting time.

To calculate time durations that pass through the hour.

Example Questions

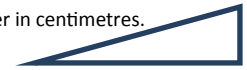
Jamie is cooking pasta. He weighs 350g of the pasta. Draw an arrow on the scale to show 350g.

Molly is making a cake. She has 450g of flour on the scale. She put in sugar and the now scale shows 600g. How much sugar did she add?

The length of a banana is about: 2cm, 20cm, 200cm, 2000cm?

Kate has a piece of ribbon one metre long. She cuts off 30cm. How many centimetres of ribbon are left?

Measure accurately the length of the shortest side of this triangle. Write your answer in centimetres.



Measure accurately the length of the diagonal of this square. (not to scale)

Here are two clock faces (show as analogue 8:20 and 4:40). Join them to the correct digital time. Show as analogue: 9:20, 8:20, 4:40, 3:40, 4:20, 8:40

A clock shows this time (show digital clock showing: 4:40pm). How long is it from this time until 6pm? What time was it quarter of an hour before the time on the clock?

How many hours are there in two days? What time is it half an hour after eleven-fifteen?

My watch show three-fifty pm. What time will it show in thirty minutes? What is the time twenty minutes after ten fifty-five?

Circle the time that is 30 minutes before midnight: 12:30am 12:30pm 11:30am 11:30pm 4 am

A film starts at 6:45pm. It lasts for 2 hours and 35 minutes. What time will the film finish?

Hassan has a jug with some water in it. He adds another 140 millilitres of water.

Draw a line to show the new level of water.

Here is a beaker with some water in it. How many millilitres of water are in the beaker?

Mental Maths

To express a relationship in words: How to find the number of days in any number of weeks.

- How to find change from £1 after buying two items.

- How to describe the short way to work out the perimeter of a rectangle.

To solve problems involving money: A game costs £4. Peter saves 40p a week. How many weeks will it take to save?

To convert pounds into pence and vice versa: How many pence is: £1.57, £10.50, £31.60

In pounds: 356p, 970p, 2040p

To calculate fractions: Harry spent 1/4 of his saving on a book. What did the book cost if he spent £4, £5, £10, £20

To solve problems involving measures: A full jug holds 2 litres. A full glass holds 1/4 of a litre. How many glasses full of water will the jug be?

To double a recipe: 125g flour, 50g fat, 75g sugar, 30ml treacle, 1 teaspoon of ground ginger. (to scale by four…)

To calculate time duration: Lunch takes 40 minutes. It ends at 1:10pm what time does it start?

- Jan went swimming on Wednesday 14th January. She went swimming again 4 weeks later. What date was it?

- The pool closed on Friday 20th March, It opened again on Friday 10th April. How many weeks was it closed for?

To know that: 1 kilometre= 1000 metres, 1 metre= 100cm or 1000millimetres, 1 centimetre= 10 millimetres, 1 kilogram= 1000 grams, 1 litre = 1000 millimetres.

To know fractions of measures: 500g is half of 1kg, 75cm is three quarters of 1m.

To write: 1.6m in cm (160cm), 5 litres in millilitres (5000ml), 8km in m (8000m), 3cm in mm (30 mm) etc.

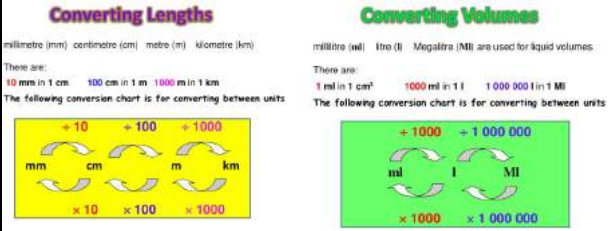
To suggest areas you would measure in mm², cm², m².

To recite the rhyme– 30 days has September… To know that a leap year has 366 days.

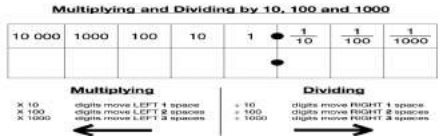
CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

Year 4- Progression (a combination of these models and images can be used for every objective)

To convert units of measure

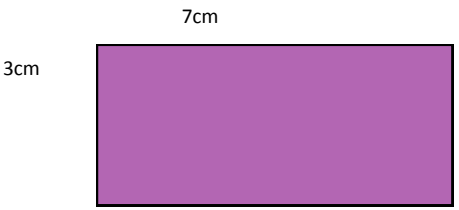


& Mass = 1000g = 1 kg



To measure and calculate the perimeter of rectilinear shapes.

Find the perimeter by adding up all it's sides:

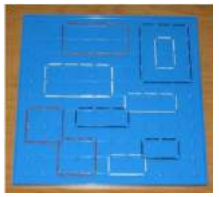


Expressing perimeter algebraically in the same units.

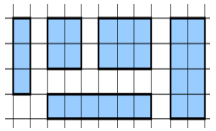
7cm + 3cm + 7cm + 3cm = 20cm (L + W + L + W)
Or if rectilinear:
(7cm+3cm) X 2 = 10cm x 2 = 20cm (2(L+W))

To find the area of rectilinear shapes (by counting squares)

Use of geo boards to find area (counting squares):

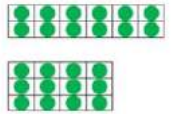


and squared paper



Area is the measure of surface (cm²).

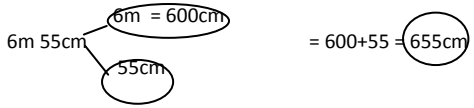
This can be linked to arrays:



To estimate, compare and calculate measures.

Estimate then measure:
Length: maths book, table, thumb nail, pencil etc around the class-room.
Capacity: pencil pot, water bottle, jam jar, can.
Mass: shoe, book, pencil case, book bag, marbles,

When comparing, convert measurements:

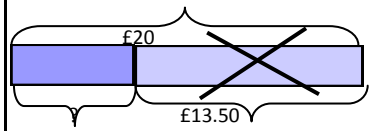


To calculate money in pounds and pence using four operations.

I buy a DVD for £13.50. How much change to I get from £20.
£20.00—£13.50

(To also investigate +, X, ÷)

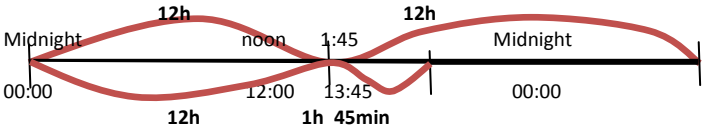
£13.50 + ? = £20.00



To do £20 - £13.50 we must write £20 as £20.00

To convert time between analogue and digital clocks (12 hour and 24 hour)

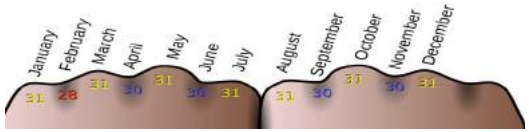
Lunch time ends at 1:45pm during a school day. How do we write this using the 24 hour clock*? (*midnight represented as 00:00).
If it is in the afternoon I start at 12 and count on… (12+1 = 13…)



1:45 is the same as 13:45.

To solve problems involving converting time.

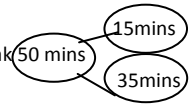
60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
4 weeks (?) = 1 month
12 months = 1 year
10 years = 1 decade
100 years = 1 century
1,000 years = 1 millenium

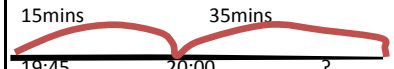


To calculate time durations that pass through the hour.

Ben left his uncle's house at 19:45. He arrived home 50 minutes later. What time was it when he arrived home?

Use part, part whole model to break 50 mins

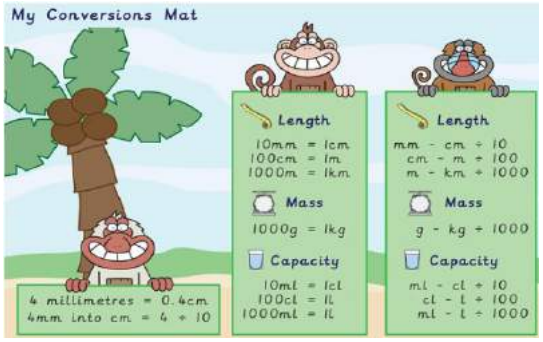




Ben reached home at **20:35**

Use the part, part, whole model to *jump to the next hour.*
Can also be used jumping backwards to find what time something started.

Conversions



Year 5 - **Measurement** (When planning ensure you track back to year 4 and forwards to year 6)

National Curriculum

Pupils should be taught to: Convert between different units of metric measure [for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre].
Understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints.
Measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres.
Calculate and compare the area of rectangles (including squares), including using standard units, square centimetres (cm^2) and square metres (m^2), and estimate the area of irregular shapes.
Estimate volume [for example, using 1 cm^3 blocks to build cuboids (inc. cubes)] and capacity [e.g. using water]
Solve problems involving converting between units of time.
Use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling.

Notes and guidance (non-statutory)

Pupils use their knowledge of place value and multiplication and division to convert between standard units.

Pupils calculate the perimeter of rectangles and related composite shapes, including using the relations of perimeter or area to find unknown lengths. Missing measures questions such as these can be expressed algebraically, for example 4 + 2b = 20 for a rectangle of sides 2 cm and b cm and perimeter of 20cm.

Pupils calculate the area from scale drawings using given measurements.

Pupils use all 4 operations in problems involving time and money, including conversions (for example, days to weeks, expressing the answer as weeks and days).

Key vocab: metric unit, measuring scale, division, square metre, square millimetre, 24 hour clock.

Key concepts

We always introduce new aspects of measurement through direct comparison and activities involving ordering.
We can convert units of measure using multiplication and division.

Potential barriers

Pupils do not apply place value knowledge to a 'measure' context.
Pupils find converting millilitres to litres a challenge because it involves multiplying and dividing by 1,000.
Pupils do not have sufficient understanding of decimals and fractions and so find it difficult to relate this to measures.
Pupils are not able to use formulas to calculate area and perimeter and therefore confuse the two concepts.
Pupils are unable to draw appropriate diagrams when calculating the area of a compound shape and so obtain incorrect dimensions.

Learning objectives (see overleaf for exemplification)

To convert between different units of metric measure.
To use multiplication and division to inter scale and calculate changing rates.
To use approximate equivalences between metric and imperial units.
To measure and calculate the perimeter of composite rectilinear shapes.
To solve missing measure questions when presented algebraically.
To calculate and compare the area of rectangles. (cm^2 and m^2)
To estimate the area of irregular shape.
To estimate and measure capacity.
To estimate volume.
To solve problems involving converting units of time.
To use all four operations to solve problems involving measure.

Example Questions

Put a ring around the approximate mass of an eating apple. 1g 5g 10g 150g 1000g
A hen's egg is likely to weigh about: 6g 60g 600g
A tin of baked beans weighs four hundred grams. How many grams less than one kilogram is this?
What time is it half an hour after nine-fifteen? How many days are there altogether in June and July?
Put a ring around the time that is the same as fourteen thirty: 2:30am 4:30pm 4:30am 1:45pm 2:30pm
What time is it ten hours after eight pm?
How would quarter past three in the afternoon be shown on a twenty-four hour digital clock?
These are the start and finish times on a DVD recorder: START 16:45 FINISH 19:25. What is the duration?
An aeroplane takes off on Tuesday at 22:47. It lands on Wednesday at 07:05. How long in hours and minutes?
The distance from London to Manchester is about: 320cm, 320m, 320km
Max jumped 2.25 metres on his second try at the long jump. This was 75cm longer than his first try. How far in metres did he jump on his first try?
How many millimetres are there in 15 centimetres? A rectangular swimming pool is 25m long and 10m wide.
David swims 5 lengths. Rosie swims 12 widths. How much further does David swim than Rosie?
A tea cup is likely to hold: 15ml, 150ml, 1500ml. A bottle holds 1 litre of lemonade. Rachel fills 5 glasses with lemonade. She puts 150ml in each glass. How much lemonade is left in the bottle?
Some children go camping. It costs £2.20 for each child to camp each night. They go for 6 nights.
How much will each child have to pay for the 6 nights?
There are 70 children. Each tent takes up to 6 children. What is the least number of tents they will need? Show your method.

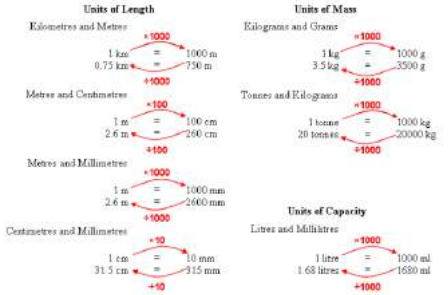
Mental Maths

To convert days to weeks.
To express duration of days as weeks and days. i.e. 2 weeks and 3 days = 17 days.
To express relationship orally: Explain how to find the number of months in any number of years.
- Explain how to find the change from 50p for a number of chews at 4p each.
- Describe a way to calculate the area of a rectangle.
To solve problems involving money: Kobi saved 15p a week for one year. How much did he save?
To calculate fractions and percentages: The deposit on a £230 bed is 50%. How much is the deposit?
-There is 25% off in the sale. If an item costs £8. how much is it in the sale? £12, £20…
To solve problems involving measures: Greg uses 5 tomatoes to make 1/2 litre of soup. How much can he make with 15 tomatoes?
- change the recipe for four people to a recipe for six (half and add on). 240g flour, 300ml milk, 2 eggs.
To calculate duration: The sun sets at 19:30 and rises again at 6:30. How many hours of daylight/ darkness?
To be fluent with these facts: 1 kilometre= 1000 metres, 1 metre= 100cm or 1000millimetres, 1 centimetre= 10 millimetres, 1 kilogram= 1000 grams, 1 litre = 1000 millimetres.
To know the equivalent of one half, one quarter, three quarters, one tenth and one hundredth of a metre, km, kg, l in m, cm, g and ml respectively. i.e. 10g is one hundredth of 1kg.
To suggest items that could be measured using: kilometres, metres, centimetres, kilograms, grams, litres, millilitres.
To know that 1 square metre = 10 000 cm^2 and that 1 square cm = 100mm^2
To recite the rhyme– 30 days has September… To know that a leap year has 366 days.

CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

Year 5- Progression (a combination of these models and images can be used for every objective)

To convert between different units of metric measure.



To use multiplication and division to inter scale and calculate changing rates.

A table of exchange rates is shown below.

Country	Exchange rate for £s sterling
Australia	2.82 Australian dollars
European countries	1.58 euros
Switzerland	2.32 Swiss francs
South Africa	16.34 rand
Turkey	2520000 New Turkish lira
U.S.A.	1.55 US dollars

To change British money to foreign money - multiply by the exchange rate.

FOREIGN CURRENCY = BRITISH CURRENCY x EXCHANGE RATE

Example

Ann is going to Spain on holiday and changes £200 to euros. How many euros does she get?

One pound is €1.58

So £200 = 200 x 1.58 = 316

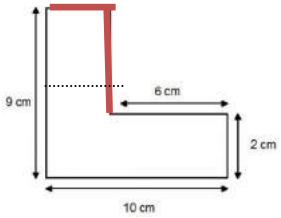
Ann gets €316

To use approximate equivalences between metric and imperial units.

IMPERIAL -> METRIC CONVERSIONS					
1 inch	=	2.54 centimetres	1 in	=	2.54 cm
1 foot	=	30.48 centimetres	1 ft	=	30.48 cm
1 yard	=	91.44 centimetres	1 yd	=	91.44 cm
1 yard	=	0.9144 metres	1 yd	=	0.9144 m
1 mile	=	1609.344 metres	1 mi	=	1609.344 m
1 mile	=	1.609344 kilometres	1 mi	=	1.609344 km

To measure and calculate the perimeter of composite rectilinear shapes.

To find missing lengths: 10cm—6cm = 4cm
9cm—2cm = 7cm

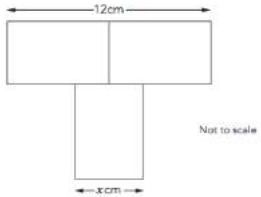


4cm + 7cm + 6cm + 2cm + 10cm + 9cm = 38cm

To solve missing measure questions when presented algebraically.

Here is a T-shape made from 3 identical rectangles.

The area of the T-shape is **90cm²**

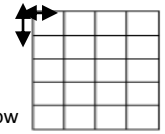


Work out the value of *x*.

90cm² ÷ 3 = 30cm²
12cm ÷ 2 = 6cm
6cm x ? = 30 cm² (30cm² ÷ 6 = 5cm)
X = 5cm

To calculate and compare the area of rectangles. (cm² and m²)

Count the number of 1cm squares covering the rectangle: 1cm
1cm



There are **5** rows of 1cm squares

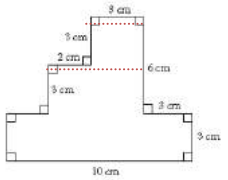
There are **4** 1cm squares in each row

There are **20** 1cm squares covering this rectangle.
Area = 20 cm²

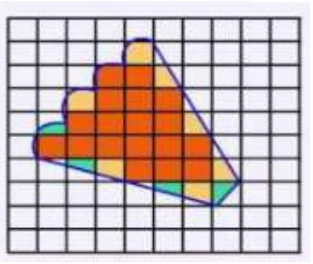
This can also be solved as area = length x breadth
= 4 cm x 5 cm
= 20 cm²

To estimate the area of a irregular shapes.

Use reasoning skills:



Count the squares:



To estimate and measure capacity.

Estimate the capacity of the bottle:
Fill bottle with liquid.
Empty water into measuring cup to find capacity.

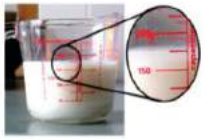
What was your estimate?
What was the actual capacity?
What was the difference?

Did you get closer each time? Why?

To estimate volume.

Capacity is the amount a container can hold.

But what about volume? This is something slightly different. Here's an example:



This **jug** has a capacity of 250 ml.

The **volume** of milk in the jug is 175 ml.

The **volume** of milk needed to **fill** the jug is 250 ml.

Can you see the difference? The volume is how much milk is in the jug.

Volume is a measure of the space taken up by something.

To solve problems involving converting units of time.

Jacques was in France for 7 days and 6 h

(i) 1 day = 24 hours

Therefore, 7 days 6 hours = (7 x 24) hours + 6 hours

= 168 hours + 6 hours

= 174 hours

To use all four operations to solve problems involving measure.

Jim and Gurpreet ran a race. Each ran from A to B and back again. A-B= 27km
Jim finished the race and Gurpreet had only completed 18km. How much further to go?

27km x 2 = 54km

A 27km B 27km A

A 18km B A

54km—18km = 36km

Gurpreet still has 36km to run to complete the race.

Can you write a two step word problem using the following words, numbers and units?

How much?	equally
Difference?	L
capacity?	2
ml	900
altogether	500
volume	12
24	5

Year 6 - Measurement (When planning ensure you track back to year 5 for progression)

National Curriculum

Pupils should be taught to: Solve problems involving the calculation and conversion of units of measure, using decimal notation up to 3 decimal places where appropriate.
Use, read, write & convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to 3 decimal places.
Convert between miles and kilometres.
Recognise that shapes with the same areas can have different perimeters and vice versa.
Recognise when it is possible to use formulae for area and volume of shapes.
Calculate the area of parallelograms and triangles.
Calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units [for example, mm^3 and km^3].

Notes and guidance (non-statutory)

Pupils connect conversion (for example, from kilometers to miles) to a graphical representation as preparation for understanding linear/proportional graphs.
They know approximate conversions and are able to tell if an answer is sensible.
Using the number line, pupils use, add and subtract positive and negative integers for measures such as temperature.
They relate the area of rectangles to parallelograms and triangles, for example, by dissection, and calculate their areas, understanding and using the formulae (in words or symbols) to do this.
Pupils could be introduced to compound units for speed, such as miles per hour, and apply their knowledge in science or other subjects as appropriate.

Key vocab: metric, imperial, estimate, leap year, century, millennium, volume.

Key concepts

We always introduce new aspects of measurement through direct comparison and activities involving ordering.
Speed = distance travelled per unit of time.
The greater the distance travelled in a constant period of time, the faster the speed.
Connecting conversion to graphical representation prepares for understanding of linear/proportional graphs.

Potential barriers

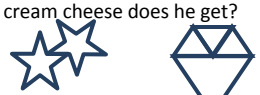
Pupils misunderstand the term 'average'.
Pupils find converting millilitres to litres a challenge because it involves multiplying and dividing by 1,000
Pupils struggle to convet between metric and imperial.
Pupils can convert measures but still fail to link this to real life examples. i.e. How many apples would weigh 1kg?

Learning objectives (see overleaf for exemplification)

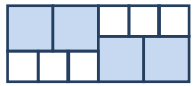
To convert measures using decimal notation (to three decimal places).
To convert between miles and kilometres.
To connect conversion to a graphical representation.
To add and subtract positive and negative integers using a numberline.
To prove that shapes with the same area can have different perimeters.
To understand when to use a formula to calculate area/volume.
To calculate the area of parallelograms
To calculate the area of triangles.
To calculate, estimate and compare the volume of cubes and cuboids
To solve problems involving money using the four operations.
To solve time duration problems using the four operations.

Example Questions

How many grams are there in two point seven kilograms?
A box contains bags of crisps. Each bag of crisps contains 25 grams. Altogether, the bags of crisps inside the box weight 1 kilogram. How many bags of crisps are inside the box?
A packet contains 1.5 kilogram of guinea pig food. Remi feeds her guinea pig 30 gram of food each day. How many days does the packet of food last?
Cream cheese costs £3.60 for 1 kg. Robbie buys some for 90p. How many grams of cream cheese does he get?
Change sixty millimetres to centimetres. Change 9 centimetres into millimetres.



Millie has some star-shaped tiles. Each edge of a tile is 5cm long.
She puts two tiles together to make this shape. Work out the perimeter of Millie's shape.
The area of a rectangle is $16cm^2$. One of the sides is 2cm long. What is the perimeter of the rectangle?
Mr Jones has two sizes of square paving stones. He uses them to make a path. The path measures 1.55m by 3.75m. Calculate the width of a small paving stone.



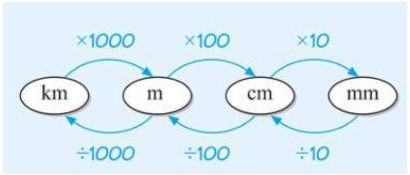
How many millilitres are there in two and a half litres?
There are two containers. One of them holds 750 millilitres and other 0.5 litres.
Which container holds the greater amount? How much more does it hold? Give your answer in millilitres.
Parveen has the same number of 20p and 50p coins. She has £7.00. How many of each coin has she?
Some children do a sponsored walk. Jason is sponsored for £3.45 for each lap. He does 23 laps. How much money does he raise? ynne wants to raise £100. She is sponsored for £6.50 for each lap. What is the least number of laps she must do?

Mental Maths

To solve problems involving money: What is the total of £110, £3.43 and £11.07?
-Three people won £363 630 on the lottery. If this is shared equally how much would each get?
To convert to a currency. There are $1.5 for every £1. How many dollars would I get for £10, £20, £60?
To calculate fractions and percentages: There is a 15% discount in a sale (divide by ten, halve and add to result)…
To solve problems involving measures: I cut 65m of a 3.5m rope. How much is left?
To calculate time durations: Lamb must be cooked for 60 minutes for every kg. How long would it need to be cooked for if the lamb was 1kg, 1.5kg, 2kg, 2.5kg, 3kg, 3.5kg. Chicken is 50 mins for every kg. Can you calculate chicken?
To know the relationships fluently: 1 kilometre= 1000 metres, 1 metre= 100cm or 1000millimetres, 1 centimetre= 10 millimetres, 1 kilogram= 1000 grams, 1 litre = 1000 millimetres.
To extend to: 1 tonne = 1000 kilograms, 1 litre = 100 centilitres, 1 centilitre = 10 millimetres
For conversion make us of rhymes:
A metre is just 3 foot three. It's longer than a yard, you see.
Two and a quarter pounds of jam. It's round about one kilogram.
A litre of water's a pint and three quarters.
To know the equivalent of one thousandth of 1km, 1kg, 1 litre in m, g and ml respectively.
To convert a larger metric unit to a smaller. 3.125km is 3125 metres
To suggest items that could be measured using: kilometres, metres, centimetres, kilograms, grams, litres, millilitres.
To understand: Greenwhich meantime, British Summer time, international date line.
To know that: 1 millennium = 1000 years, 1 century = 100 years and 1 decade = 10 years.
To recite the rhyme 30 days hath September.

Year 6- Progression (a combination of these models and images can be used for every objective)

To convert measures using decimal notation (to three decimal places).

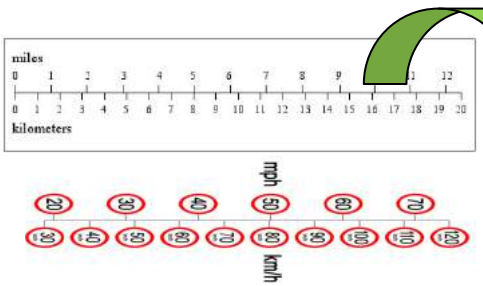


e.g metres to centimetres:

0.8m = 0.8 x 100
= 80 cm

0.06m= 0.06x100
= 6 cm

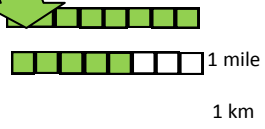
To convert between miles and kilometres.



To connect conversion to a graphical representation.

1km = $\frac{5}{8}$ of a mile

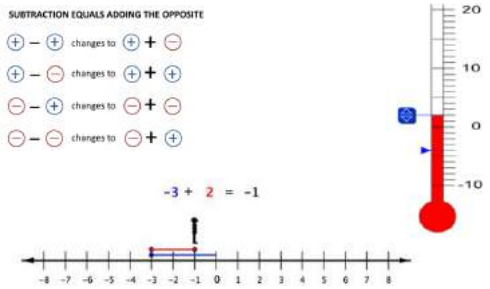
1 mile = $\frac{8}{5}$ of a km



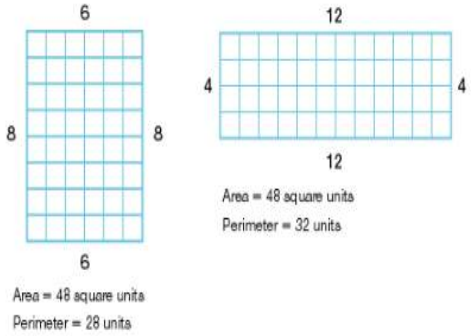
60 km in miles = 60÷ 8 = 7.5 x 5 = 37.7 miles
50 miles in km= 50 ÷ 5 = 10 x 8 = 80 km

(or to find km– divide the miles by 1.6)

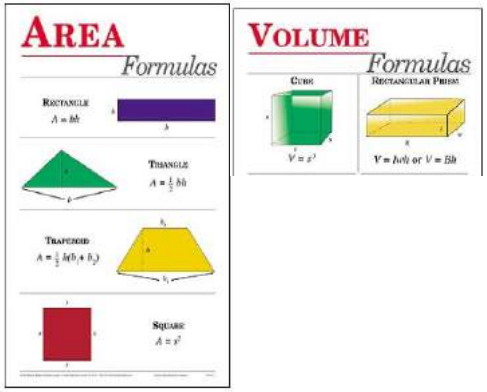
To add and subtract positive and negative integers using a number line.



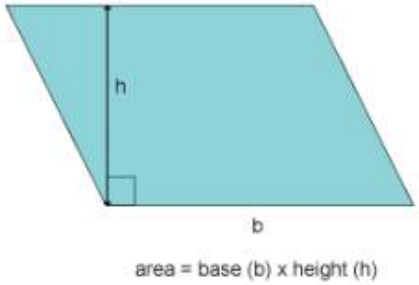
To prove that shapes with the same area can have different perimeters.



To understand when to use a formula to calculate area/volume.



To calculate the area of parallelograms

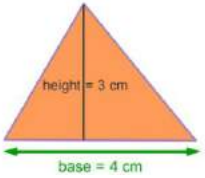


area = base (b) x height (h)

To calculate the area of triangles.

The area of a Triangle equals one half the base times the height.

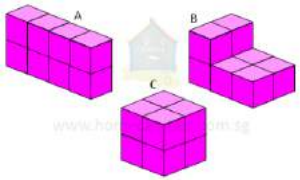
$A = \frac{1}{2} \times b \times h$ or $A = (b \times h) / 2$



$A = \frac{1}{2} \times b \times h$
$A = \frac{1}{2} \times 4 \times 3$
$A = 6\ cm^2$

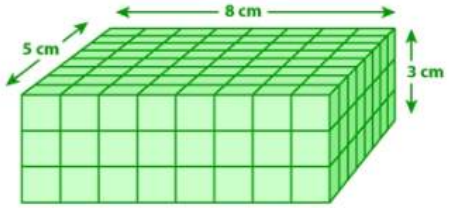
Nb. Find the area of a right angled triangle by seeing it as half a rectangle.

To calculate, estimate and compare the volume of cubes and cuboids.



Using diennes/cubes can you build these?
Which has the most cubes? The least?
Total = cubic centimetres = cm^3

To calculate, estimate and compare the volume of cubes and cuboids



Volume = Length x Width x Depth

= 8 cm x 5 cm x 3 cm
= 120 cm^3

To solve problems involving money using the four operations.

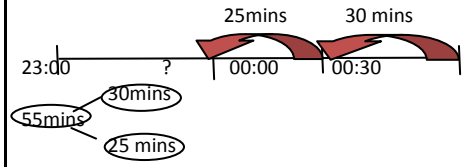
Embedded within real life conte›

Planning for school fair.

End of year production
Cake sale
How much do we need to spend?
How much will we make?
What will be the profit?

To solve time duration problems using the four operations.

Mr Mowz got off the train at 00:30 on Boxing day. He had travelled for 55 mins. What time did he board the train? What day was it?



00:00—25 mins = 23:35
It was 23:35 on Christmas Day.

Chapter 11

Geometry- Properties of Shape

EYFS 1 - Geometry - Properties of shapes (When planning ensure you track forwards to year 1)

Early Learning goal 12

Pupils use everyday language to talk about size, weight, capacity, position, distance, time and money to compare quantities and objects and to solve problems.

They recognise, create and describe patterns.

Key vocabulary: shape, pattern, flat, round, straight, solid, hollow, corner, side, end, sort, make, build, draw, circle, triangle, square, rectangle, size, bigger, larger, smaller.

Key concepts:

Children's experience should begin with play investigating plane and solid shapes, building and taking apart, drawing and talking about shapes in the world around them.

Sameness and difference is used to first explore the concept of different shapes.

Shapes can be drawn (2d) or picked up (3d).

Each shape has a different name and associated properties.

Learning objectives (see overleaf for exemplification)

Shows interest in shapes in the environment.
Shows an interest in shape and space by playing with shapes.
Can say what is different and what is the same.
Begins to categorise according to properties such as size.
Begins to categorise objects according to properties such as shape
Begins to categorise objects according to properties such as colour.
Begins to talk about the shapes of everyday objects e.g. 'round' and 'tall'.
Uses familiar objects and common shapes to build models.
Notices simple shapes and patterns in pictures.
Shows interest in shape and space by making arrangements with objects.
Shows interest in shape by talking about shapes or arrangements.
Shows interest in shape by sustained construction activity.
Shows awareness of similarities of shapes in the environment.

Potential barriers/misconceptions:

Pupils link patterns to one criteria e.g. colour or shape.

Pupils don't learn the key vocabulary when they first start discussing shape. This can be introduced from the early stages.

Example questions:

Copy this pattern 

Look at this patterns with me: big square, small square, big square, small square….
What are the next two shapes in the pattern?
Let's say together the shapes in your pattern: blue cube, green cone, blue cube, green cone. Find the next two shapes in the pattern.
I have given you four cones. Put all your cones in the order of size.

Mental Maths

To listen to and talk about stories related to size: The three bears, Three little pigs, Jack and the Beanstalk, Titch.

Put in order of size a set of teddies, dolls, towers etc.

Make 'families' of the same shape in different sizes– a family of plastic worms, monsters, houses for bears.

To recreate simple patterns made from people, beads and shapes.

To find halves of paper shapes.

To make a symmetrical model made from bricks– point out the matching sides.

To make repeated patterns from bricks or beads.

To make patterns from shapes and coloured paper.

To have rapid recall of names of shapes: triangle, circle, square,

EYFS 1— Progression (a combination of these models and images can be used for every objective)			
Shows interest in shapes in the environment.	**Shows an interest in shape and space by playing with shapes**	**Can say what is different and what is the same.**	**Begins to categorise according to properties such as size, shape and colour.**
 		Same colour? Different colour? Same shape? Different shape? 	Can you order these cones from smallest to largest?  
Begins to talk about the shapes of everyday objects e.g. 'round' and 'tall'.	**Uses familiar objects and common shapes to build models.**		**Notices simple shapes and patterns in pictures.**
 			  
Shows interest in shape and space by making arrangements with objects.	**Shows interest in shape by talking about shapes or arrangements.**	**Shows interest in shape by sustained construction activity.**	**Shows awareness of similarities of shapes in the environment.**
	It has three sides. It has four sides. 		 

EYFS 2 - Geometry - Properties of shapes (When planning ensure you track forwards to year 1)

Early Learning Goal 12

Pupils use everyday language to talk about size, weight, capacity, position, distance, time and money to compare quantities and objects and to solve problems.

They recognise, create and describe patterns.

KS1 ready

Recognise and name common 2-D and 3-D shapes, including:

2-D shapes [for example, rectangles (including squares), circles and triangles].

Key vocabulary: shape, pattern, flat, curved, round, straight, solid, hollow, corner, face, side, end, sort, make, build, draw, circle, triangle, square, rectangle, cube, cone, sphere…

Key concepts:

Children's experience should begin with play investigating plane and solid shapes, building and taking apart, drawing and talking about shapes in the world around them.

Sameness and difference is used to first explore the concept of different shapes.

Shapes can be drawn (2d) or picked up (3d).

Each shape has a different name and associated properties.

Learning objectives (see overleaf for exemplification)

Selects a particular named shape.
Beginning to use mathematical names and 'flat' 2D shapes.
Uses familiar objects and common shapes to create and recreate patterns.
Uses mathematical terms to describe 2-D shapes.
They recognise, create and describe patterns.
Uses everyday language to solve problems.
Explores characteristics of/uses mathematical language for everyday objects and shapes.

Potential barriers/ misconceptions

Pupils link patterns to one criteria e.g. colour or shape.

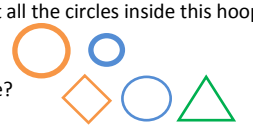
Pupils don't learn the key vocabulary when they first start discussing shape. This can be introduced from the early stages.

Example Questions

Look at this set of flat shapes. Put all the circles inside this hoop.

Point to the smaller circle.

Which of these shapes is a square?



Pack this set of solid shapes into the box. Try to fit them all in. Tell me why this shape won't fit in the box.

Look at this set of solid shapes

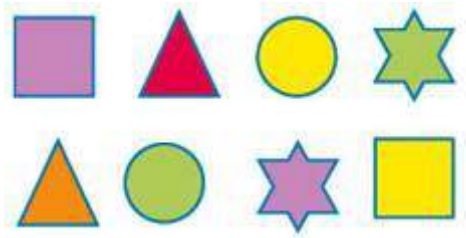
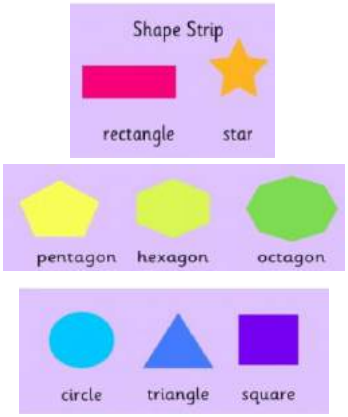
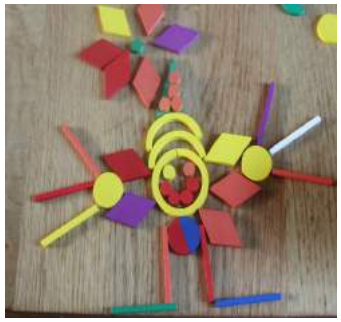
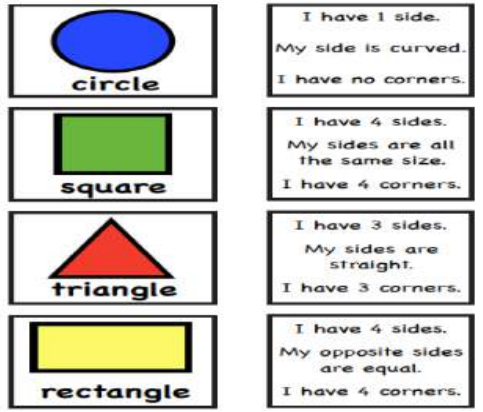
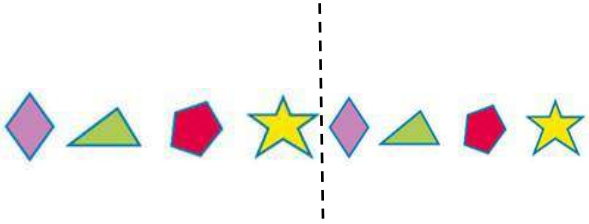
Pick up:
A shape with two faces that are triangles
A shape with six faces that are squares
A shape with two faces that are circles
A shape with three faces that are rectangles

Mental maths

Identify solid shapes that can be seen around the school
To identify that solid shapes may roll, slide etc
To guess the name of a solid shape when placed in a cloth bag.
To identify what objects will fit inside certain boxes.

To find similar shapes on faces of objects– find two circles on lids etc.
To sort or match items.
To find solid shapes around the classroom with at least one face matching a 2-D picture.
To describe a thin plastic shape hidden in a bag.
To identify a shape from it's description.
To find shapes that are not square, round etc.
To know what comes next in a sequence, pattern.

To play I spy: I spy something green with wheels… that has a curved face… lots of points…
To guess how many bricks will pack into a box.
To identify what is missing in a pattern.

EYFS 2— Progression (a combination of these models and images can be used for every objective)			
Selects a particular named shape.	**Beginning to use mathematical names and 'flat' 2D shapes.**	**Uses familiar objects and common shapes to create and recreate patterns.**	**Uses mathematical terms to describe 2D shapes.**
 Point to the: Squares Circles Triangles 			
They recognise, create and describe patterns.	**Uses everyday language to solve problems.**		**Explores characteristics of/uses mathematical language for everyday objects and shapes.**
 	**What could we try next?** **Where does it go?** **Sort** **same** **Match** **different** E.g. fit solid shapes together: **'I think X boxes will fit inside this box. What do you think? Let's try… '** 		'The garage for my car has been made from lots of blocks, and they have straight edges and flat sides and ends.'  'My shape has three corners and three sides.' 

Year 1 - Geometry - Properties of shapes (When planning ensure you track back to Reception and forwards to year 2)

National Curriculum

Recognise and name common 2-D and 3-D shapes, including:

2-D shapes [for example, rectangles (including squares), circles and triangles].

Notes and guidance (non-statutory)

Pupils handle common 2-D and 3-D shapes, naming these and related everyday objects fluently. They recognise these shapes in different orientations and sizes, and know that rectangles, triangles, cuboids and pyramids are not always similar to each other.

Key vocabulary: shape, pattern, flat, solid, hollow, side, edge, face, straight, curved, round, point, pointed, corner, sort, make, build, draw, cube, cuboid, sphere, cone, cylinder, circle, triangle, rectangle, square.

Key concepts:

Children's experience should begin with play investigating plane and solid shapes, building and taking apart, drawing and talking about shapes in the world around them.

Sameness and difference is used to first explore the concept of different shapes.

Shapes can be drawn (2d) or picked up (3d).

Each shape has a different name and associated properties.

Give children the opportunity to explore the properties of various shapes, including the different kinds of triangles and quadrilaterals, and regular and irregular shapes, by folding, tracing, matching, looking for reflective and rotational symmetries and drawing out the implication of these.

Learning objectives (see overleaf for exemplification)

Recognise and name common 2-D shapes (rectangles (including squares, circles and triangles)
Recognise and name 3-D shapes.
To recognise shapes in different orientations and sizes.
To make models, patterns and pictures using construction kits and everyday material.
To identify shapes in the environment.
To identify and make patterns.

Potential barriers/misconceptions

Pupils struggle the vocabulary associated with shapes e.g. 2d 'sides' become 3d 'faces'.

Example Questions

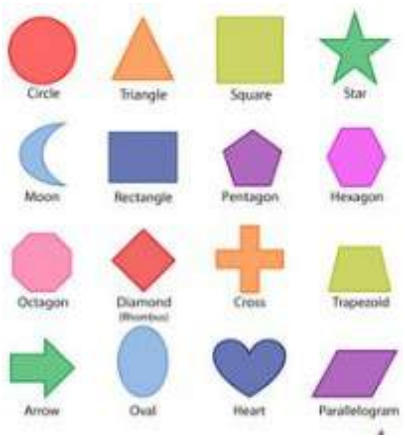
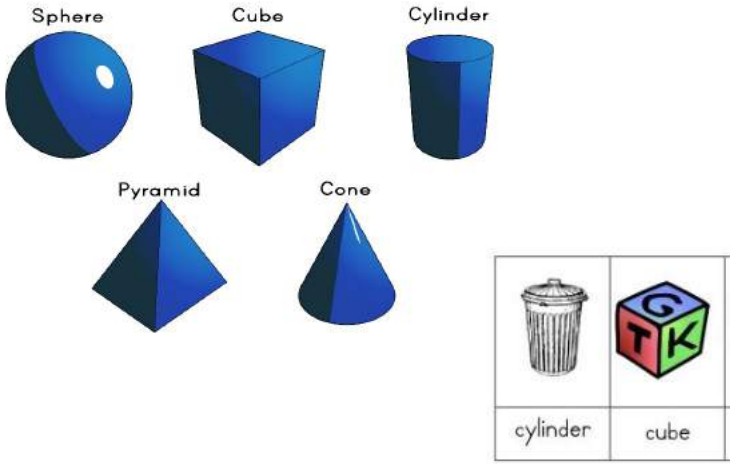
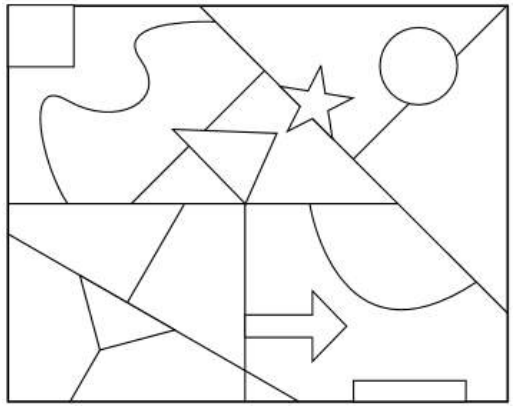
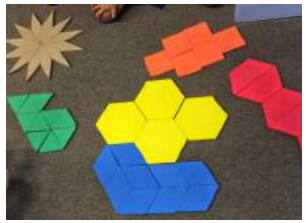
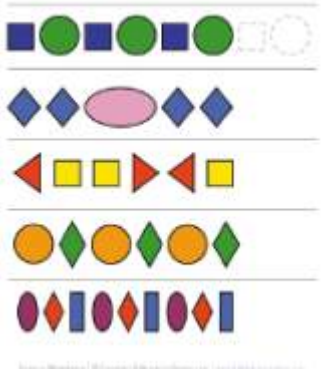
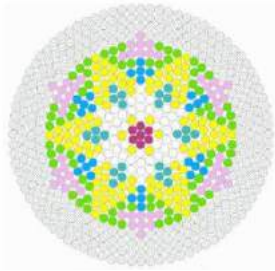
Hand each child one of these shapes: a cylinder, a triangular prism, a cone, a cube. Look at the shape I have given you. Tell me one thing about the shape.

Give each child two different shapes:
Tell me something that is the same about the two shapes. Now tell me something that is different about the shapes.

How many sides does a pentagon have?
Match shapes to their names (pentagon, hexagon, circle, triangle, rectangle)
Tim drawn round the bottom of a cone. Tick the shape that Tim draws:

Mental Maths

Identify solid shapes in the classroom.
Explain how to sort shapes according to property– It has straight edges.
To be able to name a shape by feeling it.
To identify shape based on properties described.
To talk about shapes and patterns in curtains, clothes, objects, displays.
To visualise 2-D shapes: imagine a big triangle painted on the floor. How many sides does it have? How many corners?
Visualise 3-D shapes: imagine you have a tin of beans in your hands. Turn it round and round in your hands. How many circles can you see?

Year 1— Progression (a combination of these models and images can be used for every objective)		
Recognise and name common 2-D shapes (including squares, rectangles, circles and triangles)	**Recognise and name 3-D shapes.**	**To recognise shapes in different orientations and sizes.**
		
To make models, patterns and pictures using construction kits and everyday material.	**To identify shapes in the environment.**	**To identify and make patterns.**
  		 

Year 2 - Geometry - Properties of shapes (When planning ensure you track back to year 1 and forwards to year 3)

National Curriculum

Identify and describe the properties of 2-D shapes, including the number of sides and line symmetry in a vertical line.

Identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces.

Identify 2-D shapes on the surface of 3-D shapes, [for example, a circle on a cylinder and a triangle on a pyramid]

Notes and guidance (non-statutory)

Pupils handle and name a wide variety of common 2-D and 3-D shapes including: quadrilaterals and polygons, and cuboids, prisms and cones, and identify the properties of each shape (for example, number of sides, number of faces). Pupils identify, compare and sort shapes on the basis of their properties and use vocabulary precisely, such as sides, edges, vertices and faces.

Pupils read and write names for shapes that are appropriate for their word reading and spelling.

Key vocabulary: shape, pattern, flat, solid, hollow, side, edge, face, straight, curved, round, point, pointed, corner, sort, make, build, draw, cube, cuboid, sphere, cone, cylinder, circle, triangle, rectangle, square, circular, rectangular, triangular, surface, pyramid, pentagon, hexagon, octagon.

Key concepts:

A prism has the same cross section along its length and that its two end faces are identical.

A quadrilateral is any flat shape with four straight sides.

Give children the opportunity to explore the properties of various shapes, including the different kinds of triangles and quadrilaterals, and regular and irregular shapes, by folding, tracing, matching, looking for reflective and rotational symmetries and drawing out the implication of these.

Learning objectives (see overleaf for exemplification)

To identify and describe the properties of 2-D shapes.
To identify the line symmetry in a 2-D shape.
To identify and describe properties of a 3-D shape (edges, vertices and faces)
To identify 2-D shapes on the surface of 3-D shapes.
To compare and sort common 2-D and 3-D shapes and everyday objects.

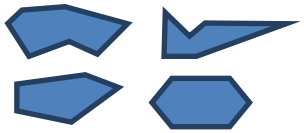
Potential barriers/ misconceptions:

Pupils struggle the vocabulary associated with shapes e.g. 2d 'sides' become 3d 'faces'.

Pupils are able to discuss shapes when physically handling them but do not have enough exposure to the process of visualisation.

Example Questions:

Two of these shapes are not hexagons. Draw a cross on the shape which is not a hexagon.



Look at this shape:

How many right angles does it have?

Look at the shape names: square, hexagon, pentagon, octagon. One of these shapes has exactly two more sides than a triangle. Tick the correct shape.

Write each word in a correct box (faces, edges, vertices)

A cube has:

6 _____

8 _____

12 _____

What shape will I get if I fold a square in half?

Draw the reflection of a shape in the mirror line. You may use a mirror.

Two of these steps have no lines of symmetry. Draw a cross on them. You may use a mirror.

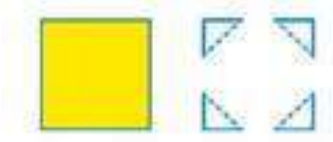
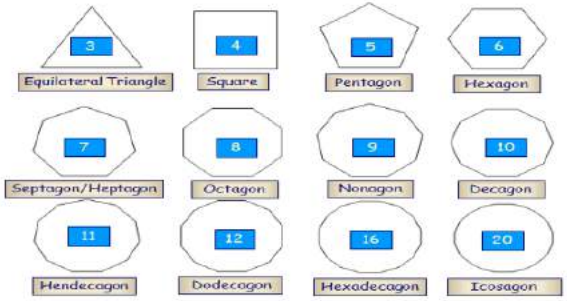
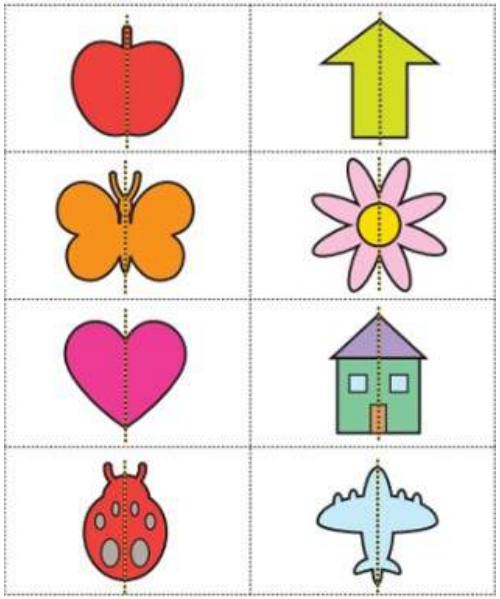
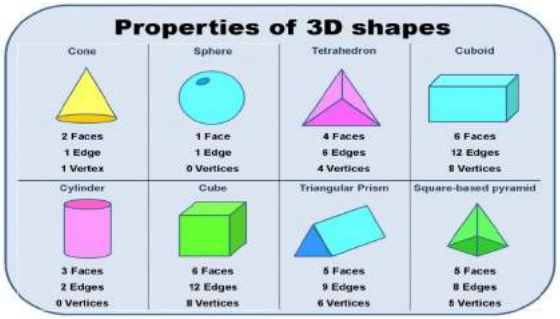
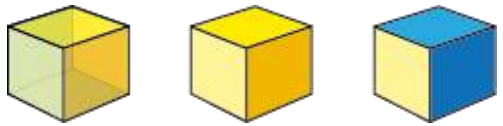
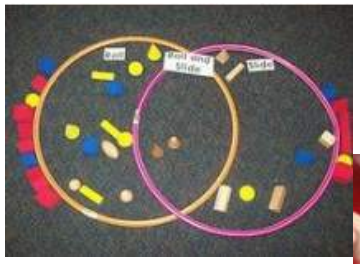
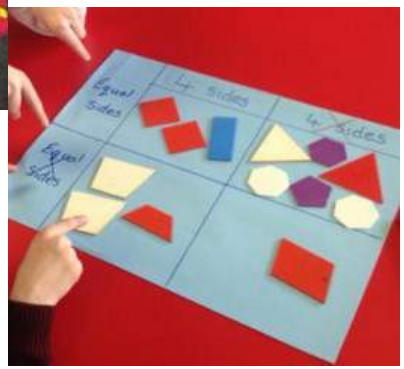
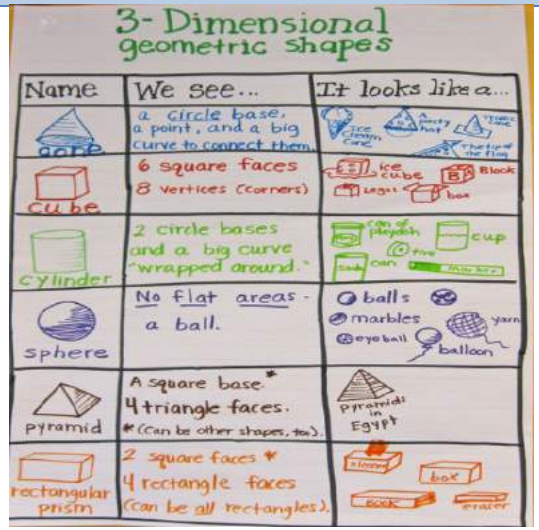
Mental Maths

Shape in my pocket. Put a shape in your pocket or a small bag. Children ask the teacher questions about that shape such as "Has it got any lines of symmetry?" "Has it got four sides?", "Is it a regular shape?". Write the answers to their questions on the board e.g. 'This shape has four sides. The sides are all equal' etc. After 5 questions, children discuss possible shapes it could be with a partner. Reveal the shape and recap its properties.

Shape bingo. Children draw three shapes on a whiteboard. The teacher describes a property of a shape e.g. two lines of symmetry, no straight sides. Children can cross off their shape if it matches the teacher's description. The first person to cross off all three shapes is the winner.

Hidden shapes. Hide a shape behind a piece of card or in a large envelope. Reveal a small part of the shape and discuss shapes it could or couldn't be. Don't always use regular shapes!

Odd one out. Draw 4 shapes on the board e.g. square, rectangle, trapezium and circle. Children discuss with a partner which shape is the odd one out and why.

Year 2— Progression (a combination of these models and images can be used for every objective)		
To identify and describe the properties of 2-D shapes.	**To identify the line symmetry in a 2-D shape.**	**To identify and describe properties of a 3-D shape (edges, vertices and faces)**
If you cut a small triangle off each corner of a square what shape would you have left? Square, hexagon, pentagon, heptagon, octagon?  		 Imagine a cube– four faces are yellow, the rest are blue. How many faces are blue? 
To identify 2-D shapes on the surface of 3-D shapes.	**To compare and sort common 2-D and 3-D shapes and everyday objects.**	
 	  	

Year 3 - Geometry - Properties of shapes (When planning ensure you track back to year 2 and forwards to year 4)

National Curriculum

Draw 2-D shapes and make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations and describe them

Recognise angles as a property of shape or a description of a turn

Identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle.

Identify horizontal and vertical lines and pairs of perpendicular and parallel lines.

Notes and guidance (non-statutory)

Pupils' knowledge of the properties of shapes is extended at this stage to symmetrical and non-symmetrical polygons and polyhedra. Pupils extend their use of the properties of shapes. They should be able to describe the properties of 2-D and 3-D shapes using accurate language, including lengths of lines and acute and obtuse for angles greater or lesser than a right angle.

Pupils connect decimals and rounding to drawing and measuring straight lines in cm in a variety of contexts.

Key vocabulary: cube, cuboid, sphere, cone, cylinder, circle, triangle, rectangle, square, circular, rectangular, triangular, surface, pyramid, pentagon(al), hexagon(al), octagon(al), right-angled, vertex, vertices, layer, diagram, prism, hemi-sphere, quadrilateral, semi-circle.

Key concepts

Shapes can be flat or solid.

Give children the opportunity to explore the properties of various shapes, including the different kinds of triangles and quadrilaterals, and regular and irregular shapes, by folding, tracing, matching, looking for reflective and rotational symmetries and drawing out the implication of these.

Emphasis the dynamic view of angle, giving plenty of experience of rotating objects– themselves, pointers, pencils etc.

Include the important stages of developing any measurement concept: comparison, ordering and the use of non standard units (turns and fractions of turns). Get pupils to compare and order angles by cutting them out and placing them on top of each other.

Learning objectives (see overleaf for exemplification)

To draw and describe 2-D shapes (reflective symmetry, regular, irregular)

To make 3-D shapes using modelling materials.

To recognise 3-D shapes in different orientations.

To recognise angles as a property of shape.

To identify angles in the environment.

To recognise angles as a description of a turn. (half turn, three quarters turn, 360')

To identify right angles, linking to turns and identifying ><= right angles. (acute, obtuse)

To sort symmetrical and non-symmetrical polygons and polyhedra.

To connect decimals and rounding to drawing and measuring straight lines.

To identify horizontal and vertical lines.

To identify pairs of perpendicular and parallel lines.

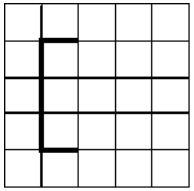
Potential barriers/misconceptions

When continually faced with regular shapes, pupils will draw the conclusion that number of lines of symmetry is equal to the number of sides – and nearly always assume lines of symmetry for rectangles.

Pupils find recognition harder with shapes than with patterns.

Example Questions:

Complete this shape so that it makes a square:



Complete the table showing information about shapes:

	Number of flat surfaces	Number of curved surfaces
Sphere	0	1
Cone		
Cuboid		
Cylinder		

Write the missing numbers in the 2 empty boxes.

	Number of square faces	Number of triangular faces	Number of circular faces
Cylinder	0	0	
Cube		0	0
Pyramid	1	4	0

Draw the reflection of the shape in the mirror line. Use a ruler.

Mental maths:

A game:

Parallelogram, concave hexagon, obtuse triangle, isosceles triangle, kite, arrowhead, scalene triangle, rectangle, rhombus, isosceles trapezium, arrowhead, concave quadrilateral, and so on…

Put the cards in a pile

A member of team A picks a card from the top

They describe the properties of the shape, without using the words on the card.

The first person to say the correct shape wins a point

If they give the wrong shape the other team/s has a go

Each person is allowed one guess per round, the game going on until each person or team has had a guess

Then the next team and so on choose the next card…

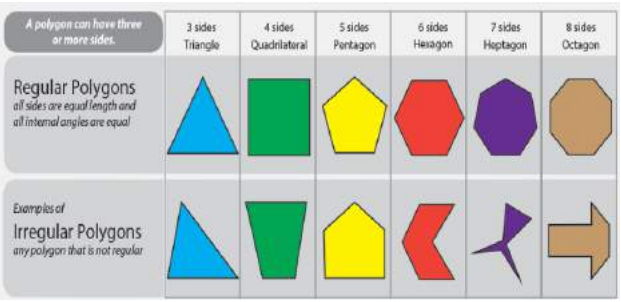
The game can be extended to include other **shapes, solids** etc…

It could be adapted to a two-team or "**Twenty Question**" scenario: **Has it got 4 sides**? **Does it have parallel sides**? And so on…

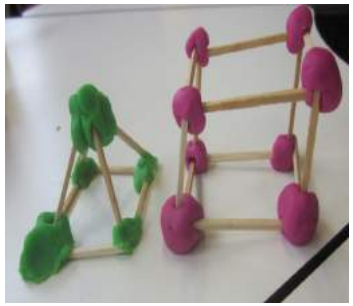
It can also be adapted so that the person at the front of the class describes the shape and other students attempt to draw the shape.

Year 3— Progression (a combination of these models and images can be used for every objective)

To draw and describe 2-D shapes (reflective symmetry, regular, irregular)



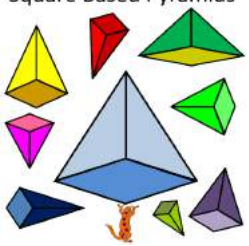
To make 3-D shapes using modelling materials



To recognise 3-D shapes in different orientations.

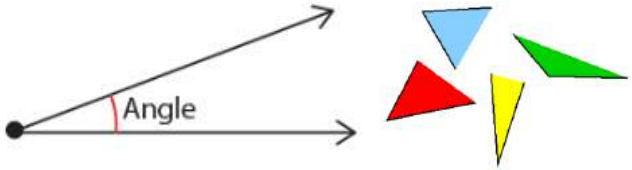
For example:

Square Based Pyramids



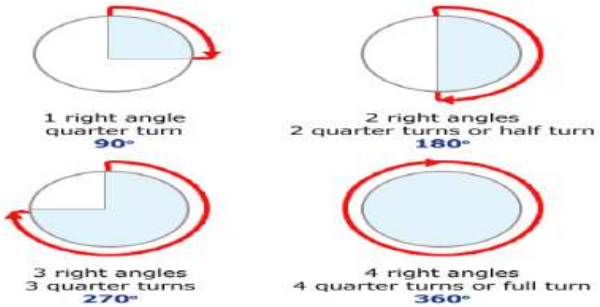
To recognise angles as a property of shape.

The measure of the amount of turn (rotation) from one direction to another, the difference in direction between two lines meeting at a point.

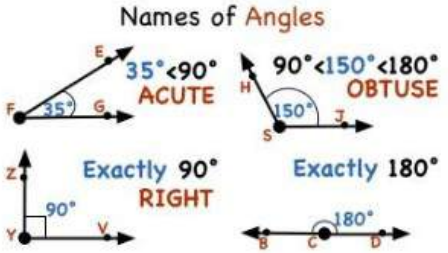


To identify angles in the environment.

To recognise angles as a description of a turn. (half turn, three quarters turn, 360')



To identify right angles, linking to turns and identifying >< = right angles. (acute, obtuse)

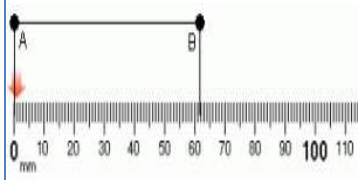


To sort symmetrical and non-symmetrical polygons and polyhedra.

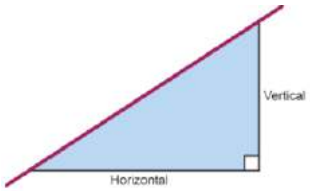
which of these shapes has no lines of symmetry. You can use a mirror.

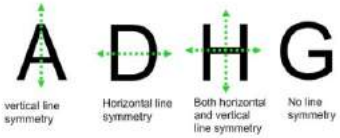
How many sides would this shape have opened:

To connect decimals and rounding to



To identify horizontal and vertical lines.

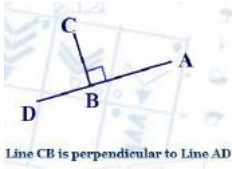




To identify pairs of perpendicular and parallel lines.

Each shape has two parallel lines. What two other properties do all these shapes share?

Parallel



Perpendicular (right angle)

Parallel lines never meet– like a railway track.

Perpendicular lines are two lines that are at 90' to eachother

Year 4- Geometry - Properties of shapes (When planning ensure you track back to year 3 and forwards to year 5)

National Curriculum

Compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes.
Identify acute and obtuse angles and compare and order angles up to two right angles by size.
Identify lines of symmetry in 2-D shapes presented in different orientations.
Complete a simple symmetric figure with respect to a specific lines of symmetry.

Notes and guidance (non-statutory)

Pupils continue to classify shapes using geometrical properties, extending to classifying different triangles (for example, isosceles, equilateral, scalene) and quadrilaterals (for example, parallelogram, rhombus, trapezium).
Pupils compare and order angles in preparation for using a protractor and compare lengths and angles to decide if a polygon is regular or irregular.
Pupils draw symmetric patterns using a variety of media to become familiar with different orientations of lines of symmetry; and recognise line symmetry in a variety of diagrams including where the line of symmetry does not dissect the original shapes.

Key vocabulary: line, construct, sketch, net, angle, base, square-based, regular, irregular, concave, convex, spherical, cylindrical, tetrahedron, polyhedron, equilateral triangle, isosceles triangle, line symmetry, reflect, translation.

Key concepts

Angles are measured in units called degrees.
Give children the opportunity to explore the properties of various shapes, including the different kinds of triangles and quadrilaterals, and regular and irregular shapes, by folding, tracing, matching, looking for reflective and rotational symmetries and drawing out the implication of these.
Remember when talking about various special quadrilaterals that a square is a special kind of rectangle and that a rectangle is a special kind of parallelogram.
Emphasis the dynamic view of angle, giving plenty of experience of rotating objects– themselves, pointers, pencils etc.
Include the important stages of developing any measurement concept: comparison, ordering and the use of non standard units (turns and fractions of turns). Get pupils to compare and order angles by cutting them out and placing them on top of each other

Learning objectives (see overleaf for exemplification)

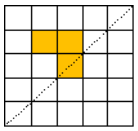
To compare and classify geometric shapes based on their properties and sizes.
To classify different triangles.
To classify different quadrilaterals.
To use a tree diagram to classify shapes
To identify acute and obtuse angles.
To compare and order angles up to two right angles, by size.
To compare length and angles to decide if a polygon is regular or irregular.
To identify lines of symmetry in 2-D shapes presented in different orientations.
To complete a simple symmetric figure.

Potential barriers/misconceptions

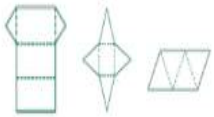
Pupils think that all polygons have the same number of lines of symmetry as they do numbers of sides/angles.
Pupils confuse the number of angles when a shape is concave.

Example Questions

On the grid join dots to make a triangle which does not have a right angle. Use a ruler.
Shade in two more squares to make this design symmetrical about the mirror line.



Here are some nets of shapes. For each net, put a tick if it folds to pyramid.
How many vertices has a cuboid?

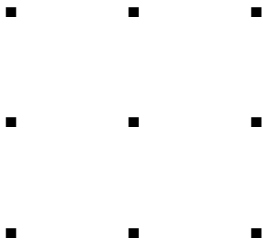


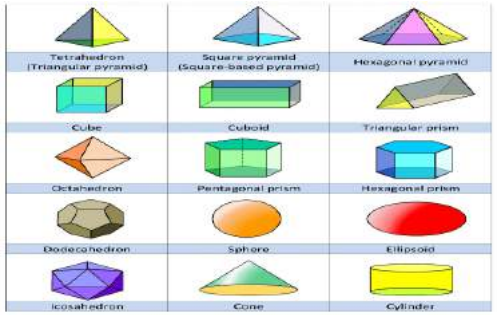
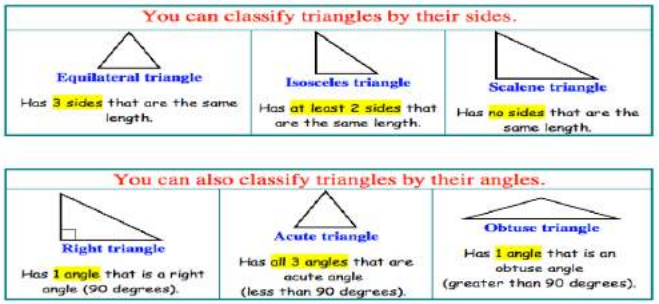
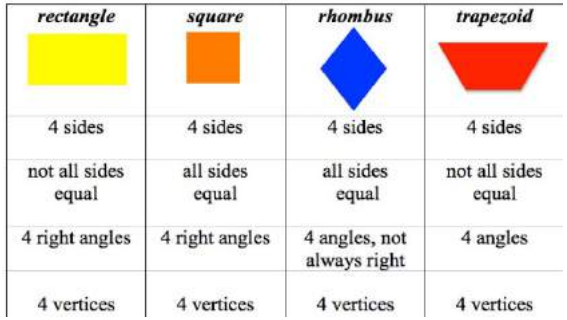
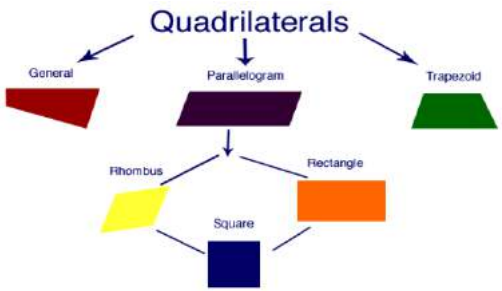
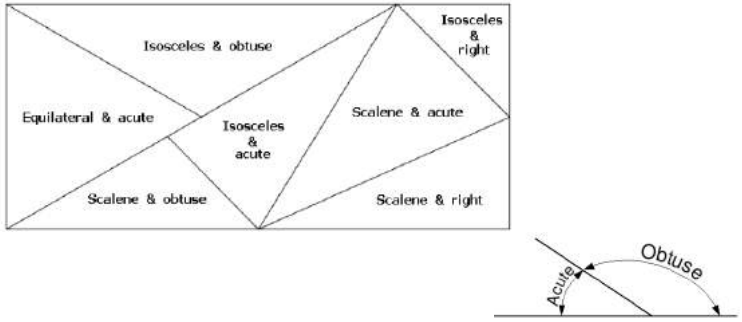
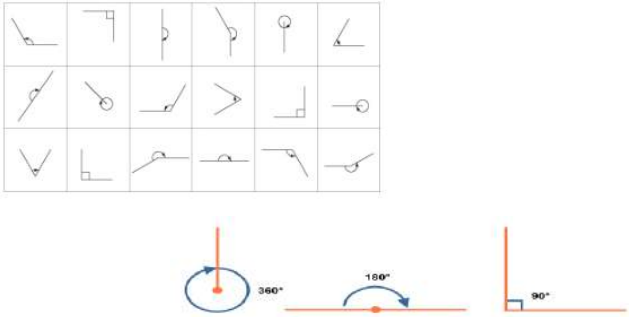
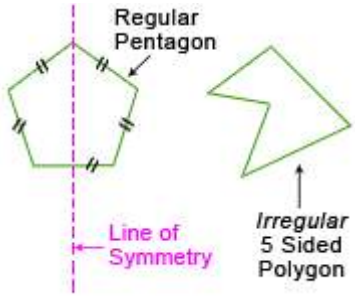
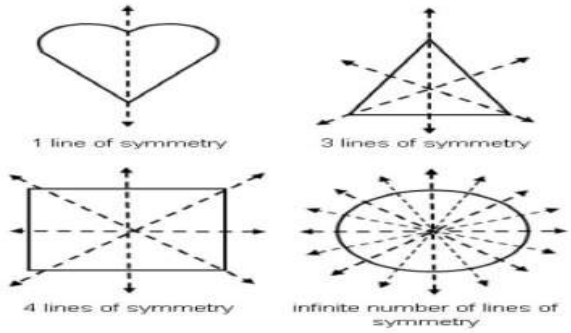
A shape has 4 right angles. It has 4 sides which are not all the same length. Write the name of this shape.

Mental Maths

To complement work on congruence, triangles, & mathematical language

How many different triangles can you draw (make if you have a geoboard) on a 3x3 grid?



Year 4— Progression (a combination of these models and images can be used for every objective)		
To compare and classify geometric shapes based on their properties and sizes.	**To classify different triangles.**	**To classify different quadrilaterals.**
		
To use a tree diagram to classify shapes	**To identify acute and obtuse angles.**	**To compare and order angles up to two right angles, by size.**
		
To compare length and angles to decide if a polygon is regular or irregular.	**To identify lines of symmetry in 2-D shapes presented in different orientations.**	**To complete a simple symmetric figure.**
		Shade in two more squares to make a symmetrical pattern. 

Year 5- Geometry - Properties of shapes (When planning ensure you track back to year 4 and forwards to year 6)

National Curriculum

Identify 3-D shapes, including cubes and other cuboids, from 2-D representations.

Know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles.

Draw given angles, and measure them in degrees (°).

Identify:

-angles at a point and one whole turn (total 360°)

-angles at a point on a straight line and 1/2 turn (total 180°)

-other multiples of 90°

Use the properties of rectangles to deduce related facts and find missing lengths and angles.

Distinguish between regular and irregular polygons based on reasoning about equal sides and angles.

Notes and guidance (non-statutory)

Pupils become accurate in drawing lines with a ruler to the nearest millimetre, and measuring with a protractor. They use conventional markings for parallel lines and right angles.

Pupils use the term diagonal and make conjectures about the angles formed between sides, and between diagonals and parallel sides, and other properties of quadrilaterals, for example using dynamic geometry ICT tools.

Pupils use angle sum facts and other properties to make deductions about missing angles and relate these to missing number problems.

Key vocabulary: line, construct, sketch, net, angle, base, square-based, regular, irregular, concave, convex, spherical, cylindrical, tetrahedron, polyhedron, equilateral triangle, isosceles triangle, line symmetry, reflect, translation, congruent, octahedron, scalene triangle, axis of symmetry, reflective symmetry.

Key concepts

Give children the opportunity to explore the properties of various shapes, including the different kinds of triangles and quadrilaterals, and regular and irregular shapes, by folding, tracing, matching, looking for reflective and rotational symmetries and drawing out the implication of these.

Remember when talking about various special quadrilaterals that a square is a special kind of rectangle and that a rectangle is a special kind of parallelogram.

Include the important stages of developing any measurement concept: comparison, ordering and the use of non standard units (turns and fractions of turns). Get pupils to compare and order angles by cutting them out and placing them on top of each other

Learning objectives (see overleaf for exemplification)

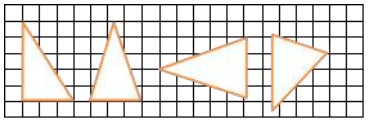
To identify 3-D shapes from 2-D representations (including cubes and other cuboids).
To estimate and compare angles. (obtuse, acute, reflect, right angle)
To draw given angles and measure them in degrees. (using a protractor)
To identify angles at a point and one whole turn.
To identify angles at a point on a straight line.
To identify missing lengths and angles. (using angle sum facts)
To sort regular and irregular polygons.
To draw lines to the nearest mm.
To label parallel lines and right angles.
To identify and use diagonal and parallel lines.

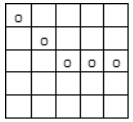
Potential barriers/misconceptions

Pupils confuse mathematical vocabulary e.g. parallel and perpendicular. Pupils think that parallel lines also need to be the same length – they are often presented with examples that are.

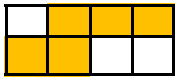
Example Questions

Draw two or more circles on this grid to make a design that has a line of symmetry.





Here are four triangles on a square grid. Put a tick next to two isosceles triangles



Here is a net from which an open top cube is made. Put a tick on a square which is its base

Lara chooses one of quadrilaterals. She says: “It has two acute angles. All four sides are the same length.” Which quadrilateral did Lara choose?

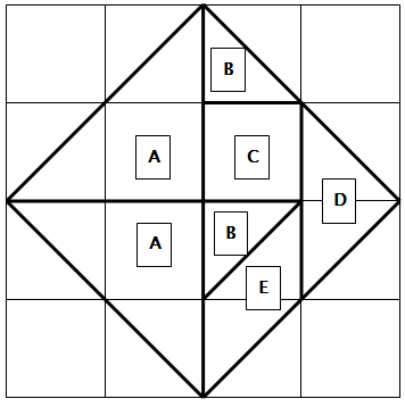
Stefan chooses one of quadrilaterals. He says: “It has more than one obtuse angle. It has no parallel sides.” Which quadrilateral d Stefan choose?

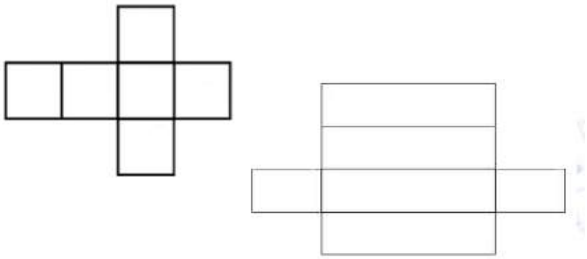
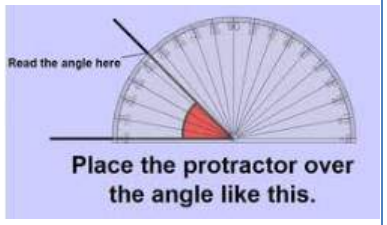
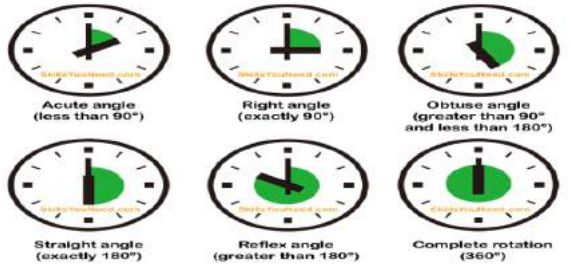
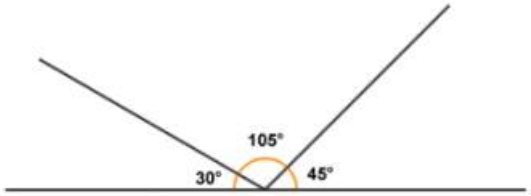
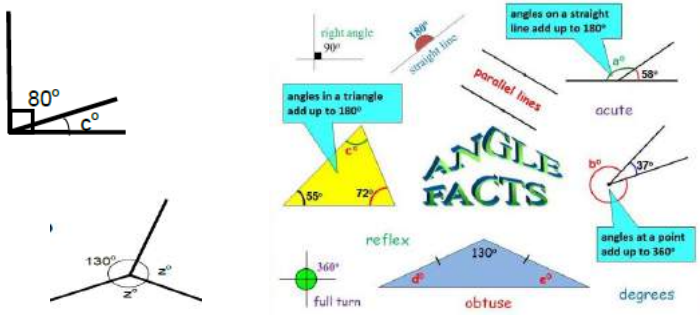
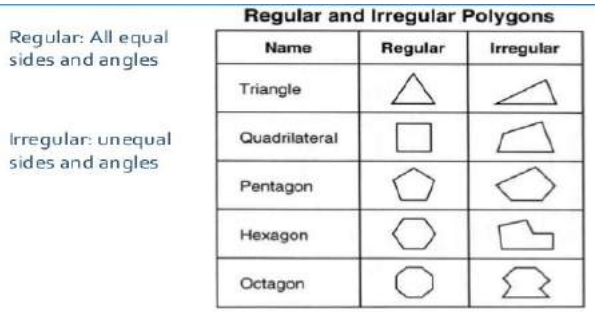
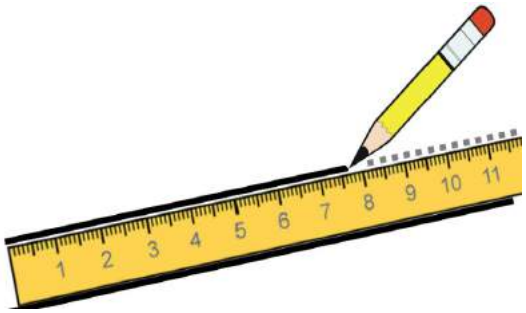
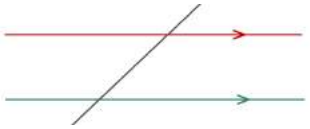
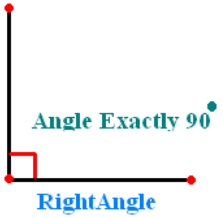
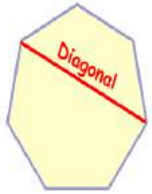
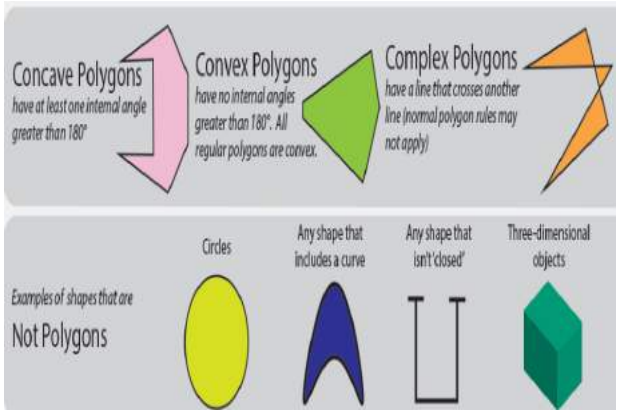
Mental maths

Tangrams:

The pupils can be asked to:

Name the individual tans.
To make all the possible convex polygons (there are 13: 1 triangle, 6 quadrilaterals, 3 pentagons and 3 hexagons)
To make all the possible squares, if not all the tans are used and so on…



Year 5 - Progression (a combination of these models and images can be used for every objective)			
To identify 3-D shapes from 2-D representations (including cubes and other cuboids).	**To estimate and compare angles. (obtuse, acute, reflect, right angle)**	**To draw given angles and measure them in degrees. (using a protractor)**	**To identify angles at a point and one whole turn.**
			
To identify angles at a point on a straight line.	**To identify missing lengths and angles. (using angle sum facts)**		**To sort regular and irregular polygons.**
Angles on one side of a straight line will always = 180’ 			
To draw lines to the nearest mm.	**To label parallel lines and right angles.**	**To identify and use diagonal and parallel lines.**	
	Parallel lines  	A straight line inside a shape that goes from one corner to another (not an edge). When we join two vertices of a polygon we get a diagonal:  **Parallel** lines are always the same distance apart 	

Year 6— Geometry - Properties of shapes (When planning ensure you track back to year 5 for progression)

National Curriculum

Draw 2-D shapes using given dimensions and angles

Recognise, describe and build simple 3-D shapes, including making nets

Compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons

Illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius.

Recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles.

Notes and guidance (non-statutory)

Pupils draw shapes and nets accurately, using measuring tools and conventional markings and labels for lines and angles.

Pupils describe the properties of shapes and explain how unknown angles and lengths can be derived from known measurements.

These relationships might be expressed algebraically for example: d=2 x r, a = 180—(b + c).

Key vocabulary: line, sketch, base, square-based, regular, irregular, concave, convex, spherical, cylindrical, tetrahedron, polyhedron, equilateral triangle, isosceles triangle, line symmetry, reflect, translation, circumference, concentric, arc, intersecting, intersection, plane, tangram, dodecahedron, rhombus, kite, parallelogram.

Key concepts

An angle is the space (measured in degrees) between two intersecting lines.

A 3d shape can be opened up to reveal a 'net'.

Construction of simple 3-D shapes from nets draws on a wide range of geometric concepts and practical skills.

Geometrical designs from different cultural traditions sucha s Islamic patterns provide a rich experience of transformations and symmetry.

Learning objectives (see overleaf for exemplification)

To draw 2-D shapes given dimensions and angles.
To recognise, describe and build simple 3D shapes.
To make nets.
To visualise a 3-D shape from it's net.
To visualise where patterns drawn on a 3-D shape will occur on its net.
To compare and classify geometric shapes.
To illustrate and name parts of a circle.
To know how to find the diameter of a circle.
To identify angles and find missing angles.
To express relationships algebraically.

Potential barriers/misconceptions

Pupils are keen to get the answers 'right'. Problems in maths should be used to a) give pupils the opportunities to apply and therefore to reinforce the knowledge and skills they have learnt. B) to develop general problem solving strategies and c) sometimes to introduce a topic by providing motivation for learning new skills.

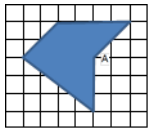
Example Questions

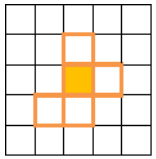
Imagine a triangular prism. How many faces does it have?

Imagine a cube. How many vertices does it have?

On the grid paper, use a ruler to draw a pentagon that has three right angles.

Here is the net of a cube with no top. The shaded square is the bottom of the cube. Draw an extra square to make the net of a cube which does have a top.





These two shaded triangles are each inside a regular hexagon. What is a name of a shaded shape?

Here are four statements. For each statement put a tick if it is possible. Put a cross if it is impossible.

A triangle can have 2 acute angles.

A triangle can have 2 obtuse angles.

A triangle can have 2 parallel sides.

A triangle can have 2 perpendicular sides.

Draw two straight lines from point A to divide the shaded shape into a square and two triangles.

Mental maths

Picturing shapes, moving, reflecting, rotating and growing.

Imagine a square: place an equilateral triangle on each side.
How many sides does the new shape have?

Imagine a triangle place a square on each side.
Imagine two dots 10m apart.
Imagine walking so that you are the same distance from each dot. Can you describe the path?
What if we had two crossing lines?
What if we had a vertical line and a dot? What would the path look like?
Imagine a line of length 3m on the floor. I wish to walk around so I am always 1m away - describe the path.

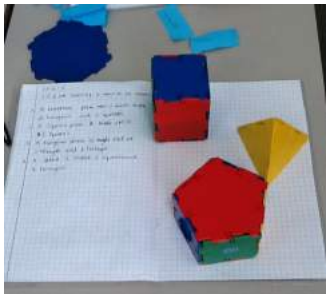
Imagine a cube. Place a blob of paint on each corner.
How many edges have one blob?
Put two blobs on the cube, on adjacent vertices.
How many edges have one blob? How many have two?
Put a blob on opposite corners Etc.

Imagine a tetrahedron. Put a blob on one vertex. How many edges have two blobs?
Same idea, try different shapes, more blobs.

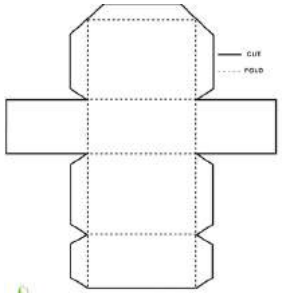
Year 6— Progression (a combination of these models and images can be used for every objective)

To draw 2-D shapes given dimensions and angles.

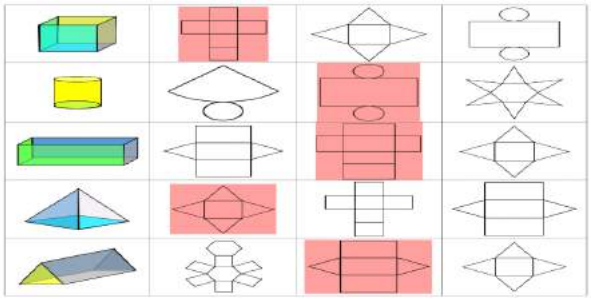
To recognise, describe and build simple 3D shapes.



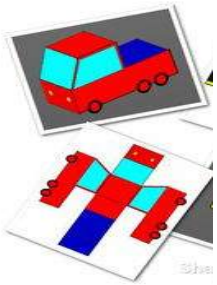
To make nets.

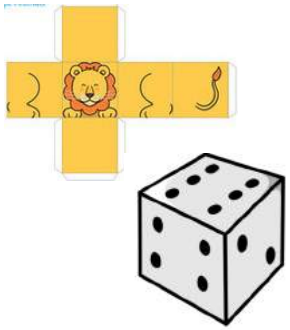


To visualise a 3-D shape from it's net.

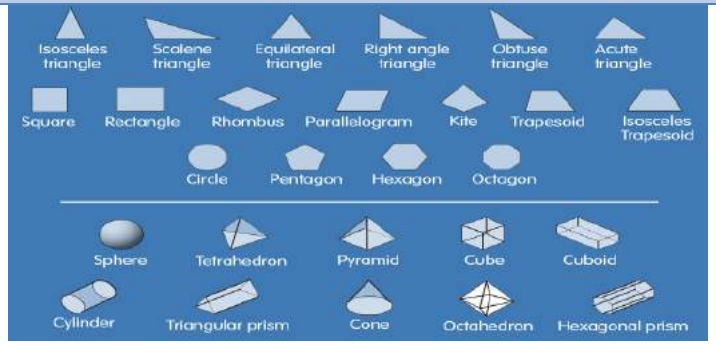


To visualise where patterns drawn on a 3-D shape will occur on its net.

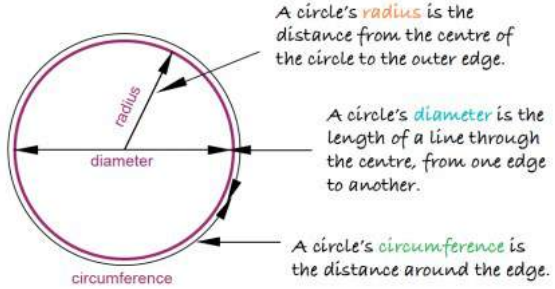




To compare and classify geometric shapes.

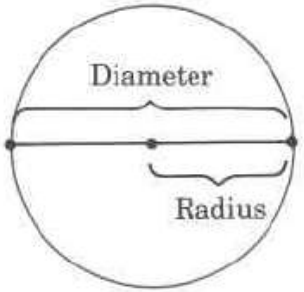


To illustrate and name parts of a circle.

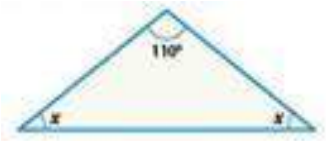


To know how to find the diameter of a circle.

The diameter if a circle is twice the radius
The radius is half the diameter.

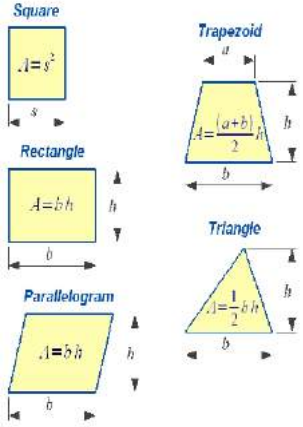


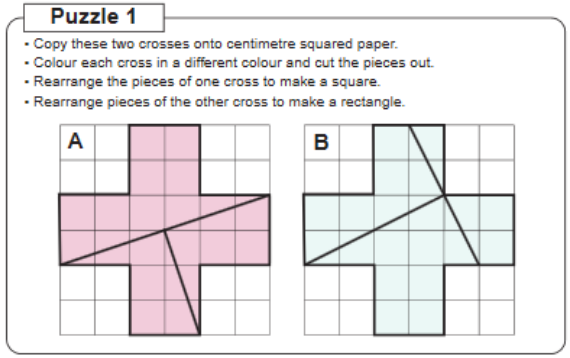
To identify angles and find missing angles.



This is an isosceles triangle. What is the size of angle x?

To express relationships algebraically.





Chapter 12

Geometry—Position and Direction

EYFS 1&2 - Geometry – Position and Direction (When planning ensure you track forwards to year 1)

Early Learning Goal 12

Pupils use everyday language to talk about size, weight, capacity, position, distance, time and money to compare quantities and objects and to solve problems.
They recognise, create and describe patterns.
They explore characteristics of everyday objects and shapes and use mathematical language to describe them.

KS1 ready

Describe position, direction and movement, including whole, half, quarter and three quarter turns.

Key vocabulary: position, over, under, above, below, on, in, outside, inside, behind , beside, before, after, next to, opposite, between, close, far, apart, middle, corner, top, bottom, front, back, side, direction, left, right, up, down, forwards, backwards, sideways, across, along, around, through, to, from towards, away from, movement, roll, slide, turn, stretch, bend.

Key concepts

Instructions can be given to a programmable toy. This would involve the use of everyday language such as in front of, next to, underneath, to describe the position of their toy.

Learning objectives (see overleaf for exemplification)

Can say what is different and what is the same.
Begins to categorise according to properties such as size.
Uses positional language ('below', 'above', 'next to', 'beside', 'in front', 'behind' and 'on top'
Describes their relative position such as 'behind' or 'next to'.
Uses everyday language to talk about position .
Uses everyday language to solve problems.
Explores characteristics of/uses mathematical language for everyday objects and shapes.

Potential barriers/misconceptions

Pupils don't have the language to confidently describe position and movement.
The computer programme used to determine movement may be a barrier if the coding/ instructions given are too complex.
Pupils are able to respond to instructions themselves but find it more challenging, for example, to move an object around a maze.

Example Questions

Use everyday words to describe the position:
Put your animal in the middle of the table. Now put the pig behind the sheep. Put the cow in front of the horse.
Stand behind the table. Now walk in a straight line to the front of the room.
Who is sitting next to, beside, in front of Ranjit?
Are the felt pens on top of, under or next to the books?
Go forwards three steps. Now go backwards three steps.
Slide the book across the table.
Roll the ball as far as you can.
Turn on the spot.

Here are pictures of a ball, a house and a boat. Put the ball above the horse. Put the boat to the felt of the ball.
Stand in front, behind, beside, opposite a partner. Stand between two other children.
Follow my instructions to get through this obstacle course. Go over the mat, through the tunnel, climb to the top of the bars….
Turn to your right and face the window.
Make half a turn on the spot.
Which of these shapes will roll in a straight line? Which will roll in a curved line?
Follow my instructions to get through the maze. Move forwards, turn left, go straight on, turn the corner…

Mental Maths

To listen and identify key vocabular y in books such as:
Rosies' walk
Bear Hunt
Spot

To talk about positions by:
Describing where things are in a picture
Follow instructions around an obstacle course in PE
Identify who they are sitting next to/ behind on the carpet

To talk about movements and direction:
Follow instructions to run forwards, backwards, turn to the left etc.
Give instructions around movement to other children.
Describe the walk to school
Explore and talk about things that turn (taps, handles etc)

Collect and sort objects that will: roll, slide, roll & slide.

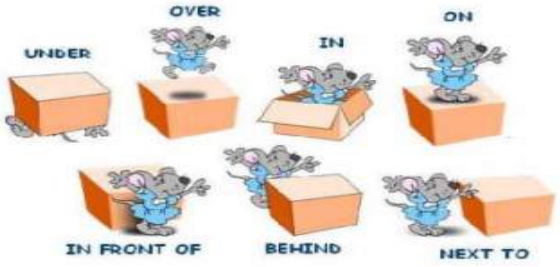
CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

EYFS 2 - Progression (a combination of these models and images can be used for every objective)

Begins to categorise according to properties such as size.

This squirrel comes next.
This squirrel goes between the yellow and the pink squirrel.
This squirrel goes at the end.

Uses positional language ('below', 'above', 'next to', 'beside', 'in front', 'behind' and 'on top'



Describes their relative position such as 'behind' or 'next to'.

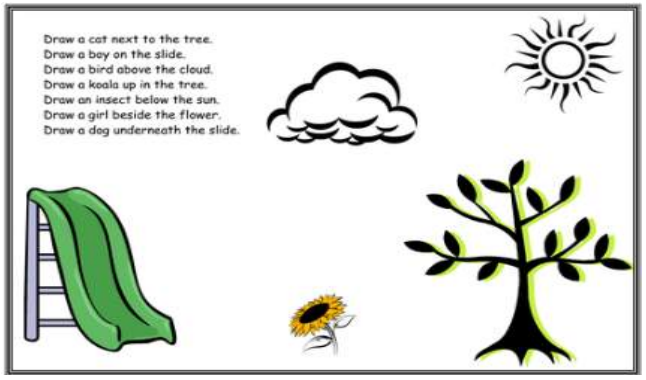
I am next to Cemal and Steven. I am behind Jessica. I am in front of Tyrone.

Uses everyday language to talk about position .

Through the house they ran
Up the stairs and into my room.
On top of my bed,
Under the covers...
I found my cat.

Anon

Uses everyday language to solve problems.



Explores characteristics of/uses mathematical language for everyday objects and shapes.

Can you describe how you will take the register back to the office?

What is next to your bed at home?

Year 1 - Geometry – Position and Direction (When planning ensure you track back to Reception and forwards to year 2)

National Curriculum	**Notes and guidance (non-statutory)**
Describe position, direction and movement, including whole, half, quarter and three quarter turns.	Pupils use the language of position, direction and motion, including: left and right, top, middle and bottom, on top of, in front of, above, between, around, near, close and far, up and down, forwards and backwards, inside and outside. Pupils make whole, half, quarter and three-quarter turns in both directions and connect turning clock-wise with movement on a clock face.

Key vocabulary: left and right, top, middle and bottom, on top of, in front of, above, between, around, near, close and far, up and down, forwards and backwards, inside and outside, half, quarter and three-quarter turns

Key concepts:

In the classroom and in PE lessons vocabulary can be reinforced around position and direction with instructions directing children to face left, right and about turn.

Pupils can give each other instructions to go round obstacles ending up facing in given directions.

Learning objectives (see overleaf for exemplification)

To describe position, direction and movement including back forward.
To identify left and right.
To use prepositional language.
To give directions
To make turns in both directions.
To link turns with the hands on a clock

Potential barriers/misconceptions

Pupils are not secure with their prepositional language.

Pupils may look at the teacher modelling clockwise/anticlockwise etc and be confused if modelled as mirror image.

Pupils unable to differentiate between their left and their right.

Pupils are unfamiliar with a clock face and unclear about the language of quarter and half.

Example Questions

Stand up and face the front wall of the classroom. Make a half turn. Which wall of the classroom are you facing now?

The big hand of the clock is pointing to 3. What number will it point to when it has made a half turn?

Sam turns the pencil one quarter turn. Tick the picture which shows how the pencil looks after the turn.

Put your finger on X. Move your finger up 1 square and then across 3 squares. Tick the fruit your finger s tops on.

Mental Maths

To use everyday language to describe positions:
- In PE stand in front of, behind, opposite a partner, or between two others.
- Describe how the furniture is arranged in a dolls house: Put a chair in front of the TV
- In the classroom name an object that is above the door, beside the sink
- describe where a smaller object is in a large area– near the edge/corner/middle etc
-describe the position of an object in relation to another. The cat is next to the tree.

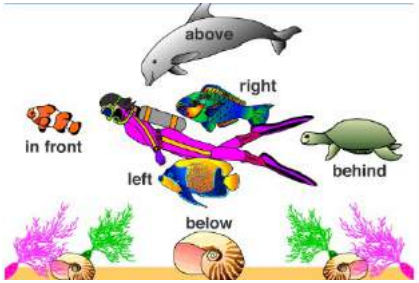
Use everyday language to describe directions:
- In PE follow and give instructions to move in particular directions: climb upwards, downwards, across…
- Talk about a journey– how to get from the school to the shop.
- To suggest instructions for how to programme robot.

To understand and use: slide, roll, turn, whole, half
To recognise and talk about movements.
- roll across the mat, slide across the floor
- Identify things that turn about a point– taps, wheels, clocks, scissors.
- Identify things that turn about a line– book, door, lid
- Make things turn– count around a clock face
- Discuss what comes next in a repeating pattern.

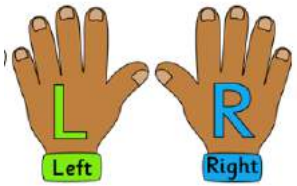
CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

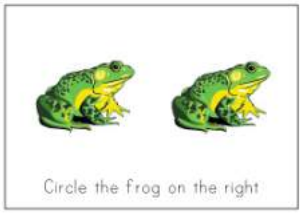
Year 1- Progression (a combination of these models and images can be used for every objective)

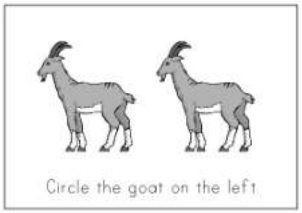
To describe position, direction and movement including backward, forward.



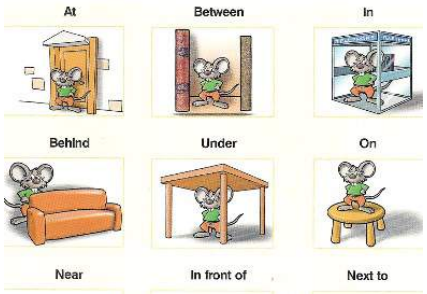
To identify left and right.

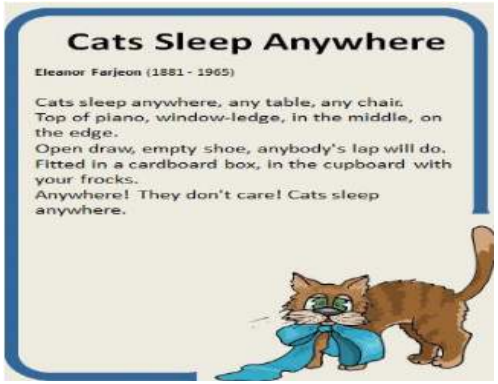




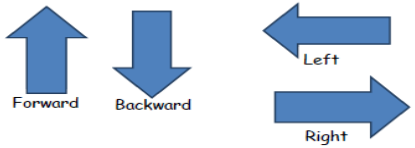


To use prepositional language.





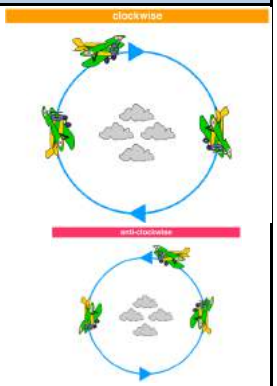
To give directions



To make turns in both directions.

Whole, half, quarter and three-quarter turns.

To link turns with the hands on a clock



Year 2 - Geometry – Position and Direction (When planning ensure you track back to year 1 and forwards to year 3)

National Curriculum

Order and arrange combinations of mathematical objects in patterns and sequences
Use mathematical vocabulary to describe position, direction and movement, including movement in a straight line and distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anticlockwise).

Notes and guidance (non-statutory)

Pupils should work with patterns of shapes, including those in different orientations. Pupils use the concept and language of angles to describe 'turn' by applying rotations, including in practical contexts (for example, pupils themselves moving in turns, giving instructions to other pupils to do so, and programming robots using instructions given in right angles).

Key vocabulary: left and right, top, middle and bottom, on top of, in front of, above, between, around, near, close and far, up and down, forwards and backwards, inside and outside, half, quarter and three-quarter turns , clockwise, anti-clockwise, route.

Key concepts:

In the classroom and in PE lessons vocabulary can be reinforced around position and direction with instructions directing children to face left, right and about turn.
Pupils can give each other instructions to go round obstacles ending up facing in given directions.

Potential barriers/misconceptions

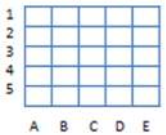
Pupils are not secure with language for position.
Pupils are still not confident with their left and right- Show how to remember 'left' and 'right'. Make L shape with forefinger and thumb of left hand – not possible with right hand.
Pupils are unfamiliar with a clock face and unclear about the language of quarter and half.

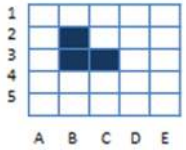
Learning objectives (see overleaf for exemplification)

To order and arrange objects in patterns and sequences.
To describe the position of objects.
To give directions.
To describe and control movement.
To describe movement in terms of right angles for turns.
To programme robots to turn.

Example Questions

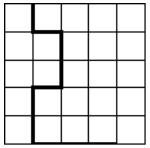
Draw a tick in square D2.





Look at the L shape on the grid. Part of it is in square B3. Write the other two squares it is in

STOP



Follow the route with your pencil. Complete this chart showing the route from START to STOP.
START
Left 3
Up 2….

START

Draw how this triangle will look after a half turn.

Mental Maths

To count along a counting stick as a scale in intervals of 1. (x-axis)
To count up a counting stick as a scale in intervals of 1 (y axis)
To count around a clock face in quarter turn, half turn, three quarter turn, full turn.

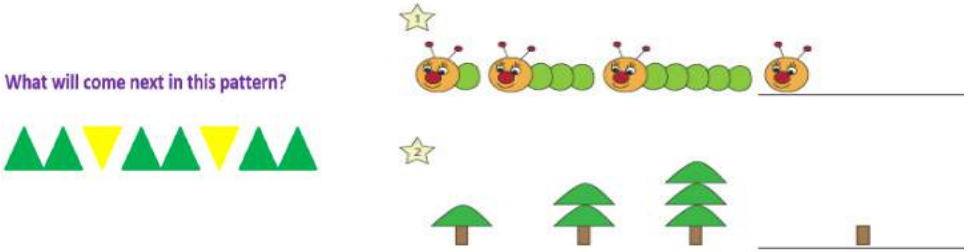
To describe positions:
Respond to oral instructions involving higher than, lower than, next to, below, further away from, on the edge of, at the corner of.
Describe the position of a feature on a map.

In PE– move clockwise, anticlockwise, face inwards, outwards.
Turn on the spot, turn through whole, half, quarter turns.
To give instructions for someone else to follow to find a route through a simple maze drawn on squared paper.

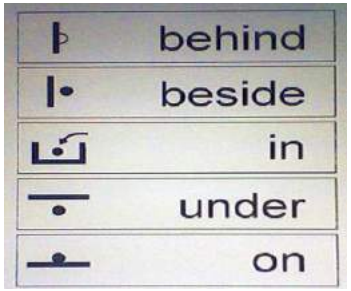
CONCRETE → PICTORIAL → ABSTRACT (2 + 1 = 3)

Year 2 - Progression (a combination of these models and images can be used for every objective)

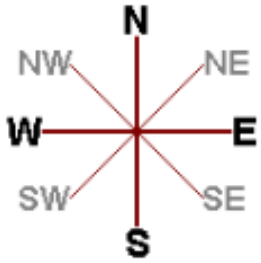
To order and arrange objects in patterns and sequences.



To describe the position of objects.

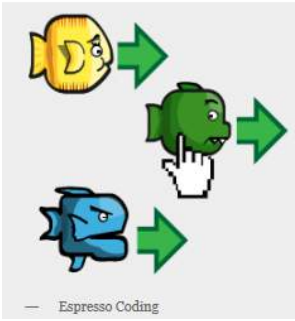


To give directions.

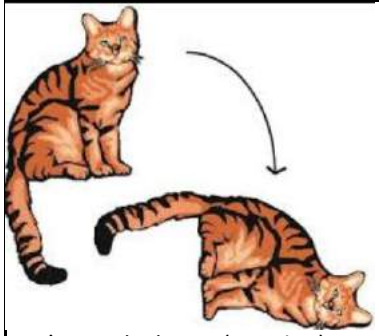


To describe and control movement.

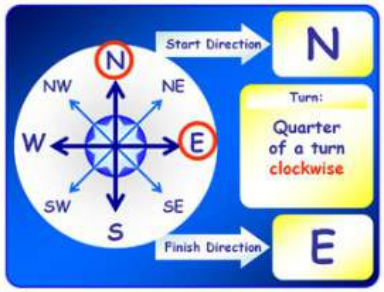
Using computer programmes to control movements of animated characters.



To describe movement in terms of right angles for turns.



90’ turn clockwise (rotation)



To programme robots to turn.

The Bee-Bot programming language consists of only five movement commands

- forward 150mm,
- backward 150mm,
- right turn 90 degrees,
- left turn 90 degrees,
- pause (II) for 1 second and make a tick sound.

Plus two device control commands.

- clear (X),
- GO - executes commands.

These are based on a small subset of the Logo programming language.

When a program has finished the Bee-Bot makes a sound and flashes its lights.

Year 3– Although not covered in the year 3 programme of study it is advised that this strand is used to get pupils year 4 'ready'.
Year 4 - Geometry – Position and Direction (When planning ensure you track back to year 2 and forwards to year 5)

National Curriculum

Describe positions on a 2-D grid as coordinates in the first quadrant
Describe movements between positions as translations of a given unit to the left/right and up/ down
Plot specified points and draw sides to complete a given polygon.

Notes and guidance (non-statutory)

Pupils draw a pair of axes in one quadrant, with equal scales and integer labels. They read, write and use pairs of coordinates, for example (2, 5), including using coordinate plotting ICT tools.

Key vocabulary:
Key concepts:

In the classroom and in PE lessons vocabulary can be reinforced around position and direction with instructions directing children to face left, right and about turn.
Pupils can give each other instructions to go round obstacles ending up facing in given directions.
Pupils can use treasure maps, battle ship games and back to back drawing to familiarise selves with the giving and receiving of instructions around positional language and directions.

Potential barriers/misconceptions

Putting y co-ordinate before the x, resulting in incorrectly placed position – due to:-
lack of knowledge of order or lack of knowledge concerning names of axes.

Pupils find it difficult to shift from co-ordinates that are presented inside the box (see below) to coordinate points on the line.

Pupils are not confident with the use of positional language.

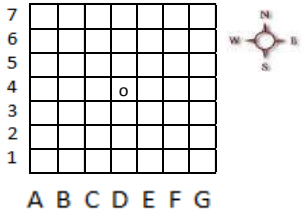
Pupils are unable to relate turns to right angles and everyday events.

If pupils do not physically engage with turns, angles etc they will not realise that after turning through half a turn or two quarter turns in the same direction they are then facing in the opposite direction.

Learning objectives (see overleaf for exemplification)

To recognise that two right angles make a half turn, three make three quarters and four complete.
To describe position on a 2-D grid as co-ordinates.(2,5)
Describe movements between positions as translations (left, right, up, down)
To plot specified points.
To draw a polygon.
To draw a pair of axes.
To use coordinate plotting ICT tools.

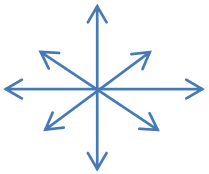
Example Questions



Lisa places a counter on a D4 square. She moves it 2 squares east and 3 squares south. Write the position of the square she moves it to.

N

The arrow labelled N is pointing north. Which arrow is pointing south-west?



Amy is facing North. She turn clockwise through 2 right angles. Which direction is she facing now?

Rashid is facing West. He turns clockwise through 3 right angles. What direction is he facing now?

Mental Maths

Practise pointing and chanting negative and positive numbers on a scale, using a 'counting stick' (forwards and backwards). Hold stick both horizontally and vertically to link to both the x and the y axes

To count along a counting stick as a scale in intervals of 1. (x-axis)
To count up a counting stick as a scale in intervals of 1 (y axis)
To count around a clock face in quarter turn, half turn, three quarter turn, full turn.
To count around a clock face in 90', 180', 270' and 360'

To have rapid recall of positions of the compass– north, south, east, west

To have rapid recall of positions of the compass, N, NE, E, SE, S, SW, W, NW

Refer to the 'symmetrical' quality of the numbers with 0 as the middle value.

Describe and find the position of a square on a grid of squares with the rows and columns labelled.

Play noughts and crosses telling partner where to place on grid.

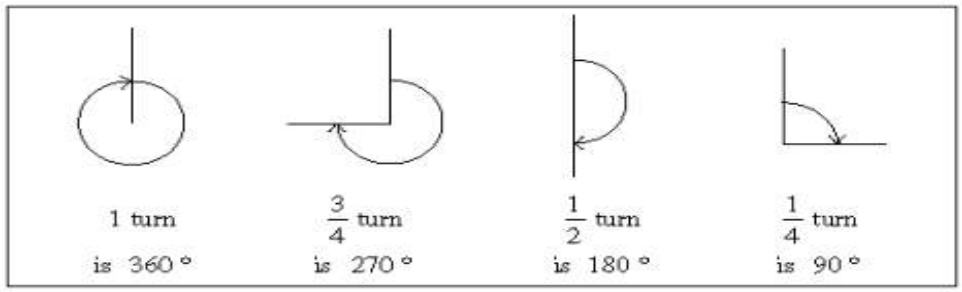
Tell a story including the words north, ascend, clockwise, left, horizontal.

To visualise and explain route from home to schools.

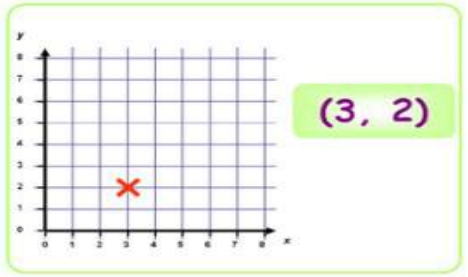
To recognise horizontal and vertical lines in the classroom environment.

Year 4- Progression (a combination of these models and images can be used for every objective)
(Year 3– Although not covered in the year 3 programme of study it is advised that this strand is used to get pupils year 4 'ready'.)

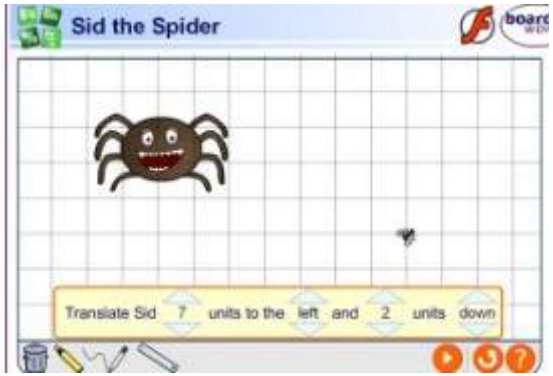
To recognise that two right angles make a half turn, three make three quarters and four complete.



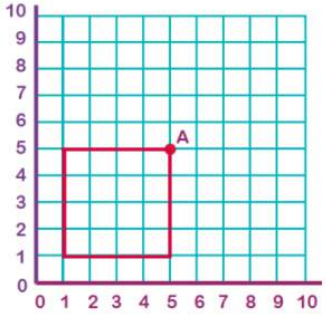
To describe position on a 2-D grid as co-ordinates.(2,5)



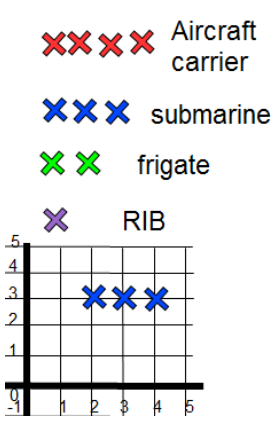
Describe movements between positions as translations (left, right, up, down)



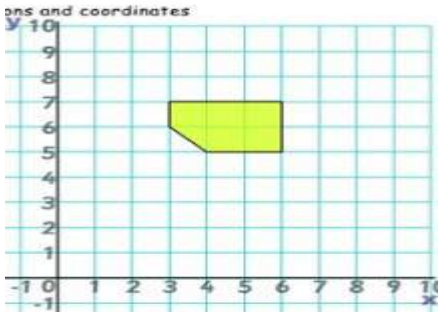
To plot specified points.



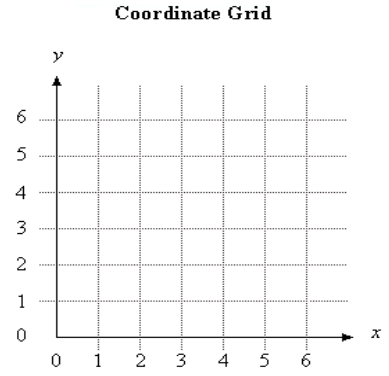
Battleships:

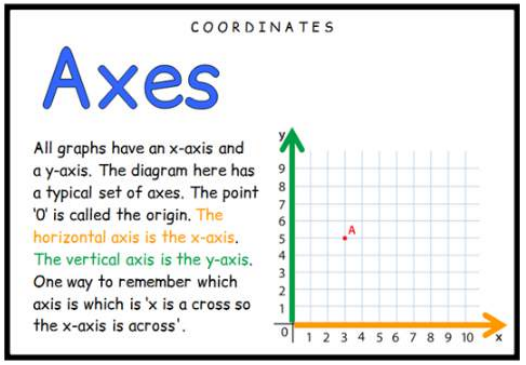


To draw polygons



To draw a pair of axes.





Year 5 - Geometry – Position and Direction (When planning ensure you track back to year 4 and forwards to year 6)

National Curriculum

Identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed.

Notes and guidance (non-statutory)

Pupils recognise and use reflection and translation in a variety of diagrams, including continuing to use a 2-D grid and coordinates in the first quadrant. Reflection should be in lines that are parallel to the axes.

Key vocabulary:
Key concepts:

When a shape has been reflected or translated it hasn't changed.
Using the colouring, cutting out, turning face down approach to explore ideas of reflection– as well as folding shapes along mirror lines and looking at shapes and their images in mirrors.
Tracing paper can be used to explore ideas of rotation and rotational symmetry.

Potential barriers/misconceptions

Pupils are unclear about the difference between reflection and translation:

Translation: a transformation in which a shape is slid from one position to another, without turning.
Reflection: A transformation in which a shape is reflected in a mirror line and changed into its mirror image.

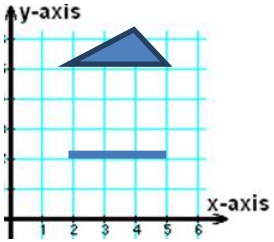
Putting y co-ordinate before the x, resulting in incorrectly placed position – due to:-
lack of knowledge of order or
lack of knowledge concerning names of axes.
When using 4 quadrants, misplaced positions due to lack of understanding of order of negative numbers on a scale.

Learning objectives (see overleaf for exemplification)

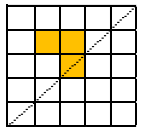
To reflect the position of a shape
To reflect the position of a shape in all four quadrants (extension)
To translate the position of a shape
To translate the position of a shape in all four quadrants (extension)
To use a 2-D grid and coordinates in the first quadrant
To use a 2-D grid and coordinates in all four quadrants. (extension).

Example Questions

Kyle has drawn triangle on this grid. Holy has started drawing identical square. What will be coordi-



On the grid join dots to make a triangle which does not have a right angle. Use a ruler.
Shade in two more squares to make this design symmetrical about the mirror line.



Mental Maths

Practise pointing and chanting negative and positive numbers on a scale, using a 'counting stick' (forwards and backwards). Hold stick both horizontally and vertically to link to both the x and the y axes
To count along a counting stick as a scale in intervals of 1. (x-axis)
To count up a counting stick as a scale in intervals of 1 (y axis)
To count around a clock face in quarter turn, half turn, three quarter turn, full turn.
To count around a clock face in 90', 180', 270' and 360'
To have rapid recall of positions of the compass– north, south, east, west
To have rapid recall of positions of the compass, N, NE, E, SE, S, SW, W, NW

Refer to the 'symmetrical' quality of the numbers with 0 as the middle value.
Sketch the position of a simple shape after it has been translated, for example 2 units to the left.
To describe to someone else the convention that (3,2) describes a point found by starting at the origin (0,0) and moving three lines across and two lines up.

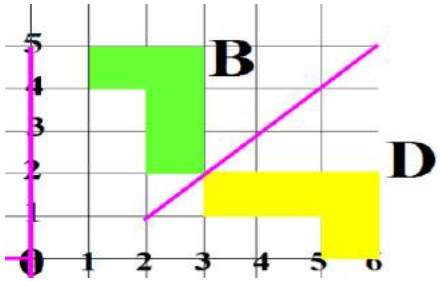
Respond to questions that involve visualisation:
-These points are the coordinates of the vertices of a shape: (1,5), (2,5), (4,3), (2,1), (1,1) What is the name of the shape?
- Three of the vertices of a square are (2,1), (2,4) and (5,4). What are the coordinates of the fourth vertex?

Know the number of diagonals in a polygon. i.e. Hexagon has 3 diagonal lines.

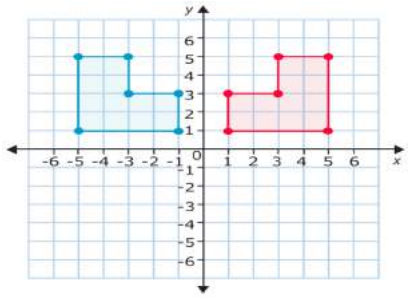
Year 5- Progression (a combination of these models and images can be used for every objective)

To reflect the position of a shape

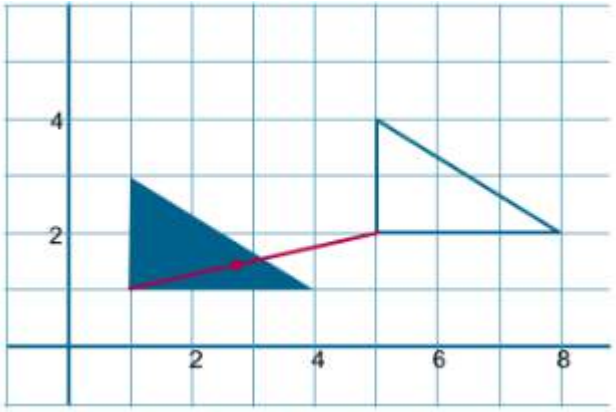
Reflected in a diagonal line:



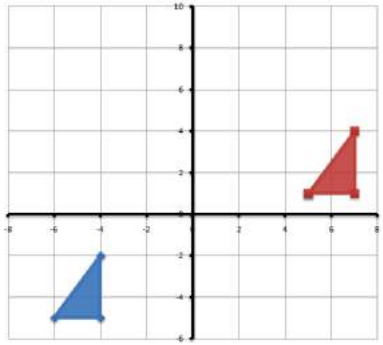
To reflect the position of a shape in all four quadrants (extension)



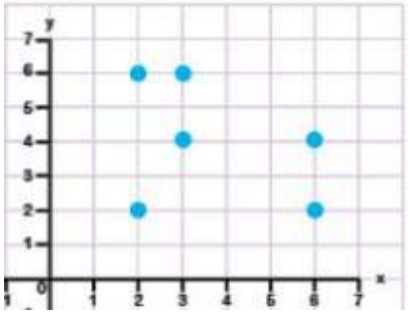
To translate the position of a shape



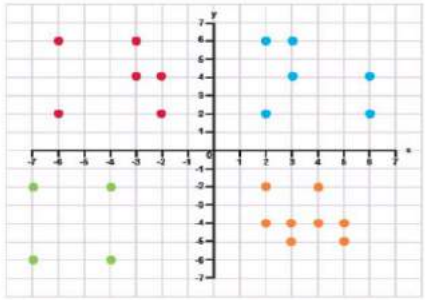
To translate the position of a shape in all four quadrants (extension)



To use a 2-D grid and coordinates in the first quadrant



To use a 2-D grid and coordinates in all four quadrants. (extension).



Year 6- Geometry – Position and Direction (When planning ensure you track back to year 5 for progression)

National Curriculum

Describe positions on the full coordinate grid (all four quadrants)
Draw and translate simple shapes on the coordinate plane, and reflect them in the axes.

Notes and guidance (non-statutory)

Pupils draw and label a pair of axes in all four quadrants with equal scaling. This extends their knowledge of one quadrant to all four quadrants, including the use of negative numbers.
Pupils draw and label rectangles (including squares), parallelograms and rhombuses, specified by coordinates in the four quadrants, predicting missing coordinates using the properties of shapes. These might be expressed algebraically for example, translating vertex (a, b) to (a – 2, b + 3); (a, b) and (a + d, b + d) being opposite vertices of a square of side d.

Key vocabulary:
Key concepts

There is no need to limit pupil's experience of coordinates to the first quadrant as the principles are the same in the other quadrants and these provide some useful experience of interpreting and applying negative numbers.
The use of coordinates to specify the location of points in a plane, rather than spaces, as in street maps, is a significant point to be explained to pupils carefully.

Potential barriers/misconceptions

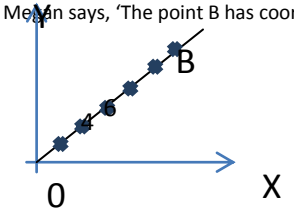
Pupils are only taught using co-ordinates in the first quadrant. They should be able to work in all 4 quadrants.
Pupils are unclear about the difference between reflection and translation:
Translation: a transformation in which a shape is slid from one position to another, without turning.
Reflection: A transformation in which a shape is reflected in a mirror line and changed into its mirror image.
Putting y co-ordinate before the x, resulting in incorrectly placed position – due to:-
lack of knowledge of order or lack of knowledge concerning names of axes.
When using 4 quadrants, misplaced positions due to lack of understanding of order of negative numbers on a scale.

Learning objectives (see overleaf for exemplification)

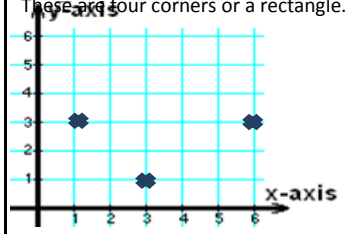
To describe positions on all four quadrants
To draw and translate simple shapes on the coordinate plane
To reflect simple shapes in the axes.
To draw and label all four quadrants with equal scaling.
To use the properties of shapes to predict missing coordinates
To express translations algebraically.

Example Questions

Here is a graph. The dots on the line are equally spaces. What are the coordinates of the point A?
Megan says, 'The point B has coordinates (11,5).' Use the graph to explain why she cannot be correct.



These are four corners or a rectangle. What are the coordinates of the fourth corner?



These are the coordinates of three points of a parallelogram: (20,27), (15,17) and (35,17). Write the coordinates of point A.

Mental Maths

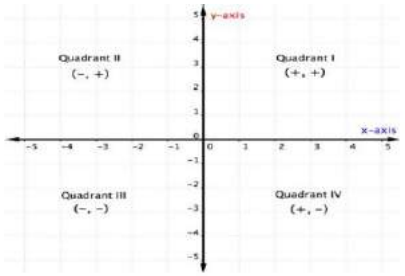
Practise pointing and chanting negative and positive numbers on a scale, using a 'counting stick' (forwards and backwards). Hold stick both horizontally and vertically to link to both the x and the y axes
To count along a counting stick as a scale in intervals of 1. (x-axis)
To count up a counting stick as a scale in intervals of 1 (y axis)
To count around a clock face in quarter turn, half turn, three quarter turn, full turn.
To count around a clock face in 90', 180', 270' and 360'
To have rapid recall of positions of the compass– north, south, east, west
To have rapid recall of positions of the compass, N, NE, E, SE, S, SW, W, NW
Refer to the 'symmetrical' quality of the numbers with 0 as the middle value.

To sketch the position of a simple shape after it has been translated, ie. 3 units to the right and 2 units down.
Respond to questions such as:
-The points (-1,1), (2,5) and (6,2) are three of the four vertices of a square. What are the coordinates of the fourth vertex?
- Draw a polygon with each vertex lying in the first quadrant. Plot its reflection in the y axis and name the coordinates of the reflected shape.
- Identify parallel and perpendicular lines in quadrilaterals.
-To know that two lines that cross eachother are intersecting lines and the point at which they cross is an intersection.

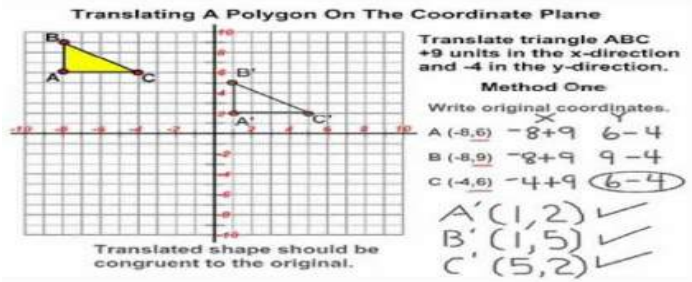
CONCRETE PICTORIAL ABSTRACT 2 + 1 = 3

Year 6 - Progression (a combination of these models and images can be used for every objective)

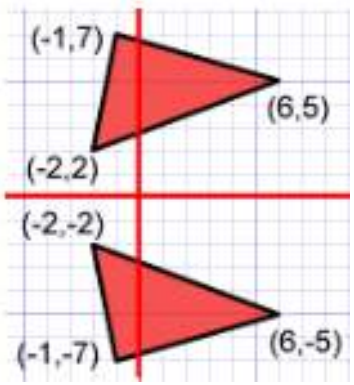
To describe positions on all four quadrants



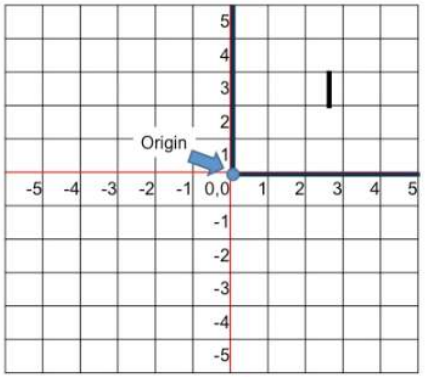
To draw and translate simple shapes on the coordinate plane



To reflect simple shapes in the axes.



To draw and label all four quadrants with equal scaling.



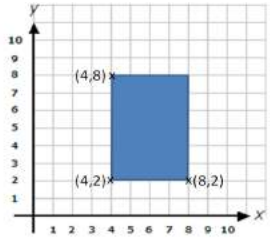
To use the properties of shapes to predict missing coordinates

A square has vertices at (0,0), (3,0) and (3,3). What is the co-ordinate of the fourth vertex?

A square has vertices at (3,0), (0,3) and (-3,0). What is the co-ordinate of the fourth vertex?

A square has vertices at (0,0), (2,0). Give two possible answers for the positions of the other two vertices.

A square has vertices at (-1, 1) and (-2, -3). Give two possible answers for the positions of the other two vertices.



To express translations algebraically.